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DESIGN PROCEDURES FOR SHOCK ISOLATION SYSTEMS OF UNDERGROUND PROTECTIVE STRUCTURES

Volume III

Response Spectra of Single-Degree-of-Freedom Elastic and Inelastic Systems

Final Report

June 1964

TECHNICAL DOCUMENTARY REPORT NO. RTD TDR-63-3096, Vol III

Research and Technology Division
AIR FORCE WEAPONS LABORATORY
Air Force Systems Command
Kirtland Air Force Base
New Mexico

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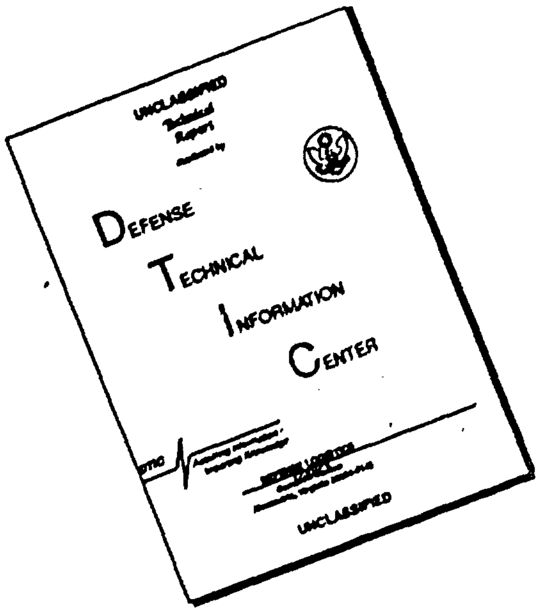

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Research and Technology Division
Air Force Systems Command
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New Mexico

FOREWORD

This report is one of five volumes presenting the results of a series of studies carried out for the Air Force by General American Transportation Corporation and Newmark-Hansen Associates. The five volumes comprise RTD TDR-63-3096 and are organized as follows:

Vol. I	Structure Interior Motions Due to Air Blast Induced Ground Shock
Vol. II	Structure Interior Motions Due to Directly Transmitted Ground Shock
Vol. III	Response Spectra of Single-Degree-of-Freedom Elastic and Inelastic Systems
Vol. IV	Response Spectra of Two-Degree-of-Freedom Elastic and Inelastic Systems
Vol. V	Response Spectra of Multi-Degree-of-Freedom Elastic Systems

Volumes I and II are authored by General American Transportation Corporation. Volumes III, IV, and V are authored by Newmark-Hansen and Associates. Volumes II, IV, and V will be published early in 1965.

Acknowledgment is made to Captain H. Auld, Captain D. H. Merkle, and Lt J. F. Flory of AFWL for their continued cooperation during the course of the project.

ABSTRACT

A discussion is presented of response spectra for single-degree-of-freedom systems subjected to different forms of ground excitation.

In the study of elastic systems, the sensitivity of the response to variations in the detailed characteristics of the input motion is discussed. For each class of forcing function, simple approximate rules are presented for the construction of response spectra for undamped systems. Simple rules are described for the construction of spectra for complex input functions by compounding the spectra for the "dominant" component pulses of the input function.

In the studies of inelastic systems, primary attention is given to elastoplastic systems and, in an exploratory way, to bilinear systems of the softening type. Response spectra are presented from which the yield resistance required to limit the maximum deformation of the system to a prescribed multiple of its limiting elastic deformation can be determined directly.

The maximum deformation of an inelastic system is related to that of an elastic system having the same initial slope in its resistance-deformation diagram and, for certain conditions, simple design rules are formulated for the construction of deformation spectra for elastoplastic systems in terms of the corresponding spectra for the associated elastic systems.

PUBLICATION REVIEW

This report has been reviewed and is approved.

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INTRODUCTION

1.1 Objectives of Program

The broad objectives of this program were to develop information regarding the response of equipment in underground installations when subjected to the effects of the ground motions induced by a nuclear detonation, to evaluate the influence and relative importance of the various factors affecting the response, and to present simplified design rules for specific conditions.

If the input motion for a system is prescribed as a function of time, it is generally recognized that the response of the system can be computed in a straightforward manner by integration of the governing differential equations of motion, no matter how complicated the motion or the system may be. However, such computations are generally time-consuming and are not very appropriate for purposes of preliminary design. The principal aim of this study was to establish a body of basic information and simplified rules which would enable the designer to arrive at a reasonable estimate of the significant effects of a prescribed motion, and to assess the engineering significance of the various parameters influencing the response of the system without the need for elaborate computations. Inasmuch as the detailed characteristics of the input motion are affected by a large number of uncontrollable factors, a special effort has been made to investigate the sensitivity of the response to the uncertainties involved in defining the input data.

The study is based on the concept of the response spectrum, and covers both elastic and inelastic systems with or without damping. In this report, only systems having a single degree of freedom are considered. However, the information presented can also be used in conjunction with the modal method of analysis to evaluate the response of multi-degree-of-freedom elastic systems for which the natural modes of vibration can be uncoupled. The study of inelastic systems is devoted mainly to elastoplastic systems and, in an exploratory way, to systems with a bilinear resistance of the softening type.

There are two important considerations in the design of the foundation for a piece of equipment. First, the foundation itself must have sufficient strength to withstand the forces that are developed in it without failure, and secondly, the accelerations or motions that are transmitted to the equipment and its parts must not be so severe as to cause damage to it, or to interfere with its operation. It follows then that response spectra are needed both for the maximum deformation of the system and for the absolute displacement, velocity and acceleration of its mass. The latter information may also be used to define the peak values of the motion experienced by the base of a light system mounted on a structure that may itself respond under the action of the shock.

If the piece of equipment is attached to a part of the structure that experiences essentially the same motion as the base of the structure in which it is housed, then the equipment may be designed for the shock spectrum applicable to the input motion. However, if it is mounted on a flexible element such as a beam or floor, which may itself respond under the influence of the shock, then both the intensity and the time-history of the motion at the base of the equipment may be significantly different from the original input motion, and the maximum response of the system can no longer be determined from the shock spectrum corresponding to the base input. It is obvious that the extent to which the input motion is modified is a function both of the characteristics of the structure and of the equipment and of its mode of attachment to the structure. This problem of the interaction between the motions of the equipment and of the supporting structure has been given some attention under this program, but the results of this effort will be reported separately. Throughout this report, the characteristics of the motion at the base of the system under investigation are assumed to be known.

1.2 Outline of Studies

The studies described here can be classified into two groups. The first group is concerned with the response of elastic systems, with or without damping, having a single degree of freedom. The input motions considered included several pulse-type excitations, approximating the primary or main component of the ground motion associated with a nuclear explosion, and two strong-motion earthquake records representing examples of extremely complex ground motions. The pulse-type of excitations include acceleration functions composed of from one-half to four cycles of oscillation, with corresponding displacement functions having from one-quarter of a cycle to one complete cycle of oscillation. The response quantities studied include the spring deformation, the relative velocity between the mass and the ground, and the absolute displacement, absolute velocity, and absolute acceleration of the mass.

The objectives of these studies were:

(a) To assess the sensitivity of the various response quantities to variations in such parameters as the shape, rise time, and periodicity of the input function.

(b) To develop simplified design rules for the construction of response spectra for the various response quantities to a greater degree of accuracy than has been possible previously.

(c) To formulate procedures for the construction of response spectra for fairly involved input functions by synthesizing the spectra for a series of simple component inputs.

(d) To study the effect of viscous damping.

The approach used was briefly as follows. First, the response of undamped systems to pulse-like excitations and to a combination of simple pulses was investigated for a wide range of the parameters involved, and, on the basis of the information obtained, simple approximate rules were formulated for the construction of response spectra for the various response quantities. Next, the effect of viscous damping was studied for systems with coefficients of damping up to 100 percent critical subjected to pulse-like excitations. Finally, to check the applicability of the approximate rules developed to inputs of extreme complexity, the response of undamped and damped systems subjected to earthquake motions was studied. The earthquake motions were used in preference to ground shock records because they are of greater complexity than those associated with a nuclear explosion, and consequently provide a severe test on the adequacy of the approximate rules.

The response spectra for the earthquake motions were evaluated for a much wider range of natural frequencies than has been customary in previous studies of earthquake effects, so that these spectra could be correlated with those corresponding to the simple pulses. It is shown that, even for ground motions of the complexity of strong motion earthquake records, the response spectra are similar to those for the simple pulses, and that their salient features can be estimated with reasonable accuracy from the spectra for the simple pulses, provided the gross characteristics of the acceleration, velocity and displacement diagrams of the ground are known.

In the analysis of multi-degree-of-freedom elastic systems by the use of response spectra, an upper bound to the maximum response can be obtained by taking the sum of the absolute values of the maximum response in the various natural modes. This approach overestimates the response. Under some conditions, a better estimate can be made by taking the square root of the sum of the squares of the modal responses, but the sign of the error cannot be determined with this approach. A much lower upper bound may be obtained if the maximum positive and the maximum negative values of the response of single-degree-of-freedom systems are known separately, both for the forced-vibration and the free-vibration eras of the motion. A representative number of such spectra is included in this report. In addition to being useful in the analysis of systems with more than one degree of freedom, these generalized spectra provide a great deal of insight into the behavior of the system, and enable one to synthesize the response spectrum corresponding to a sequence or combination of simple pulses from the spectra applicable to the individual pulses of the input motion.

The second group of studies performed was concerned mainly with the response of single-degree-of-freedom elastoplastic systems having equal yield levels in the two directions of deformation. Some consideration was also given to systems with a bilinear resistance of the softening type. For these systems, only the maximum deformation of the spring was investigated. The parameters studied include the characteristics of the input motion, and the natural frequency, yield point deformation, and damping of the systems. The input motions considered include several pulse-like excitations and the two strong-motion earthquake records used in the study of elastic systems. Furthermore, as an aid in the interpretation of the results, the effects of an instantaneous displacement change, an instantaneous velocity change, and an instantaneous acceleration change were studied in detail.

The results are summarized in the form of response spectra from which the yield resistance required to limit the maximum deformation of the system to a prescribed multiple of its limiting elastic deformation can be determined directly. On the basis of the information presented, simple design rules are formulated under certain conditions for the construction of deformation spectra for elastoplastic systems in terms of the corresponding spectra for elastic systems having the same initial slope in their resistance-deformation diagrams.

All response quantities are presented in dimensionless form, in terms of the maximum value of the appropriate input motion, so that revised input data can be treated readily as they become available.

Section 2 deals with the response of elastic systems, and Section 3 with the response of inelastic systems. The numerical data used to construct the response spectra presented in this report are tabulated in Appendix A. Included in this Appendix is also a brief account of the method of solution used. Finally, in Appendix B is given a summary of expressions for the computation of various response quantities of single-degree-of-freedom elastic systems with damping.

1.3 Notation

The symbols used are defined where they are first introduced, and the most important ones are summarized here.

A = pseudo-acceleration, defined as p^2U for an elastic system and as p^2u_y for an inelastic system

A_o	= maximum value of A
A_r	= maximum value of A corresponding to motion during free vibration
c	= Q_y/Q_o = reduction factor; see also Eq. 3.1. Its reciprocal is defined as the overload factor
f	= undamped natural frequency of system, in cps; for an inelastic system, it represents the frequency corresponding to the initial elastic range of behavior
g	= acceleration of gravity
k	= spring constant
m	= mass
p	= $\sqrt{k/m}$ = undamped circular natural frequency
p_d	= $p\sqrt{1 - \beta^2}$ = damped circular natural frequency
Q	= spring force
Q_m	= absolute maximum value of Q for an inelastic system
Q_o	= absolute maximum value of Q for an elastic system
Q_y	= yield value of Q
T	= undamped natural period of system
T_d	= natural period of damped system
t	= time
t_d	= total duration of a pulse
$t_{d,a}$, $t_{d,v}$	= durations of an acceleration pulse and a velocity pulse, respectively; used only when confusion may arise
t_r	= rise time to maximum value of a half-cycle pulse

$t_{t,a}$, $t_{r,v}$, $t_{r,d}$ = rise times for an acceleration, velocity and displacement half-cycle pulse, respectively; used only when confusion may arise

$\bar{t}_r$ = effective rise time, defined in paragraph preceding Eq. 2.31

$\bar{t}_{r,a}$, $\bar{t}_{r,v}$ = values of $\bar{t}_r$ for an acceleration and velocity pulse, respectively

t_o = time of occurrence of the absolute maximum value of a response quantity

$t_{o,a}$, $t_{o,v}$, $t_{o,d}$ = effective duration of an acceleration, velocity and displacement half-cycle pulse, respectively

t_1 = duration of dominant half-cycle pulse in an input function

$t_{1,a}$, $t_{1,v}$, $t_{1,d}$ = values of t_1 for an acceleration, velocity and displacement function, respectively

U = $|u_o|$ = absolute maximum value of u without regards to sign

U_o = maximum value of U

u = x-y = relative displacement between mass and ground = spring deformation

u_m = absolute maximum deformation of an inelastic system without regards to sign

u_{max} = maximum positive value of deformation

u_{min} = maximum negative value of deformation

u_o = for an elastic system, the numerically greater of the values of u_{max} and u_{min}; in Section 3 it is used in lieu of U to denote the absolute maximum value of u without regards to sign

u_y	=	yield point deformation
V	=	relative pseudo-velocity, defined as pU for elastic systems and as pu_y for inelastic systems
V_o	=	maximum value of V
x	=	absolute displacement of mass
x_o	=	peak extremum value of x, with its appropriate sign; in Section 3 it refers to its absolute value
X	=	$\lvert x_o \rvert$ = absolute maximum value of x without regards to sign
y	=	displacement of ground
y_{av}	=	average value of y for a half-cycle pulse
y_f	=	residual or final value of y
y_o	=	absolute maximum value of y without regards to sign
$y_{o,p}$	=	maximum value of y for the primary component of an earthquake motion
β	=	c/c_{cr} = fraction of critical coefficient of damping
μ	=	u_m/u_y = ductility factor

1.4 Acknowledgment

The studies reported herein were made with the assistance of Dr. C. V. Chelapati and Messrs. W. H. Walker and J. A. Nieto. A portion of the numerical data presented were obtained by Dr. Chelapati, as a supplement to his doctoral disseration prepared under Dr. Veletsos' direction at the University of Illinois.

This contract was technically monitored by Capt. Harry Auld and Capt. Douglas M. Merkle, in succession; their guidance and suggestions in the course of this study are acknowledged with appreciation.

SECTION 2

RESPONSE OF SINGLE-DEGREE-OF-FREEDOM ELASTIC SYSTEMS

2.1 System Considered

The system considered consists of a rigid mass, m, connected to a base by a weightless elastic spring and a dashpot exerting a resisting force which is proportional to the relative velocity between the mass and the base, as shown in Fig. 2.1. The spring constant is denoted by k and the coefficient of viscous damping by c. It is assumed that the mass can move only in the direction of the spring so that the system has a single degree of freedom. The base of the system will also be referred to as the ground.

The absolute displacement of the mass is denoted by x, the absolute displacement of the ground by y, and the relative displacement between the mass and the ground, the spring deformation, is denoted by u, i.e.,

$$u = x - y \tag{2.1}$$

Both x and y refer to the same inertial frame of reference, and their positive directions coincide. The quantity u is taken as positive when it produces tension in the spring. For a fixed-base system acted upon by an exciting force, the spring deformation, which is also equal to the absolute displacement of the mass, is designated by x. A dot superscript denotes differentiation with respect to time. For example, $\dot{u}$ denotes the relative velocity between the mass and the ground, and $\ddot{x}$ denotes the absolute acceleration of the mass.

2.2 Response Quantities of Interest

Throughout this report, the term response is used in a generalized sense to include any response quantity, such as a force, stress, displacement or velocity. In a similar manner, the term disturbance may refer to a loading, such as a force or pressure, or to a ground motion, which may be described as a time function of acceleration, velocity or displacement. Whatever its form, the forcing function is presumed to be known and independent of the motion of the system itself.

When the source of excitation is a force, the response quantities of interest are the displacement, velocity and acceleration of the mass. For a ground excitation, both the absolute and the relative values of these quantities may be needed. Of greatest importance is the relative displacement between the mass and the ground, which is proportional to the force or stress in the responding structure. The relative velocity, which is proportional to the rate of straining of the material of the spring, is also of interest, as it may be used to estimate the possible increase in the yield level of the material under dynamic conditions and to determine the magnitude of the maximum force due to viscous damping. The relative acceleration between the mass and the ground, although it does not appear to have any special practical significance, is of interest, because it may be used in conjunction with certain analogies to obtain response quantities for modified forms of ground excitation. This matter is discussed further in Section 2.4. The absolute displacement, velocity and acceleration of the mass are needed because the design of the system may be governed by limitations on the motion of its mass rather than by strength considerations, and because these quantities may be used to define the characteristics of the input motion for a secondary light mass that may be a part of the main mass or may actually be attached to it.

2.3 Equations of Motion

For use in subsequent developments, it is desirable to record here the governing differential equation of motion. For a system subjected to a ground excitation, this equation is

$$\ddot{x} + 2\beta p\ \dot{u} + p^2 u = 0 \tag{2.2}$$

where $\beta(=c/c_{cr})$ denotes the fraction of critical coefficient of damping, and $p(=\sqrt{k/m})$ denotes the undamped circular natural frequency of the system. The natural frequency of the system in cycles per second is denoted by f, and is given by the expression

$$f = \frac{p}{2\pi}$$

Equation 2.2 can be written in one of the following alternate forms:

$$\ddot{x} + 2\beta p\ \dot{x} + p^2 x = p^2 y(t) + 2\beta p\ \dot{y}(t) \tag{2.3}$$

or

$$\ddot{u} + 2\beta p\ \dot{u} + p^2 u = -\ddot{y}(t) \tag{2.4}$$

The latter form is the more convenient of the two when the ground motion is specified as an acceleration function. Obviously, the solution of these equations depends on the characteristics of the disturbing function, the degree of damping in the system, as represented by the parameter β, and the natural frequency of the system. Actually, the latter parameter enters in the solution as a dimensionless product of f and a characteristic time of the disturbing function. The expressions for the various response quantities are given in Appendix B in terms of Duhamel's integral.

For a fixed-base system acted upon by an external force $P(t)$ applied at the mass, the governing differential equation is

$$\ddot{x} + 2\beta p\ \dot{x} + p^2 x = p^2 x_{st}(t) \tag{2.5}$$

where

$$x_{st}(t) = \frac{P(t)}{k} \tag{2.6}$$

denotes the deflection that would be produced by the force $P(t)$ were to be applied gradually to the system. This quantity will be referred to as the static deflection of the system, and its maximum value will be designated as $(x_{st})_o$.

Equation 2.5 is analogous to Eq. 2.4, and its solution may be obtained from that of Eq. 2.4 simply by replacing u by x and the quantity $\ddot{y}(t)$ by minus $p^2 x_{st}(t)$. It follows that, if the external force P(t) has the same shape as the acceleration function for the ground input problem and if, <u>in addition</u>, the initial conditions on x and $\dot{x}$ for the force input are the same as those on u and $\dot{u}$, for the acceleration input, then the amplification factors $x(t)/(x_{st})_o$ and $[-p^2 u(t)]/\ddot{y}_o$ for the two cases will be identical. Similar analogies also exist between the derivatives of these quantities. In particular,

$$\left[\frac{\dot{x}(t)}{p(x_{st})_o} \text{ due to } P(t)\right] = \left[-\frac{p\dot{u}(t)}{\ddot{y}_o} \text{ due to } y(t)\right] \tag{2.7}$$

and

$$\left[\frac{\ddot{x}(t)}{p^2(x_{st})_o} \text{ due to } P(t)\right] = \left[-\frac{\ddot{u}(t)}{\ddot{y}_o} \text{ due to } y(t)\right] \tag{2.8}$$

Thus, if the response histories for one set of quantities, say x, $\dot{x}$ and $\ddot{x}$, are available, the histories for the corresponding set, u, $\dot{u}$ and $\ddot{u}$, can be obtained directly. Obviously, these analogies are also applicable to the maximum values of the response quantities.

For a system that is initially at rest, the initial conditions for the two problems considered above will be the same if

$$y(0) = \dot{y}(0) = 0$$

2.4 <u>Analogies Between Response Quantities Corresponding to Different Forms of Ground Excitation</u>

Let $y_1(t)$, $y_2(t)$ and $y_3(t)$ be three different motions, such that the displacement history of the first, the velocity history of the second, and the acceleration history of the third have the same shape. That is,

$$\frac{y_1(t)}{(y_1)_o} = \frac{\dot{y}_2(t)}{(\dot{y}_2)_o} = \frac{\ddot{y}_3(t)}{(\ddot{y}_3)_o} \tag{2.9}$$

where the subscript o denotes the maximum value of the function to which it is attached. Also, let $x_j(t)$ be the absolute displacement of the mass of the system subjected to $y_j(t)$.

The equations of motion for x_1, x_2, and x_3 can be expressed in terms of y_1, $\dot{y}_2$ and $\ddot{y}_3$ as follows:

$$\frac{d^2x_1}{dt^2} + 2\beta p \frac{dx_1}{dt} + p^2x_1 = p^2y_1(t) + 2\beta p\, \dot{y}_1(t) \tag{2.10a}$$

$$\frac{d^2\dot{x}_2}{dt^2} + 2\beta p \frac{d\dot{x}_2}{dt} + p^2\dot{x}_2 = p^2\dot{y}_2(t) + 2\beta p\, \ddot{y}_2(t) \tag{2.10b}$$

$$\frac{d^2\ddot{x}_3}{dt^2} + 2\beta p \frac{d\ddot{x}_3}{dt} + p^2\ddot{x}_3 = p^2\ddot{y}_3(t) + 2\beta p\, \dddot{y}_3(t) \tag{2.10c}$$

the last two equations being obtained formally from Eq. 2.3 by differentiation. Now, if the initial conditions for these equations are the same, the three solutions will be identical, and it may be concluded that:

$$\left[\frac{x_1(t)}{(y_1)_o} \text{ due to } y_1(t)\right] = \left[\frac{\dot{x}_2(t)}{(\dot{y}_2)_o} \text{ due to } \dot{y}_2(t)\right] = \left[\frac{\ddot{x}_3(t)}{(\ddot{y}_3)_o} \text{ due to } \ddot{y}_3(t)\right] \tag{2.11}$$

By subtracting from the three parts of this equation, the corresponding parts of Eq. 2.9, and recalling that $u_j = x_j - y_j$, one concludes further that

$$\left[\frac{u_1(t)}{(y_1)_o} \text{ due to } y_1(t)\right] = \left[\frac{\dot{u}_2(t)}{(\dot{y}_2)_o} \text{ due to } \dot{y}_2(t)\right] = \left[\frac{\ddot{u}_3(t)}{(\ddot{y}_3)_o} \text{ due to } \ddot{y}_3(t)\right] \tag{2.12}$$

Similar analogies also exist between the higher derivatives of these quantities.

The initial conditions of Eqs. 2.10 are specified in terms of

$x_1(0)$ and $\dot{x}_1(0)$ for Eq. 2.10a

$\dot{x}_2(0)$ and $\ddot{x}_2(0)$ for Eq. 2.10b, and

$\ddot{x}_3(0)$ and $\dddot{x}_3(0)$ for Eq. 2.10c.

The last three quantities can be related to the initial values of the input motion by application of Eq. 2.2 as follows:

$$\ddot{y}_2(0) = -p^2u_2(0) - 2\beta p\,\dot{u}_2(0) \tag{2.13}$$

$$\ddot{y}_3(0) = -p^2u_3(0) - 2\beta p\,\dot{u}_3(0) \tag{2.14}$$

and

$$\dddot{y}_3(0) = 2\beta p^3u_3(0) - (1-4\beta^2)p^2\dot{u}_3(0) + 2\beta p\,\ddot{y}_3(0) \tag{2.15}$$

As an illustration, consider the special case of a system that is initially at rest. If the initial values of y_1, $\dot{y}_2$ and $\ddot{y}_3$ are zero, the initial conditions for each of Eqs. 2.10 are likewise zero, and the analogies of Eqs. 2.11 and 2.12 are valid. On the other hand, if the initial values of y_1, $\dot{y}_2$ and $\ddot{y}_3$ are different from zero, it can readily be verified that only the initial conditions for Eqs. 2.10b and 2.10c are identical, with the result that in Eqs. 2.11 and 2.12 only the analogies represented by the equality of the second and third terms are valid.

For the special case of a system without damping, it follows from Eq. 2.2 that

$$\ddot{x}(t) = -p^2u(t) \tag{2.16}$$

and the analogies described in Eqs. 2.11 and 2.12 can therefore be extended accordingly.

Of special interest is the following set of analogies applicable to the free vibration era of the motion for systems without damping. If the terminal value of $\dot{y}_2(t)$ in Eq. 2.9 is zero, the maximum values of the velocity and the displacement during free vibration are related by the equations

$$\dot{x}_2(t) = px_2(t) = pu_2(t)$$

since the motion is of the simple harmonic type. It follows then that the maximum values of

$$\left[\frac{x_1(t)}{(y_1)_o} \text{ due to } y_1(t)\right] = \left[\frac{pu_2(t)}{(\dot{y}_2)_o} \text{ due to } \dot{y}_2(t)\right] = \left[-\frac{p^2u_3(t)}{(\ddot{y}_3)_o} \text{ due to } \ddot{y}_3(t)\right] \quad (2.17)$$

2.5 Response Spectra and Spectral Quantities

For design purposes, it is generally necessary to know both the maximum positive and the maximum negative values of the response. In certain applications, it may also be desirable to know the magnitudes of these quantities separately both for the interval that the forcing function acts on the system and for the time following the end of the disturbance. If the direction of the excitation cannot be predicted, or if the characteristics of the exciting function are such that the maximum positive and the maximum negative values of the response are equally likely, then it may suffice to know only the absolute maximum value of the response.

The subscript "max" will be used to designate the absolute maximum positive value of a response quantity, and the subscript "min" will refer to the corresponding maximum negative value. Thus

u_{max} = the absolute maximum positive deformation

x_{min} = the absolute maximum negative value of the absolute displacement

The numerically greater of the maximum positive and the maximum negative response quantities will be identified with the subscript o, and the absolute maximum value of the quantity, without regards to sign, will be denoted by the capital letter of the symbol used to designate that quantity. Thus

$$U = |u_o| \quad (2.18a)$$

$$\dot{U} = |\dot{u}_o| \quad (2.18b)$$

$$\ddot{X} = |\ddot{x}_o| \quad (2.18c)$$

A plot of the maximum value of a response quantity as a function of the natural frequency of the system, or a quantity which is related to the frequency, constitutes the response spectrum or shock spectrum for that quantity. It is assumed that the system has a single degree of freedom, and that the excitation is known and independent of the motion of the system itself. For example, the diagram expressing the variation of $\dot{x}_{max}$ with frequency represents the response spectrum for the absolute maximum positive value of the absolute velocity of the mass of the system.

It is convenient to express the various response quantities in dimensionless form by normalizing them with respect to the maximum value of the corresponding input quantity. For a ground motion, displacements may conveniently be expressed in terms of the maximum ground displacement, velocities in terms of the maximum ground velocity, etc. The ratio of the instantaneous value of a response quantity to the corresponding maximum input value will be referred to as the amplification factor for that quantity. The normalized spectral quantities are the peak values of the amplification factors.

The term deformation spectrum will be used to designate the response spectrum for the absolute maximum spring deformation, U, or a quantity used as a measure of U.

In many instances, the maximum spring deformation may be expressed more conveniently by the quantity V, defined as

$$V = pU \tag{2.19}$$

where p is the undamped circular natural frequency of the system. The quantity V has units of velocity, and is related to the maximum strain energy of the system, E_{max}, by the equation

$$E_{max} = \frac{1}{2} mV^2 \tag{2.20}$$

which follows from the fact that

$$E_{max} = \frac{1}{2} kU^2 = \frac{1}{2} mp^2U^2$$

Under certain conditions to be discussed subsequently, the quantity V is identical to, or approximately equal to, the maximum relative velocity, $\dot{U}$, and these quantities have, at times, been used interchangeably. However, they are generally different from one another, and care should be exercised in replacing one for the other. To avoid possible confusion, the quantity V will be referred to as the relative pseudo-velocity, or simply pseudo-velocity, and the term relative velocity will be reserved for the true relative velocity of the system.

Another convenient measure of the maximum spring deformation is the pseudo-acceleration of the mass A, defined as

$$A = pV = p^2U \tag{2.21}$$

and related to the maximum spring force, Q_o, as follows:

$$Q_o = kU = mp^2U = \frac{W}{g} A$$

where W is the weight of the system, and g is the gravitational acceleration. The force Q_o may also be written in the form

$$Q_o = C\,W \tag{2.22}$$

where C, the so-called lateral force coefficient or dynamic load factor, represents the number of times the system must be capable of supporting its own weight in the direction of motion, and is equal to the pseudo-acceleration of the system expressed in units of gravity.

For a system without damping, the acceleration $\ddot{x} = -p^2u$, whence it follows that the quantity A also represents the absolute maximum value of the true acceleration of the mass, $\ddot{X}$. For a damped system, A is only approximately equal to $\ddot{X}$, but the difference between these two quantities is of practical significance only for large values of damping, as will be seen subsequently. It may finally be noted from Eq. 2.2 that, for a damped system, the value of p^2u at the instant that u is an extremum represents the true acceleration of the mass, since the second term in this equation vanishes by virtue of the fact that $\dot{u} = 0$ at that instant. It is to be emphasized, however, that the maximum values of these two quantities are equal only for $\beta = 0$.

2.6 Ground Motions of Interest

2.6.1 General. Whereas the detailed characteristics of the ground motions resulting from two nuclear explosions under comparable conditions may differ significantly because of unavoidable differences in the values of the physical parameters involved, the gross or smoothed-out characteristics of such motions are generally quite similar. These similarities are particularly noticeable in the records of ground velocity and ground displacement.

Examination of available field test data (Ref. 1)* reveals that the time-history of the ground velocity induced by a nuclear explosion is characterized by a low-frequency, pulse-type of disturbance on which are superimposed oscillations of higher frequencies and usually smaller applitudes of more or less random character. The pulse-like disturbance will be referred to as the primary component of the motion, and the oscillatory component as the secondary or random component. The general shape, the peak value, and the duration of the primary component can generally be estimated with fair accuracy in terms

*Listed at the end of the text.

of the yield of the weapon, the distance of the point of observation from the source of the explosion, and the direction of the motion. In contrast, the random component cannot be defined reliably. This component arises mainly from reflections of the transmitted shock wave and is influenced significantly by the detailed properties of the medium through which the shock is transmitted. Since the properties of the soil may vary in a more or less arbitrary manner with depth or with distance from ground zero, the characteristics of this component can at best be described in statistical terms. It can generally be said, however, that the less uniform the soil conditions, or the greater the ground range, the more prominent is the contribution of the random component to the total input motion.

It is convenient to consider the effects of the two components of the input motion separately, and to estimate the maximum effect of the actual input by a combination of the corresponding effects produced by the two component inputs. The greater part of this report is concerned with the effect of the primary component of the motion. However, the manner in which the random component may modify the effects produced by the primary component, may also be estimated from the data to be presented.

2.6.2 Wave Forms of Primary Component. In the immediate vicinity of ground zero and at shallow depths, the velocity of the ground in the vertical direction has the characteristic shape of the overpressure curve, as shown in the lower part of Fig. 2.2a. This is essentially a half-cycle pulse with a sharp rise to a maximum value followed by a gentler decay. The corresponding displacement-time curve, shown in the upper part of the figure, is a pulse with a quarter of a cycle and a final or permanent displacement equal to the maximum value of the ground displacement.

As the distance from ground zero increases, the primary component of the ground velocity changes into a more nearly full-cycle pulse with both positive (downward) and negative (upward) parts. At the shorter ranges, the area under the negative part of the velocity diagram is smaller than under the positive part, and, consequently, the displacement-time diagram shows only partial recovery from its maximum value, as indicated in Fig. 2.2b. At the greater ranges, the two areas become equal to each other, and the ground displacement is represented by a half-cycle pulse with complete recovery, as shown in Fig. 2.2c. In general, the duration of the negative phase of the velocity pulse is longer than of the positive phase, and the corresponding displacement pulse is very similar to the velocity pulse applicable in the immediate vicinity of ground zero. At still greater ranges, the velocity diagram may consist of three or more half cycles, and the associated displacement diagram may have either one complete cycle, as shown in Fig. 2.2d, or several half-cycles as discussed in Ref. 2.

Half-cycle displacement pulses with complete recovery may also be expected in the immediate vicinity of ground zero if the intensity of the shock or the strength of the ground material are such that no permanent displacement results.

Evidently, the intensity of the ground motion decreases with increasing ground range, but this reduction in intensity may not be sufficiently great to compensate for the increased dynamic effects resulting from the greater number of oscillations present in the input function.

The characteristics of the ground motion in the horizontal direction are generally similar to those for vertical motion at great ranges. The time-history of the displacement is represented either by a half-cycle pulse with complete recovery, or a pulse with both positive and negative parts. It should be noted, however, that the available data for this case are not as conclusive as those for motion in the vertical direction.

In summary then, the following forms of ground motion are of interest.

(1) Half-cycle velocity pulses,

(2) Half-cycle displacement pulses, or displacement pulses with partial recovery from their maximum value.

(3) Full-cycle displacement pulses composed of both positive and negative parts, and displacement functions with several half-cycles.

Inasmuch as it is physically impossible to have instantaneous changes of displacement, velocity or acceleration of the ground, the displacement-time diagrams and their first and second derivatives must be continuous functions. In the following discussion, primary emphasis is given to the effects of continuous functions as indicated above; however, some discontinuous pulses are also considered as limiting forms of ground excitation. In addition, for the sake of completeness and for the purpose of developing the various concepts in an orderly fashion, consideration is first given to the effects of motions represented by a half-cycle acceleration pulse. For this class of excitation, the velocity of the ground after termination of the pulse has a constant value different from zero, and the ground displacement increases linearly, as shown in Fig. 2.3. This type of motion is of course of interest in the design of equipment mounted in a moving vehicle.

It must be noted here that the characteristics of the ground motions induced by a nuclear blast are not unlike those obtained for some strong motion earthquakes, and that, when properly interpreted, the dynamic response of systems to the two sources of excitation is generally quite similar. The earthquake motion is of course of longer duration than the blast induced motion, and the random component of the motion is more pronounced for an earthquake record. However, insofar as their effects on systems with moderate amounts of damping are concerned, these differences are found to be of minor consequence.

2.7 Deformation Spectra for Undamped Systems Subjected to Half-Cycle Acceleration Pulses

2.7.1 Presentation of Data. In Figs. 2.4 and 2.5 are given response spectra for the maximum positive and the maximum negative acceleration of the mass of a system subjected to a ground acceleration in the form of a half-sine pulse and a versed-sine pulse, respectively. The system is considered to be initially at rest. In each figure, the response acceleration, $\ddot{x}$, normalized with respect to the maximum input acceleration, $\ddot{y}_o$, is plotted against the dimensionless product of the natural frequency of the system in cycles per second, f, and the duration of the pulse, t_d. The results for the forced-vibration era of the motion, i.e. the period during which the pulse acts on the system, are given separately from those applicable to the period of free vibration. In particular, the solid line represents the spectrum for the maximum positive acceleration during forced vibration, the dashed-dotted line represents the spectrum for the corresponding maximum negative acceleration, and the dashed line represents the spectrum for the maximum acceleration during free vibration. In the latter case, the positive and negative values of the response are numerically equal. Since the system has no damping, by virtue of Eq. 2.16, these spectra can also be interpreted as deformation spectra.

In Figs. 2.6 and 2.7, the absolute maximum values of the positive and the negative values of the response acceleration, without regards as to their times of occurrence, are replotted on logarithmic scales. On such a plot, diagonal lines sloping upward to the right are lines of constant values of the ratio of the quantities plotted on the ordinate and the abscissa, and diagonal lines sloping downward to the right are lines of constant values of the product of these two quantities. The diagonal scales in these figures have been normalized such that they represent the dimensionless ratios

$$\frac{\ddot{x}}{p\dddot{y}_o} = -\frac{pu}{\dddot{y}_o} \quad \text{and} \quad \frac{p\ddot{x}}{\dddot{y}_o}$$

respectively, where $\dddot{y}_o$ is the maximum value of the derivative of the input acceleration function, the so-called "jerk".

For input acceleration functions having more than a single half-cycle, such as those to be considered subsequently, it is convenient to plot the relative pseudo-velocity pu on the vertical axis of the diagram instead of on the diagonal axis as was done in Figs. 2.6 and 2.7, and for the sake of uniformity, the quantity pu will be plotted on the vertical axis for all deformation spectra given in the remainder of this report.

In Fig. 2.8 the upper envelope of the spectra presented in Fig. 2.6 is replotted in this manner with the pseudo-velocity V normalized by the maximum ground velocity $\dot{y}_o$. The corresponding spectrum for the versed sine pulse is given in Fig. 2.9 along with those for two "skewed versed sine" pulses having rise times, t_r, equal to 1/4 and 1/8 the pulse duration. The latter pulses consist of two half-segments of a versed sine with unequal lengths. For $t_r/t_d = 1/4$, the duration of the second segment is three times as long as that of the first. Included in Fig. 2.9, is also a sketch of the derivative of the input acceleration function.

On a plot such as that given in Fig. 2.8 diagonal lines sloping upward to the right are lines of constant displacement U, and diagonal lines sloping downward to the right are lines of constant acceleration, $A = \ddot{X}$. Accordingly, with the scales for the diagonal axes established, from a plot of V alone, one can also read the values of U and A. In Figs. 2.8 and 2.9 the scales for A have been normalized with respect to the maximum value of the input acceleration $\ddot{y}_o$. The relationship between $V/\dot{y}_o$ and $A/\ddot{y}_o$ may be stated as

$$\frac{V}{\dot{y}_o} = \frac{1}{2\pi f}\frac{\ddot{y}_o}{\dot{y}_o}\frac{A}{\ddot{y}_o} \tag{2.23}$$

For a half-cycle acceleration pulse,

$$\dot{y}_o = \frac{2}{\pi}\ddot{y}_o t_d$$

and Eq. 2.23 reduces to

$$\frac{V}{\dot{y}_o} = \frac{1}{4\ f t_d}\frac{A}{\ddot{y}_o}$$

For a versed-sine pulse of arbitrary rise-duration ratio,

$$\dot{y}_o = \frac{1}{2}\ddot{y}_o t_d$$

and

$$\frac{V}{\dot{y}_o} = \frac{1}{\pi\ f t_d}\frac{A}{\ddot{y}_o}$$

For the class of input functions considered in this section, the scale for the relative displacement U cannot be normalized with respect to the maximum ground displacement, since this displacement is not defined in this case. For use in later sections, it is noted that, when y_o is defined, the relationship between $V/\dot{y}_o$ and U/y_o may be stated as

$$\frac{V}{\dot{y}_o} = 2\pi\ f\ \frac{y_o}{\dot{y}_o}\frac{U}{y_o} \tag{2.24}$$

2.7.2 Discussion of Results. From the information that has been presented and from a study of the additional data summarized in References 3 through 7, the following observations can be made.

a. Low Frequency Systems. This term describes the condition in which the duration of the excitation is small relative to the natural period of the system, i.e. $t_d f$ is small. From Figs. 2.4 through 2.7 it can be seen that for values of $t_d f$ less than about 0.6, the absolute maximum value of the deformation occurs during free vibration, with the result that both u_{max} and u_{min} are numerically the same. As $t_d f$ approaches zero, the curves approach the limiting values of

$$\frac{V_{max}}{\dot{y}_o} = \frac{pu_{max}}{\dot{y}_o} = 1 \tag{2.25a}$$

and

$$\frac{V_{min}}{\dot{y}_o} = \frac{pu_{min}}{\dot{y}_o} = -1 \tag{2.25b}$$

That this result is as it should be may be appreciated physically by noting that, with $t_d f$ approaching zero, the disturbing function approaches a velocity step of infinite duration, for which it is well known that the maximum and minimum values of the deformation are as given by Eqs. 2.25. For use subsequently, this result is derived below by application of the simple impulse theory.

Consider first a half-cycle force pulse applied to the mass of a fixed-base system. For small values of $t_d f$, the pulse may be approximated by an instantaneous velocity change of the mass, v_o, the magnitude of which may be obtained by application of the impulse-momentum relation

$$mv_o = \int_0^{t_d} P(\tau)\, d\tau \tag{2.26}$$

The displacement x may then be determined from the expression*

$$x(t) = \frac{v_o}{p} \sin pt \tag{2.27a}$$

which yields

$$x(t) = p\left[\int x_{st}(\tau)\, d\tau\right] \sin pt \tag{2.27b}$$

The maximum value of this expression constitutes an <u>upper bound</u> to the true maximum displacement, since the effectiveness of the impulse has been overestimated by assuming it to be concentrated at t = 0 instead of being spread over a finite time.

*Unless otherwise noted, the limits of integration for the integral expressions presented are from 0 to t_d, and the resulting equations are applicable for values of $t \geq t_d$.

If the disturbing function is a ground acceleration $\ddot{y}(t)$ of the same shape as P(t), the resulting spring deformation may be obtained from Eqs. 2.26 and 2.27a by replacing $P(\tau)$ by $-m\ddot{y}(\tau)$ and $x(t)$ by $u(t)$. The resulting expression is

$$u(t) = -\frac{1}{p}\left[\int \ddot{y}(\tau)\, d\tau\right] \sin pt = -\frac{1}{p}\, \dot{y}_o \sin pt \tag{2.28a}$$

from which Eqs. 2.25 follow directly. In particular,

$$U \leq \frac{1}{p} \int \ddot{y}(\tau)\, d\tau = \frac{1}{p}\, \dot{y}_o \tag{2.28b}$$

Note that u_{min} occurs at $T/4$ and u_{max} at $3T/4$, where T is the natural period of the system.

From the available data, the error incurred by the use of this simple impulse theory is estimated to be less than 10 percent if

$$t_{o,a} f \leq \frac{1}{\pi}$$

where $t_{o,a}$ is the <u>effective duration</u> of the acceleration pulse. This quantity is defined as the duration of a <u>triangular</u> pulse having the same peak value and the same area as the actual pulse, and it is given by the expression

$$t_{o,a} = 2\,\frac{\dot{y}_o}{\ddot{y}_o} = 2\,\frac{\ddot{y}_{av}}{\ddot{y}_o}\, t_d \tag{2.29}$$

in which $\ddot{y}_{av}$ is the average value of the input acceleration. For the versed sine pulses considered, the effective duration $t_{o,a} = t_d$. The concept of an effective pulse duration is introduced to account for circumstances in which the exciting function includes low-intensity regions, which, on purely physical grounds, can be expected to be relatively ineffective. From Eq. 2.23 it can be verified that the limiting value of $t_{o,a} f = 1/\pi$ referred to above, corresponds to the value of $t_d f$ for which both $V/\dot{y}_o$ and $A/\ddot{y}_o$ are equal to one. In general, the effect of pulse shape is unimportant for values of $t_{o,a}f$ as high as 0.5.

b. High Frequency Systems. Figs. 2.4 through 2.7 show that for values of $t_d f$ greater than that corresponding to the peak value of the curve for free vibration, the maximum positive value of $\ddot{x}$ is greater than the maximum negative value, the difference becoming progressively greater with increasing value of $t_d f$. As $t_d f$ approaches infinity, $\ddot{x}_{max}$ and $\ddot{x}_{min}$ approach the maximum positive and the maximum negative value of the input function, respectively. For the class of pulses considered in this section, the negative value is of course zero. These limiting values are valid only if the input acceleration is a continuous function.

In general, $\ddot{x}(t)$ can be expressed as the sum of two components: a function that is proportional to the input acceleration, and a sinusoidal component, the frequency of which is equal to the natural frequency of the system, f. As f tends to infinity, the magnitude of the first component becomes numerically equal to the input acceleration, and, in the absence of any discontinuities in the input acceleration, the amplitude of the periodic component reduces to zero. The response of the system then approaches that obtained under "static" conditions, and the limiting value of A becomes equal to the maximum input acceleration.

The effect of a discontinuity in the input acceleration is to make the amplitude of the periodic component in the expression for $\ddot{x}(t)$ equal to the magnitude of the discontinuity. If the input function has several discontinuities, the amplitude of the periodic component at any instant is equal to the numerical sum of the discontinuities up to that instant. This condition is illustrated in Fig. 2.10 for a series of acceleration pulses, including two full-cycle functions. The dashed line curves in this figure represent the input acceleration and the solid lines the acceleration of the system. The curves are drawn on the assumption that the natural period of the system, as represented by the period of the oscillatory component of the response, is small in comparison to the smallest time interval between consecutive discontinuities. These plots show that the limiting value of A must be either equal to the sum of the absolute maximum value of the input acceleration and the numerical sum of the discontinuities preceding this maximum, or equal to the numerical sum of a greater number of discontinuities and the magnitude of the following maximum, whichever combination gives the numerically greater value. For example, for the input function considered in Fig. 2.10f, the limiting value of A is

$$[(\ddot{y}_o)_1 + a_1] \text{ or } [a_1 + a_2 + (\ddot{y}_o)_2] \text{ or } [\text{zero} + a_1 + a_2 + a_3]$$

whichever is greatest. From these plots, the limiting values of $\ddot{x}_{max}$ and $\ddot{x}_{min}$ can also be determined, as shown.

It must be remembered that in the preceding discussion, the system was presumed to be completely undamped. Obviously, the effect of damping is to reduce the amplitude of the periodic component of the motion, the magnitude of the reduction being more pronounced in cases such as those shown in Figs. 2.10c and 2.10f, where the maximum response occurs at a considerable distance from the major discontinuity, instead of when the maximum occurs immediately after the discontinuity.

For an input acceleration pulse without any discontinuities, the range of frequencies within which the quantity A may be considered to be equal to the maximum input acceleration depends on the shortest rise time of the pulse rather than on its duration. From available data, and particularly those given in Fig. 4.19 of Ref. 3, it is concluded that the peak value of the input and the response accelerations may be considered to be equal for values of

$$\bar{t}_{r,a} f \geq 1.25 \qquad (2.30)$$

where $\bar{t}_{r,a}$ is the shortest "effective" rise time to the peak acceleration. This quantity is defined as the horizontal projection of a straight line extending from zero to the maximum value of the input acceleration with a slope equal to the maximum slope of the original curve, and is given by the equation

$$\bar{t}_{r,a} = \frac{\ddot{y}_o}{\dddot{y}_o} \qquad (2.31)$$

The error incurred by the use of the approximation referred to in Eq. 2.30 is estimated to be less than about 15 percent.

The general procedure described in the preceding paragraphs for the computation of the response acceleration of high-frequency systems, in combination with the analogies presented in Art. 2.4, can also be used to define the limiting values of other response quantities and to obtain other useful information. For example, the limiting value of $\dddot{x}(t)$ may be determined by considering $\dddot{y}(t)$ to be the associated input function. Now, if $\dddot{y}(t)$ is a discontinuous function, the amplitude of $\dddot{x}(t)$ during free vibration will be different from zero, and from the magnitude of this amplitude, it is also possible to define the manner in which $\dddot{x}_{min}$ approaches its limiting value. For the half-cycle acceleration pulses considered in this section, $\dddot{x}_{min}$ occurs during free vibration, and is therefore, related to $\ddot{x}_{min}$ by the equation

$$\dddot{x}_{min} = p\, \ddot{x}_{min}$$

As an illustration, consider the spectra for the half-cycle acceleration pulse presented in Fig. 2.6. In this case, $\dddot{y}(t)$ is a cosine function, and the limiting value of $\dddot{x}_{min} = -2\dddot{y}_o$, as may be appreciated from the diagram in Fig. 2.10b. It follows that at the limit

$$\frac{p|\dot{x}_{min}|}{\dot{y}_o} \rightarrow 2 \qquad \text{or} \qquad \frac{|\dot{x}_{min}|}{\dot{y}_o} \rightarrow \frac{1}{ft_d}$$

this result being substantiated by the data in Fig. 2.6.

For the versed sine acceleration pulse considered in Fig. 2.7, the limiting amplitude of $\ddot{x}(t)$ during free vibration is zero since $\ddot{y}(t)$ is a continuous function. However, by working with the <u>second derivative</u> of $\ddot{y}(t)$, which is discontinuous, and its associated response quantity $\ddddot{x}(t)$, one finds that the limiting value of the residual amplitude of $\ddddot{x}(t)$ is $2\ddddot{y}_o$. Accordingly,

$$|\ddddot{x}_{min}| = p^2|\ddot{x}_{min}| \rightarrow 2\ddddot{y}_o$$

and by noting that

$$\ddddot{y}_o = 2\frac{\pi^2}{t_d^2}\ddot{y}_o,$$

one obtains the result

$$\frac{|\ddot{x}_{min}|}{\ddot{y}_o} \rightarrow \frac{1}{(t_d f)^2}$$

which agrees with the data presented in Fig. 2.7.

c. <u>Maximum Values of A</u>. In Table 1 are listed the values of A_o and A_r with the associated values of $t_d f$ for the pulses considered in the preceding sections and for three triangular pulses discussed in Ref. 3. The quantity A_o denotes the absolute maximum value of A, and A_r denotes the maximum corresponding to the residual or free-vibration motion. The rise times of these pulses, t_r, are also listed along with the effective rise times, $\bar{t}_r$, as defined by Eq. 2.31.

In Table 1 the smallest value of $A_o/\ddot{y}_o = 1.26$ is obtained for a triangular pulse with vertical termination, and the greatest value of 2 is obtained for pulses with a vertical front. The results show clearly that the rise time of the pulse is the most important single parameter influencing the magnitude of the absolute maximum response, the detailed shape of the pulse being of secondary significance. Between pulses having the same peak value and the same duration, the greater value of $A_o/\ddot{y}_o$ can generally be expected to occur for the pulse with the shorter effective rise time.

For pulses with a smooth rise, the value of A_r is equal to or slightly less than the absolute maximum value A_o. However, the difference between the two sets of values increases with decreasing rise time, with the maximum difference obtained for a pulse with vertical front and a smooth decay.

The value of $t_d f$ corresponding to $A_o/\ddot{y}_o$ generally increases with decreasing rise time. Although there does not appear to exist a simple way of defining this value, it is worth noting that it is consistently greater than or equal to the value corresponding to the peak residual acceleration, A_r, the difference between the two values becoming greatest for the pulses with a sharp rise. The value of $t_d f$ corresponding to A_r can most reliably be approximated in terms of the effective duration of the pulse, $t_{o,a}$, as follows

$$t_{o,a} f \simeq 0.8 \qquad \text{or} \qquad t_d f \simeq 0.4 \frac{\ddot{y}_o}{\ddot{y}_{av}} \tag{2.32}$$

For an acceleration pulse with a vertical front and a smooth decay, the response spectrum for $A/\ddot{y}_o$ increases monotonically with $t_d f$, and approaches the value of 2 as a limit. For such pulses, for values of $t_d f$ greater than 1.0, the quantity $A/\ddot{y}_o$ can be approximated by the expression

$$\frac{A}{\ddot{y}_o} \simeq 1 + \frac{\ddot{y}(0.5T)}{\ddot{y}_o} \tag{2.33}$$

where $\ddot{y}(0.5T)$ is the value of $\ddot{y}(t)$ at a time equal to one half the natural period of the system. When expressed as a fraction of the total pulse duration, this time is equal to $0.5/(t_d f)$. In the following table, the approximate and exact values of $A/\ddot{y}_o$ are compared for an initially peaked triangular pulse and for a cosine function of one-quarter of a cycle. The exact values for the latter pulse were obtained from a plot included in Ref. 5.

$t_d f$	Triangular Pulse		Cosine Pulse	
	Approx.	Exact	Approx.	Exact
1.0	1.50	1.57	1.71	1.82
1.5	1.67	1.72	1.87	1.91
2.0	1.75	1.79	1.92	--

2.7.3 Design Rules. For design purposes, the response spectra for the absolute maximum deformation of undamped systems subjected to half-cycle acceleration pulses without any discontinuities can be approximated as shown in Fig. 2.11. For specific numerical applications, it is convenient to plot this diagram on a four-way logarithmic grid similar to that given in Fig. 2.12, in which the scales are expressed in absolute units instead of the dimensionless ratios used up to this point. To a first approximation, the spectrum may be defined by the straight line segments ab, bc, de, and the curved segment cd. Improved accuracy can be obtained by use of the smooth transition curve, as shown by the dotted line.

The spectrum is defined as follows:

(a) Along the horizontal line ab, the relative pseudo-velocity V is equal to the maximum value of the ground velocity.

(b) Along the diagonal line bc, the acceleration A is approximately equal to 1.5 times the maximum ground acceleration.

(c) Along the diagonal line de, the acceleration A is equal to the maximum ground acceleration.

(d) The curve cd is tangent to the line bc and intersects line de at an angle, as shown in the figure. The frequency corresponding to point d is determined from the expression $\bar{t}_{r,a}f = 1.25$, and that of point c can best be estimated from the data given in Table 1. For a symmetrical pulse, the location of c may be determined from Eq. 2.32.

For a discontinuous input function, the diagram must be modified in accordance with the general observations made previously.

2.8 Synthesis of Spectra for a Sequence of Half-Cycle Acceleration Pulses.

In addition to providing a great deal of insight into the behavior of the system, the detailed spectra of the type presented in Figs. 2.4 and 2.5 can also be used to synthesize the response spectrum corresponding to a sequence of half-cycle pulses. This possibility is described with reference to the full-cycle acceleration pulse shown in Fig. 2.13, the individual pulses of which may be of any shape for which detailed spectra are available.

The basic idea is to consider the motion produced by each half-cycle pulse acting independently, and to combine the resulting maximum effects, taking into consideration both the shape and duration of the individual pulses and also the times at which these effects take place. Fig. 2.13 shows the motion produced by each component pulse acting alone, along with the notation used. The symbol $\ddot{x}_{o,1}$ denotes the maximum value of the acceleration produced by the first pulse during forced vibration, i.e. in the interval $t < t_1$, and $\ddot{x}_{r,1}$ denotes the corresponding residual maximum. The remaining symbols are self-explanatory.

The absolute maximum response due to the actual pulse will naturally occur in one of the following regions:

Region 1, corresponding to $t \leq t_1$

Region 2, corresponding to $t_1 < t < t_d$

Region 3, corresponding to $t \geq t_d$

Let R_j denote the magnitude of the maximum response for the jth region, and R be the absolute maximum response.

For the first region, R_1 is evidently equal to $\ddot{x}_{o,1}$, the latter value being determined from the appropriate spectrum for the first half-cycle pulse and the specified value of the frequency parameter $t_1 f$.

For the third region, R_3 is obtained by a combination of the amplitudes of the two residual oscillations, $\ddot{x}_{r,1}$ and $\ddot{x}_{r,2}$, these quantities being again determined from the appropriate spectra with the appropriate values of $t_1 f$ and $t_2 f$. These amplitudes may be combined in a number of different ways, of which the following two appear to be the more appropriate:

(a) Take the numerical sum of $\ddot{x}_{r,1}$ and $\ddot{x}_{r,2}$ (2.34)

(b) Use the expression

$$R_3 = \sqrt{(\ddot{x}_{r,1})^2 + (\ddot{x}_{r,2})^2} \tag{2.35}$$

The first approach, which assumes the two residual oscillations to be in phase, obviously leads to an upper bound. The sign of the error committed by the second approach, which amounts to assuming the two residual oscillations to be 90° out of phase, cannot be determined in general.

For the intermediate region, the response R_2 is computed by combining the quantity $\ddot{x}_{o,2}$ with the amplitude of the residual oscillation due to the first pulse, $\ddot{x}_{r,1}$. Two alternative procedures are noted which are analogous to those used for region 3.

An upper bound may be obtained by linear superposition as follows:

$$R_2 \leq \ddot{x}_{o,2} \pm \ddot{x}_{r,1} \tag{2.36}$$

where the signs are selected so as to yield the maximum possible numerical value.

Alternatively, one may use the expression

$$R_2 \approx a_2 + \sqrt{(\ddot{x}_{o,2} - a_2)^2 + (\ddot{x}_{r,1})^2} \tag{2.37}$$

where $(\ddot{x}_{o,2} - a_2)$ approximates the oscillatory component of the motion induced by the second pulse, and the square root quantity approximates the amplitude of the oscillatory component due to both pulses.

As an illustration, consider an acceleration function composed of a sequence of two half-sine waves such that $t_1/t_d = 1/4$ and $a_2/\ddot{y}_o = t_1/t_2 = 1/3$. Assume further that $t_1 f = 0.5$; whence it follows that $t_2 f = 1.5$.

By entering Fig. 2.4 with the appropriate values of the frequency parameter, one finds that

$$\ddot{x}_{o,1} = 1.57\, \ddot{y}_o \qquad \ddot{x}_{r,1} = \pm 1.57\, \ddot{y}_o$$

$$\ddot{x}_{o,2} = 1.5\, a_2 = 0.5\, \ddot{y}_o \qquad \ddot{x}_{r,2} = 0$$

It follows that

$$R_1 = \ddot{x}_{o,1} = 1.57\, \ddot{y}_o$$

$$R_2 = \begin{cases} (-1.57 - 0.5)\, \ddot{y}_o = -2.07\, \ddot{y}_o, \text{ by Eq. 2.36} \\ \left[-0.33 - \sqrt{(0.5 - 0.33)^2 + (1.57)^2}\right] \ddot{y}_o = -1.91\, \ddot{y}_o, \text{ by Eq. 2.37} \end{cases}$$

and

$$R_3 = \begin{cases} (1.57 + 0)\, \ddot{y}_o = 1.57\, \ddot{y}_o, \text{ by Eq. 2.34} \\ \sqrt{(1.57)^2 + 0}\;\; \ddot{y}_o = 1.57\, \ddot{y}_o, \text{ by Eq. 2.35} \end{cases}$$

The absolute maximum value of the response is, therefore, $R = -2.07$ by linear superposition, and $R = -1.91$ by the square root rule. The latter value happens to coincide with the exact value.

In Fig. 2.14a the response spectrum for the acceleration function considered in the preceding example is compared with the results obtained by the two versions of the approximate procedure presented. A similar comparison is made in Fig. 2.14b for a full-cycle sinusoidal function. As might have been expected, the agreement is better in the first case where one of the pulses dominates the response.

It must be noted here that the response of the system in the low-frequency range can reliably be predicted by simple relations to be presented later, and therefore this procedure need not be used for this frequency range. The procedure is recommended especially for the computation of the absolute maximum value of A, and will be used for this purpose later.

2.9 Deformation Spectra for Undamped Systems Subjected to Half-Cycle Velocity Pulses

The pulses considered in this section are of the type shown in Fig. 2.2a for which the areas under the positive and negative parts of the acceleration function are equal. The system is presumed to have no damping and to be initially at rest.

2.9.1 Low Frequency Systems. If the duration of the velocity pulse, t_d, is short in comparison to the natural period of the system, the maximum ground displacement, y_o, will be attained before the mass of the system has had an opportunity to respond, and the ground motion will literally be "absorbed" by the spring. It is physically apparent that the first extremum value of the deformation will occur approximately at $t = t_d$, and will be nearly equal to the negative value of y_o. The subsequent motion of the mass will be essentially that of a fixed-base system subjected to an initial deformation $-y_o$. It follows, therefore, that

$$u_{min} = -y_o \qquad u_{max} = y_o \qquad U = y_o$$

$$x_{min} = 0 \qquad x_{max} = 2\,y_o \qquad X = 2\,y_o$$

and that the first values of u_{max} and x_{max} will occur approximately at $t = t_d + 0.5T$.

The limiting value of U for this case can also be determined from Eq. 2.28b by making use of the analogy expressed by the second and third terms in Eq. 2.17. Noting that $A = p^2 U$, one obtains

$$V \leq p \int \dot{y}(\tau)\, d\tau = p\, y_o \tag{2.38a}$$

or

$$\frac{U}{y_o} \leq 1$$

If the maximum ground displacement y_o is expressed in terms of the ground acceleration, Eq. 2.38 can be written alternatively as

$$U \leq \int \ddot{y}(\tau)\, \tau\, d\tau \tag{2.38b}$$

The latter integral represents the moment of the acceleration diagram about the end of the pulse.

2.9.2 Presentation and Discussion of Results.

a. Characteristics of Representative Spectra. In Fig. 2.15 are given response spectra for the relative displacement U, the relative pseudo-velocity V, and the pseudo-acceleration A, of a system subjected to a versed-sine pulse of ground velocity. A sketch of this pulse and of the associated acceleration and displacement functions are included in the figure. It must be emphasized that these curves are interrelated by Eq. 2.21, and that if one of them is known, the remaining two can be determined.

It can be seen that the deformation U never exceeds the maximum ground displacement, and that, for small values of $t_d f$, it is essentially equal to y_o. As far as the acceleration A is concerned, at large values of $t_d f$, it is approximately equal to the maximum ground acceleration $\ddot{y}_o$, but the peak value of A is greater than $\ddot{y}_o$ and occurs in the intermediate range of $t_d f$ values. These limiting values of U and A are in agreement with those discussed in the preceding section.

Of special significance is the relative order of magnitude of U/y_o and $A/\ddot{y}_o$ for the extreme values of $t_d f$. At small values of $t_d f$, the amplification factors for A are a fraction of those for U, whereas at the large values, the order of the curves if reversed, and the amplification factors, for U are a fraction of those for A. It would appear that, for low-frequency systems, the maximum deformation is insensitive to the details of the acceleration and velocity records, whereas for high-frequency systems, it is insensitive to the characteristics of the input displacement. In the intermediate range, the response appears to be sensitive to the characteristics of both the velocity and the acceleration traces. Because of the general shape of these curves, the maximum deformation of medium-frequency systems can more conveniently be expressed in terms of V instead of directly in terms of U. Similarly, for high-frequency systems, the quantity A is a more convenient measure of the maximum spring deformation than either U or V. It is essantially for this reason that V and A are used as alternative measures of U.

The maximum positive and the maximum negative values of the spring deformation are shown separately in Figs. 2.16 and 2.17a in the form of acceleration spectra and pseudo-velocity spectra, respectively.

The following characteristics of the curves are worth noting.

(a) For values of $t_d f \leq 1$, the maximum deformation occurs during free vibration, with the result that the positive and negative values of the response are numerically equal.

(b) The absolute maximum value of the pseudo-velocity, V_o, occurs during free vibration.

(c) For values of $t_d f \geq 1$, the maximum deformation during forced vibration is equal to, or constitutes a good approximation to, the absolute maximum value. Furthermore, the positive and negative values of the response are close to one another, but this agreement is believed to be valid only for symmetrical velocity pulses for which the positive and negative parts of the input acceleration have the same general shape and magnitude. If the peak magnitudes of the two parts of the acceleration function are different, $\bar{X}_{max}$ will converge to the corresponding negative value. This condition is illustrated in Fig. 2.17b which refers to a skewed versed-sine velocity pulse with a rise time of 0.25 t_d. The ratio of the minimum and maximum values of the input acceleration function being 1/3, the limiting value of $|\bar{X}_{min}| = \ddot{y}_o/3$.

(d) For the pulses considered, the peak values of the acceleration A for the forced vibration and the free vibration eras of the motion are close to each other. Pertinent data are summarized below for a class of skewed versed-sine velocity pulses having rise-duration ratios of 1/2, 1/4 and 1/8.

$\frac{t_{r,v}}{t_d}$	Absolute Maximum Value of $A/\ddot{y}_o$	
	Forced Vibration	Free Vibration
1/2	3.25	3.21
1/4	1.97	1.74
1/8	1.85	1.71

b. <u>Effects of Rise Time and Discontinuities in Acceleration</u>. The spectra in Fig. 2.18 are for a family of skewed versed-sine velocity pulses with rise times ranging from 1/2 to 1/8 the duration of the pulse.

It can clearly be seen from this figure that for values of $t_d f$ less than one the effect of rise time is almost imperceptible. Similarly, at the right hand end of the diagram, the limiting value of A for each curve can be shown to be equal to the maximum input acceleration $\ddot{y}_o$. By virtue of the fact that the magnitude of the ground acceleration for a fixed value of the maximum ground velocity increases with decreasing rise time, on a plot such as that given in Fig. 2.18, the location of the limiting value of A shifts to the right as $t_{r,v}/t_d$ decreases. Thus, the principal effect of a decrease in the rise time is to increase the width of the nearly flat portion of the V-spectrum.

In Figs. 2.19 through 2.22c are given deformation spectra for several velocity pulses the derivatives of which are discontinuous functions. The velocity pulses considered include a symmetrical parabolic pulse (Figs. 2.19 and 2.20), a skewed sinusoidal pulse with a rise time equal to one third the total duration (Fig. 2.21), and a series of triangular pulses with different rise-duration ratios (Figs. 2.22a through 2.22c). The velocity pulses and the corresponding acceleration histories are shown in the inset diagrams. The values of $t_d f$ below which the absolute maximum response consistently occurs during free vibration are also indicated.

For values of $t_d f$ between zero and a value slightly greater than that for which V is maximum, the spectra presented in these figures are almost identical to those presented earlier, verifying the prediction that, for flexible systems, the maximum deformation is dependent on the shape of the ground displacement alone, rather than on the shapes of the corresponding velocity or acceleration traces. In each case, the spectrum is bounded on the left by a line of constant displacement equal in magnitude to the maximum ground displacement. For values of $t_d f$ less than that for which $V/\dot{y}_o = 1$, the maximum error due to taking $U = y_o$ is for all practical purposes negligible.

In contrast, for values of $t_d f \geq 1$, the magnitude and general appearance of the curves are influenced to a rather significant degree by the detailed features of the input motion. In Figs. 2.20 and 2.21 the limiting value of A is equal to twice the maximum ground acceleration, and in Figs. 2.22 it is equal to twice the maximum value of the discontinuity in the input acceleration function. These limiting values are in agreement with those predicted by the procedure described in Art. 2.7.2b. In Fig. 2.22c the curve for $t_{r,v}/t_d = 0$ approaches asymptotically the line $V/\dot{y}_o = 1$, because the maximum input acceleration is infinite in this case, and the velocity function approaches a step pulse of infinite duration.

For an input acceleration without any discontinuities, the response of a high-frequency system may be considered to be the same as that obtained under static conditions if Eq. 2.30 is satisfied for each component pulse in the input acceleration. However, if the amplitudes of the individual pulses are significantly different from one another, it may be sufficient to satisfy this relation only for the pulse with the greatest ordinate, since the effect of the remaining pulse or pulses may be negligible.

c. Maximum Values of V and A. The maximum values of V, for the velocity pulses considered in the preceding sections and for several additional pulses considered in Ref. 2, are listed in Table 2 together with their corresponding values of $t_d f$. The results for the pulses identified with an asterisk correspond to maxima that occur during free vibration, but these maxima are expected to be equal to or very close to the absolute maximum values. Pulses 5a and 5b are defined by Eq. 4.6 of Ref. 2 as the product of a versed sine function, a skewing constant, and a decaying exponential function.

It can be seen that the value of $V_o/\dot{y}_o$ ranges between 1 and 2, the lower bound corresponding to a rectangular velocity pulse of infinite duration, and the upper bound to a rectangular pulse of finite duration. These limiting values suggest that, for velocity pulses of other shape, the smaller values of $V_o/\dot{y}_o$ would occur for pulses having a sharp rise and a gradual decay, and that the larger values, would correspond to pulses having a sharp rise, a sharp decay, and a fairly flat intermediate region. It follows further that, for symmetrical pulses, the values of $V_o/\dot{y}_o$ can be expected to be greater than those for unsymmetrical pulses, and that among symmetrical pulses of the same duration and the same peak value, the greater values of $V_o/\dot{y}_o$ would correspond to the pulse having the shortest effective rise time combined with the flattest top. These conclusions are substantiated by the numerical data presented in Table 2. The effective rise time, $\bar{t}_{r,v}$, defined in a manner analogous to that used for a half-cycle acceleration pulse, is given by the equation

$$\bar{t}_{r,v} = \frac{\dot{y}_o}{\ddot{y}_o} \tag{2.3}$$

In the absence of specific information about the shape of the velocity pulse, the value of V_o for the unsymmetrical velocity pulses encountered in ground shock problems may be taken as 1.5 times the maximum input velocity.

It may be recalled that in discussing the effects of half-cycle acceleration pulses, it was noted that the maximum value of A depends primarily on the effective rise time of the pulse, and that the detailed shape of the pulse, including its decay time, were relatively unimportant. That the significant parameters for an acceleration input are different from those for a velocity input can best be appreciated by considering the response of high-frequency systems to a rectangular forcing function. For an acceleration input, the amplification factor for A_o is one, irrespective of the duration of the pulse, whereas for a velocity input, the amplificator factor for V_o is one only for a pulse of infinite duration, and becomes two for a pulse of finite duration.

In Table 2, the values of $t_d f$ corresponding to V_o range from 0.50 to 1.9, with the majority of the values being of the order of 0.7. The maximum value is obtained for the decaying skewed versed-sine pulse, No. 4a, for which it is physically apparent that the "effective duration", $t_{o,v}$, which excludes the low-intensity tail end of the pulse, is shorter than the actual duration. The location of V_o can more reliably be expressed in terms of the effective duration parameter $t_{o,v} f$, the values of which, as can be seen from the table, are considerably less dependent on the details of the pulse shape than are those of the parameter $t_d f$. The quantity $t_{o,v}$, defined in a manner analogous to that presented earlier for an acceleration pulse, is given by the expression

$$t_{o,v} \cong 2 \frac{y_o}{\dot{y}_o} = 2 \frac{\dot{y}_{av}}{\dot{y}_o} t_d \tag{2.40a}$$

In the absence of detailed information about the shape of the velocity pulse, V_o may be considered to occur at a value of

$$t_{o,v} f \simeq 0.8 \qquad \text{or} \qquad t_d f \simeq 0.4 \frac{\dot{y}_o}{\dot{y}_{av}} \tag{2.40b}$$

For the pulses considered, the values of $A_o/\ddot{y}_o$ range from a maximum value of 4 to a value of less than 2. In general, the larger values are obtained for the acceleration pulses for which the positive and negative half-cycles are of the same shape and duration (i.e., for symmetrical velocity pulses). In the following table, the exact values of $A_o/\ddot{y}_o$ for the class of skewed versed-sine velocity pulses considered are compared with the values obtained by application of the two versions of the approximate procedure described in Section 2.8. For the values given in the third column, the contributions of the individual pulses were combined linearly, and for the values given in the fourth column the square root rule was used. The agreement between the exact and the approximate values is considered to be quite adequate for all practical applications.

$\frac{t_{r,v}}{t_d}$	Maximum Values of $A_o/\ddot{y}_o$		
	Exact	Approximate	
1/8	1.85	1.85	1.84
1/4	1.97	2.17	2.06
1/2	3.30	3.46	2.86

Strictly speaking, the value of A_o depends not only on the relative amplitudes and durations of the individual pulses in the input acceleration function, but also on the pulse shapes themselves, as may be appreciated from the discussion presented in Section 2.7.2. However, in the absence of information about the detailed shape of these pulses, the value of A_o may be taken approximately as 1.5 times the numerical sum of the maximum and minimum values of the input acceleration.

The location of A_o can best be defined in terms of the duration $t_{1,a}$ of the dominant acceleration half-cycle rather than the total duration of the pulse. The quantity $t_{1,a}$ is, of course, equal to shorter rise time in the associated velocity pulse, $t_{r,v}$. For continuous functions, A_o may be considered to occur at a value of

$$t_{1,a} f = 0.6 \tag{2.41}$$

2.9.3 Design Rules. For design purposes, the deformation spectra for systems subjected to half-cycle velocity pulses may be approximated by the diagram given in Fig. 2.23, provided the ground acceleration is a continuous function. To a first approximation this spectrum may be defined by the straight line segments ab, bc, cd, ef and the curved segment de, as follows:

(a) Along the diagonal line ab, the displacement U is equal to the maximum value of the ground displacement.

(b) Along the horizontal line bc, the relative pseudo-velocity V is equal to 1.5 times the maximum ground velocity. If the detailed shape of the input velocity is known, a more precise estimate for this upper bound on V may be obtained from the data presented in Table 2.

(c) Along the diagonal line cd, the acceleration A is equal to 1.5 times the sum of the absolute values of the maximum and minimum ground accelerations. If the input function is known exactly and response spectra for the component pulses are available, a somewhat better estimate of this value of A may be obtained by the procedure described in Section 2.8.

(d) Along the line ef, the acceleration A is equal to the maximum ground acceleration.

(e) The curve de is tangent to the line cd and intersects the line ef at an angle, as shown in the diagram. The frequencies corresponding to points d and e are determined approximately from the expression shown in the figure. The frequency for point d should not be smaller than the frequency corresponding to point c of the diagram.

(f) The transition curves shown in dotted lines are tangent to the straight line segments at points g, h and d. Point g corresponds to a value of $V = \dot{y}_o$, and point h corresponds to a frequency determined from Eq. 2.40b. The latter frequency should not be greater than that corresponding to point c.

2.10 Deformation Spectra for Undamped Systems Subjected to Half-Cycle Displacement Pulses and Pulses with Partial Recovery

The pulses considered in this section are of the type shown in Figs. 2.2c and 2.2b. As before, the system is considered to have no damping and to be initially at rest.

2.10.1 Low Frequency Systems. From a physical argument entirely analogous to that used in Section 2.9.1, one concludes that for low-frequency systems the first extremum value of the deformation will occur at or near the instant that the ground attains its maximum value and will be approximately equal to the negative value of y_o, i.e.

$$u_{min} = - y_o \quad \text{at} \quad t = t_d \tag{2.42}$$

The second extremum will occur after termination of the pulse. By virtue of the similarity of Eqs. 2.3 and 2.5, the absolute displacement of the system for $t \geq t_d$ can be determined from Eq. 2.27b by replacing $x_{st}(\tau)$ by $y(\tau)$, as follows

$$x(t) = p \left[\int y(\tau)\, d\tau \right] \sin pt = pt_d \, y_{av} \sin pt \tag{2.43a}$$

where y_{av} is the average value of the displacement in the interval between 0 and t_d. Equation 2.43a also represents the relative displacement $u(t)$, since $y(t) = 0$ for $t \geq t_d$. It follows then that

$$u_{max} = x_{max} \leq pt_d \, y_{av} \tag{2.43b}$$

or

$$\frac{u_{max}}{y_o} = \frac{x_{max}}{y_o} \leq \left[2\pi \frac{y_{av}}{y_o} \right] t_d f \tag{2.44}$$

The first values of u_{max} and x_{max} occur at a time roughly equal to one-half the natural period of the system. Incidentally, Eq. 2.44 could also have been obtained from Eq. 2.38a by utilizing the analogy expressed by the first two terms in Eq. 2.17.

For a ground displacement $y(t)$ with partial recovery, the relative displacement for $t \geq t_d$ is given by the expression

$$u(t) = pt_d \, y_{av} \sin pt - y_f \cos[p(t-t_d)],$$

in which the first term represents the contribution of the pulse within $0 \leq t \leq t_d$, and the second term represents the contribution of the residual or final displacement of the ground, y_f. For small values of pt_d, taking $\sin pt_d = pt_d$ and $\cos pt_d = 1$, one obtains

$$u(t) \simeq pt_d[y_{av} - y_f] \sin pt - y_f \cos pt,$$

whence

$$\frac{u_{max}}{y_o} \simeq \sqrt{4\pi^2 \left[\frac{y_{av}}{y_o}\right]^2 \left[1 - \frac{y_f}{y_{av}}\right]^2 (t_d f)^2 + \left[\frac{y_f}{y_o}\right]^2} \qquad (2.45)$$

For a displacement pulse with complete recovery, the absolute maximum value of the deformation, U, is the numerically larger of the values given by Eqs. 2.42 and 2.44, and for a pulse with partial recovery, it is the larger of the values given by Eqs. 2.42 and 2.45. For a displacement pulse with complete recovery, Eq. 2.42 governs for values of

$$t_d f < \frac{1}{2\pi} \frac{y_o}{y_{av}} \qquad (2.46)$$

2.10.2 Presentation and Discussion of Data. In Fig. 2.24 are presented response spectra for the maximum and minimum deformations of undamped systems subjected to a half-cycle displacement pulse. The acceleration diagram of the input motion consists of a sequence of three half-sine pulses of the same amplitude and of durations t_1, $2t_1$ and t_1, respectively, as shown in the inset diagram. The maximum deformation, u_{max}, corresponds to an extension of the spring and is a positive quantity, whereas u_{min} corresponds to compression and is a negative quantity. It should be noted that the abscissa in this figure is the quantity $2t_1f$ instead of the quantity t_df used in previous figures. The quantity $2t_1$ is also equal to the duration of each velocity pulse and to the rise time of the associated displacement function. Included in this figure as dotted line curves are also the results obtained from Eq. 2.42 and from the right member of Eq. 2.44.

In Figs. 2.25a and 2.25b are presented similar curves for a versed-sine displacement pulse and for a skewed versed-sine displacement pulse with a rise-duration ratio of $t_1/t_d = 1/4$. The upper envelopes of these curves are compared in Fig. 2.26 with the corresponding curve for a displacement pulse of the same family but with a value of $t_1/t_d = 1/8$. The displacement pulses considered in these figures are identical to the velocity pulses considered in Figs. 2.17a through 2.18.

The following characteristics of the curves are worthy of note:

(a) Unlike the spectra for the half-cycle velocity pulses considered previously, which were bounded by a value of U equal to the maximum ground displacement, the spectra presented in this section have values of U exceeding the maximum ground displacement over a considerable range of the frequency parameter.

(b) For low-frequency systems, the results obtained from Eqs. 2.42 and 2.44 are in good agreement with the exact values. Note, in particular, that the approximate results define with reasonable accuracy the initial position and the initial slope of the "hump" on the left-hand portion of the diagram. In the following table the values of t_1f corresponding to this break in the spectra are compared with the values obtained from Eq. 2.46 for the family of skewed versed-sine displacement pulses.

$\frac{t_1}{t_d}$	Value of t_1f	
	Exact	From Eq. 2.46
1/2	0.15	0.16
1/4	0.084	0.080
1/8	0.042	0.039

(c) For the motion considered in Fig. 2.24 the limiting value of $A/\ddot{y}_0$ for high-frequency systems is 1.0, whereas for the motions considered in Figs. 2.25 it is equal to 2.0. This is due to the fact that the acceleration function of the first motion is continuous, whereas of the second motion it is discontinuous. These limiting values are in agreement with those predicted by the procedure described in Section 2.7.2b.

(d) The absolute maximum value of the deformation, U_o, occurs during free vibration, and the maximum value of the pseudo-velocity, V_o, either occurs during free vibration (as in Figs. 2.24 and 2.25a), or is a close approximation to the corresponding maximum obtained during free vibration (as in Fig. 2.25b). It may be recalled that for the half-cycle velocity pulses considered in Section 2.9 similar results were obtained for the quantities V_o and A_o. Accordingly, the analogies given in Eq. 2.17 are applicable, and it follows that the value of U_o/y_o for a half-cycle displacement function must be equal to the value of $V_o/\dot{y}_o$ for a velocity input of the same shape, and they must occur at the same values of $t_d f$. Similarly, the coordinates of $V_o/\dot{y}_o$ for the displacement input may be considered to be the same as those for $A_o/\ddot{y}_o$ for the corresponding velocity input. That this is indeed true can be verified by comparing the corresponding coordinates of the spectra presented in Figs. 2.17 and 2.25. The magnitude of A_o for the displacement pulses cannot be obtained by analogy of the results presented previously, but it is clear from the material already presented that this quantity depends primarily on the number of half-cycles in the input acceleration, on the degree of regularity of the individual pulses, and, to a lesser extent, on the shape of the individual pulses. If the spectra corresponding to the component pulses are available, then the value of A_o may be determined with good accuracy by use of the procedure described in Section 2.8. In the absence of detailed information about the characteristics of the ground acceleration, the approximate design rule given in the next section may be used.

In Fig. 2.27 are given deformation spectra for a half-sine displacement pulse. It is important to note that, whereas the left-hand portions of these spectra are quite similar to those presented in Figs. 2.24 and 2.25a, a result that might have been anticipated from the similarity of the three displacement functions, the right-hand portions differ radically. It should be apparent that the medium-frequency and high-frequency regions of a deformation spectrum are functions of the detailed or "microscopic" features of the displacement function, which are generally difficult to ascertain from the displacement diagram itself. These features are most clearly depicted in the velocity and acceleration diagrams of the input motion. In Fig. 2.27 the spectrum approaches a horizontal asymptote because the velocity diagram of the input motion is a discontinuous function (i.e. the acceleration function has infinite discontinuities).

By utilizing the procedure described in Section 2.7.2b and the analogy expressed by the second and third terms in Eq. 2.11, it can readily be shown that as $t_d f \rightarrow \infty$ the maximum and minimum values of the velocity of the mass, $\dot{x}$, approach a value equal to twice the maximum ground velocity. This relation is also valid during free vibration. But, since $\dot{y}(t) = 0$ for $t \geq t_d$,

$$|\dot{x}| = |\dot{u}| = |pu|$$

whence it follows that

$$|pu_{max}| = |pu_{min}| = 2\,\dot{y}_o$$

Some data for displacement pulses with partial recovery are given in Ref. 8.

2.10.3 Design Rules. For design purposes, the deformation spectrum corresponding to a half-cycle displacement pulse, or a pulse with partial recovery, may be approximated by the diagram abcdefgh, as shown in Fig. 2.28. For improved accuracy, the portion of the diagram between points b and f may be replaced by smooth transition curves as shown by the dotted lines. The time histories of the displacement, velocity and acceleration of the ground are considered to be continuous functions. The characteristics of this diagram are as follows:

(a) Along the limiting lines ab and gh, the relationship between the input and response quantities are the same as for the corresponding lines ab and ef in Fig. 2.23. Furthermore, the frequency corresponding to point g is the same as that for point e in Fig. 2.23.

(b) For a displacement pulse with complete recovery, the displacement U along line bc is given by the right-hand member of Eq. 2.44, and for a displacement pulse with partial recovery, it is given by Eq. 2.45.

(c) Along line cd, the displacement U may be approximated by the equation

$$U \approx y_o \left[1.5 - 0.5 \frac{y_f}{y_o}\right] \tag{2.47}$$

(d) Along de, the relative pseudo-velocity V is equal to 1.5 times the sum of the absolute values of the maximum and minimum ground velocities. This relationship is the same as that between the response acceleration A and the input acceleration for line cd in Fig. 2.23. If the velocity of the ground is known accurately and the deformation spectra corresponding to the component pulses of the velocity function are available, then an improved estimate of this maximum value of V may be obtained in a manner analogous to that described in Section 2.8.

(e) Along line ef, the acceleration A is proportional to the maximum input acceleration, the ratio of proportionality depending on the degree of regularity of the input function. If the durations for the individual pulses of the ground acceleration are approximately equal to each other, then the value of A along this line may be determined from the expression

$$A_o \simeq 1.5 \sum_{j=1}^{n} |(\ddot{y}_o)_j| \tag{2.48}$$

where $(\ddot{y}_o)_j$ is the amplitude of the jth half-cycle, and n is the number of half-cycles present. If the amplitudes of the individual pulses are of the same order of magnitude but their durations differ appreciably, then the result obtained from this equation may be quite conservative. If the deformation spectra for the component pulses are available, an improved estimate of A_o may be obtained by the procedure described in Section 2.8.

(f) The frequency corresponding to point f is determined from Eq. 2.41, where $t_{1,a}$ should be interpreted as the average duration of the acceleration half-cycles contributing over fifty percent of the value of A_o. The curve fg is similar to the curve de in Fig. 2.23.

(g) The transition curves represented by the dotted line are tangent to the corresponding straight line segments at points b, i, j and f. Frequently, the lengths of the straight line segments between b and f are small, and the transition curves can be drawn without having to evaluate the location of points i and j. When this is not the case, the frequency corresponding to point i may be determined from the following expression, obtained by analogy to Eq. 2.40b,

$$t_d f \simeq 0.4 \frac{y_o}{y_{av}} \tag{2.48x}$$

and the frequency corresponding to point j, may be determined from the expression

$$t_{1,v} f \simeq 0.6 \tag{2.49}$$

where $t_{1,v}$ is the duration of the velocity half-cycle with the maximum amplitude. Strictly speaking, Eq. 2.40b is applicable only to displacement pulses with complete recovery, but, for want of any better information, it may also be used for pulses with partial recovery. It should be noted that points i and j, which are analogous to points h and d in Fig. 2.23, should be within the limits of the lines cd and de, as shown in the figure.

2.11 Deformation Spectra for Undamped Systems Subjected to Full-Cycle Displacement Pulses

For the typical full-cycle displacement pulse shown in Fig. 2.29, let $(y_o)_1$ and $(y_o)_2$ denote the numerical values of the first and second extremums, and $(t_o)_1$ and $(t_o)_2$ denote the corresponding times. In addition, let t_d denote the total duration of the function, and t_1 and t_2 denote the durations of the first and the second half-cycles.

2.11.1 Low Frequency Systems. As previously explained, in the interval $0 \leq t \leq t_d$ the time history of the deformation for systems with small values of $t_d f$ may be taken equal and opposite to that of the ground. Accordingly, the first extremum value of the deformation, $(u_o)_1$, will occur at $t \doteq (t_o)_1$, and will be given by the expression

$$(u_o)_1 \doteq - (y_o)_1 \tag{2.50}$$

The second extremum, $(u_o)_2$, will occur at $t \doteq (t_o)_2$ and will be given by the expression

$$(u_o)_2 \doteq (y_o)_2 \tag{2.51a}$$

Eq. (2.51a) is valid for very soft systems, or more precisely, for values of $t_d f \rightarrow 0$. For somewhat stiffer systems, a better estimate of $(u_o)_2$ may be obtained from the expression

$$(u_o)_2 \doteq pt_1 \, (y_{av})_1 \sin [p(t_o)_2] + (y_o)_2 \tag{2.51b}$$

where, for values of $p(t_o)_2$ greater than $\pi/2$, the term $\sin[p(t_o)_2]$ should be taken as one. The quantity $(y_{av})_1$ in this equation denotes the average value of the displacement between $t = 0$ and $t = t_1$, and the term involving $(y_{av})_1$ represents the simple impulse theory approximation to the contribution of the first pulse.

The third extremum value of the deformation, $(u_o)_3$, occurs during free vibration. For a displacement function for which the areas under the positive and negative parts are equal, the value of this extremum is given by the following expression which, by virtue of the similarity of Eqs. 2.3 and 2.4, may be obtained directly from Eq. 2.38b by replacing $\ddot{y}(\tau)$ by $p^2 y(\tau)$

$$|(u_o)_3| \leq p^2 \int_0^{t_d} y(\tau)\, \tau\, d\tau \qquad (2.52)$$

The absolute maximum value of the deformation, U, is the numerically greatest of the values given by Eqs. 2.50, 2.51b and 2.52. When $(y_o)_2 \geq (y_o)_1$, the second extremum is numerically greater than the first, and Eq. 2.50 need not be considered if only the absolute maximum value of the deformation is required.

2.11.2 Presentation and Discussion of Data. In Fig. 2.30 is given the deformation spectrum corresponding to a full-cycle displacement function which is identical in shape to the velocity diagram of the half-cycle displacement pulse considered in Fig. 2.24. In the extreme left-hand portion of the figure, the first three extremum values of the deformation are shown separately, along with the corresponding results obtained from the approximate equations of the preceding section. Similar information is presented in Figs. 2.31a through 2.32 for a family of displacement functions composed of a sequence of two half-sine waves. These functions are identical to the velocity diagrams considered in Figs. 2.25a through 2.26.

The following results are worth noting:

(a) For low-frequency systems, the results obtained by the approximate equations are in reasonable agreement with the exact values.

(b) The magnitude and location of the absolute maximum deformation U_o for the spectrum presented in Fig. 2.30 are identical to those of V_o for the spectrum presented in Fig. 2.24. This result is a consequence of the analogy expressed by the first two terms in Eq. 2.17. The same is also true of the coordinates of U_o in Figs. 2.31a through 3.32 and the coordinates of V_o of the corresponding spectra in Figs. 2.25a and 2.26.

(c) The coordinates of the absolute maximum pseudo-velocity, V_o, for the spectra in Figs. 2.30 through 2.32 are identical to, or approximately equal to, the coordinates of the absolute maximum pseudo-acceleration, A_o, of the corresponding spectra in Figs. 2.24 through 2.26. The approximation in this case arises from the fact that these maxima generally do not occur during free vibration, and the analogy expressed by the last two terms in Eq. 2.17 is not strictly applicable.

(d) In contrast to the spectrum in Fig. 2.30 which at high frequencies approaches a diagonal asymptote, the spectra in Figs. 2.31 and 2.32 approach a horizontal asymptote. This difference can best be explained by reference to the acceleration diagrams of the ground motion. In Fig. 2.30 the acceleration function has finite discontinuities, and the limiting value of the deformation spectrum is $A/\dot{y}_o = 3.0$, as might have been predicted by the procedure presented in Section 2.7.2b. On the other hand, the curves in Figs. 2.31 and 2.32 approach a horizontal asymptote because the <u>velocity</u> diagram of the input motion is discontinuous, i.e. the acceleration function has infinite discontinuities. If both the ground velocity and the ground acceleration were continuous, the limiting value of A would have been equal to the maximum ground acceleration.

2.11.3 Design Rules. On the basis of the information presented, it is concluded that the response spectrum for a full-cycle displacement function may be approximated by the diagram shown in Fig. 2.33. Both the displacement function and its first two derivatives are assumed to be continuous.

Along the straight line aa', the displacement U is equal to the first maximum ground displacement. Curve a'b is defined by Eq. 2.51b, and point b is the intersection of this curve and the curve represented by the right hand member of Eq. 2.52. Along the straight line cd, the displacement U is equal to 1.5 times the sum of the absolute values of the maximum ground displacements. Point i, which is equivalent to point j in Fig. 2.28, corresponds to a frequency determined from the expression

$$t_{1,d} f \cong 0.6 \tag{2.53}$$

where $t_{1,d}$ is the duration of the displacement pulse with the maximum amplitude. This point should lie to the left of point d. The velocity V along the horizontal line de may be approximated by the expression

$$V = 1.5 \sum_{j=1}^{n} |(\dot{y}_o)_j| \tag{2.54}$$

If the deformation spectra corresponding to the component velocity pulses are available, a more accurate estimate of V may be obtained with the aid of the procedure described in Section 2.8. The remaining features of this spectrum are similar to those of the spectrum given in Fig. 2.28.

2.12 Relationship of Computed Results to Field Test Data

The approximate rules presented in the preceding section are substantiated by the results of the field tests analyzed in Ref. 1. Included in this reference are response spectra for systems with a very small amount of damping (0.5 percent critical) subjected to the horizontal and vertical components of the ground motions measured in a number of field tests. These spectra are given in dimensionless plots similar to those used in the present study. The relative pseudo-velocity V is normalized with respect to the so-called "velocity jump" which may be considered to be equal to the maximum ground velocity, and the frequency scale is taken as the product of the natural frequency of the system and the "duration" of the velocity pulse. The latter quantity is taken as the time from arrival of the velocity jump to the first zero value of the ground velocity.

Three classes of spectra are distinguished in Ref. 1 depending on the direction of the motion and the distance of the point under consideration from ground zero. These correspond to:

1. Vertical motions in the superseismic region of the blast.
2. Vertical motions in the subseismic region of the blast, and
3. Horizontal motions in general.

From the discussion in Section 2.6 it follows that the spectra in Item 1 should be compared with those for half-cycle velocity pulses, and the spectra of Item 2 should be compared with those for half-cycle displacement pulses or for displacement pulses with partial recovery. Finally, the spectra in Item 3 should be compared with those for half-cycle displacement pulses and possibly for displacement pulses having both positive and negative parts. On making these comparisons, one finds that the agreement between corresponding spectra is in general very good.

The important features of the spectra given in Ref. 1 are as follows. For vertical motions in the superseismic region of the blast, the values of U are consistently smaller than the maximum ground displacement, and the maximum value of V is about 1.5 times the maximum ground velocity. It is particularly noteworthy that, in the regions of the spectrum where U or V may be considered to be constant, the results fall within a very narrow band, whereas for the high-frequency region of the diagram the "scatter" is considerable. These trends correspond almost exactly to those of the spectra for half-cycle velocity pulses presented in Section 2.9, as may readily be appreciated by referring to Figs. 2.18 or 2.22c. The "scatter" in the high frequency range emphasizes the difficulties involved in specifying precisely the value of the maximum ground acceleration. The value of $t_d f$ corresponding to the peak value of the spectra in Ref. 1 ranges between a value of 1 and 3, from which it may be inferred that the dominant velocity pulse of the ground is highly unsymmetrical.

For vertical motions in the subseismic regions of the blast and for horizontal motions in general, the maximum values of U in Ref. 1 range from 1.5 to slightly more than 2 times the maximum input displacement, the greater values corresponding to horizontal motions. The maximum values of V lie between 2 and 2.5 times the maximum ground velocity for motions in the vertical direction, and between 2 and 2.8 for horizontal motions.

2.13 Deformation Spectra for Damped Systems

In Figs. 2.34 through 2.43b are given deformation spectra for damped systems with coefficients of viscous damping between zero and 100 percent critical for the following classes of ground motion:

(a) A family of skewed versed-sine velocity pulses with rise-duration ratios of 1/2, 1/4 and 1/8 (Figs. 2.34 through 2.36)

(b) The following half-cycle displacement pulses: A pulse for which the acceleration diagram consists of a sequence of three half-sine waves of the same amplitude but different durations (Fig. 2.37), a family of displacement pulses having the shape of the velocity pulses considered under item (a) (Figs. 2.38 through 2.40), and a half-sine displacement pulse (Fig. 2.41).

(c) The following full-cycle displacement functions: A displacement function having the shape of the velocity diagram for the first motion considered under item (b) (Fig. 2.42), and two functions composed of a sequence of two half-sine waves each (Figs. 2.43a and 2.43b).

It can clearly be seen from these figures that the overall effect of damping is to reduce the magnitude of the maximum deformations, and to smooth out the humps and undulations of the spectra. It is also clear that the extent of the reduction is generally different for the different frequency regions, and that, within a given range of frequencies, it is different for the different ground motions.

For the simple pulses considered, the effectiveness of damping in reducing the magnitude of the maximum deformation can be related to:

(a) the number of oscillations that the system undergoes before attaining its maximum deformation, and

(b) the amplitude of the oscillatory component of the response.

The latter component corresponds to the solution of the homogeneous part of the governing differential equation of motion.

In general, other things being equal, the greater the number of oscillations or the amplitude of the oscillatory component, the greater is the reduction achieved with a given amount of damping.

In the low-frequency region of the spectrum, the effectiveness of damping is generally small because the maximum value of the deformation is reached at a small fraction of the natural period of the system. Since the maximum deformation in this region occurs at or near the instant that the ground displacement attains its maximum value, it follows that in comparing the spectra for the different ground motions considered, the comparisons should be made for fixed values of $t_{r,d}f$, where $t_{r,d}$ is the rise time to the peak ground displacement.

Fig. 2.36 shows that, for small values of the frequency parameter, damping has a greater effect on the maximum positive deformations than on the corresponding negative deformations. This condition arises from the fact that u_{min} corresponds to the first extremum, whereas u_{max} corresponds to the second. Similar results are indicated in Fig. 2.40 for a versed-sine displacement pulse, but in this case the reduction for u_{max} is comparatively less pronounced than in the preceding case. This difference can again be explained in terms of the times at which the respective maxima occur. For the versed-sine velocity pulse considered in Fig. 2.36, u_{max} occurs approximately at one-half the natural period of the system after the time of maximum ground displacement, whereas for the versed-sine displacement pulse considered in Fig. 2.40, it occurs at one-quarter the natural period of the system. These values were noted before in connection with undamped systems.

In the high-frequency region of the spectrum, the effect of damping depends on the amplitude of the oscillatory component of the motion, and this amplitude depends, in turn, on whether the ground acceleration is a continuous or a discontinuous function. For the input functions considered in Figs. 2.34 through 2.37, for which the ground acceleration is continuous, the effect of damping can be seen to be negligible. In contrast, for the pulses considered in Figs. 2.38 through 2.40, for which the ground acceleration is discontinuous, the maximum reductions achieved are of the order of 50 percent for values of $\beta = 1.00$. The reductions for a given amount of damping are even greater for the full-cycle displacement pulse considered in Fig. 2.42. For the pulses in Figs. 2.38 through 2.40, the absolute maximum value of the deformation corresponds to the first extremum value, and it may be approximated by the following expression, which is applicable to a step acceleration function of long duration,

$$A = \left[1 + \exp\left(-\frac{\beta}{\sqrt{1-\beta^2}}\pi\right)\right] \ddot{y}_o \qquad (2.55)$$

This equation is valid for values of β less than one. Note that its first term, which corresponds to the particular solution of the governing differential equation of motion, is independent of damping and equals the maximum input acceleration. Equation 2.55 is also applicable to the full-cycle displacement pulse considered in Fig. 2.42, provided the amount of damping in the system is sufficiently large such that the first extremum value of deformation represents the absolute maximum deformation. The results presented indicate that this condition holds true for values of β equal to or greater than about 0.05.

In the high frequency region of Fig. 2.40, it is of interest to note that, whereas for undamped and for critically damped systems, u_{max} and u_{min} are numerically equal to each other, for systems with $\beta = 0.20$, u_{min} is numerically greater than u_{max}. This result can be explained with reference to Fig. 2.10e which shows the input acceleration together with the response acceleration for a high frequency undamped system. It can be seen that the first maximum positive acceleration (corresponding to u_{min}) occurs at a very early stage of the motion, whereas the first maximum negative acceleration (corresponding to u_{max}) occurs near the middle of the pulse, after the system has executed several cycles of oscillation. Although for an undamped system these two maxima are numerically equal, for a system with damping u_{min} governs because it occurs earlier than u_{max}. For a critically damped system, the two maxima are nearly the same because the oscillatory component of the deformation is negligible in this case, and the remaining component, which is proportional to the input acceleration, has the same positive and negative parts.

For the displacement pulses considered in Figs. 2.43a and 2.43b, the maximum deformation of a high-frequency undamped system occurs mostly during free vibration, following the second discontinuity in the velocity diagram. For a system with a substantial amount of damping, however, the oscillatory component of the motion induced by the first discontinuity is generally damped out by the time the second discontinuity is applied, with the result that the maximum deformation during free vibration is no larger than that attained during forced vibration. Under these circumstances, the maximum deformation of the system can be approximated by the following expression that gives the effect of a sudden velocity change without rebound. The equation is valid for values of $\beta < 1$.

$$\frac{up}{\dot{y}_o} = \exp\left[-\frac{\beta}{\sqrt{1-\beta^2}} \tan^{-1} \frac{\sqrt{1-\beta^2}}{\beta}\right] \tag{2.56}$$

The results obtained from this equation are found to be in good agreement with the exact results for systems with values of $t_1 f$ greater than about 2 and values of β greater than about 0.10. For smaller values of β, Eq. 2.56 defines the lower envelope of the response spectra.

Excepting ground motions for which the acceleration and/or velocity diagrams are discontinuous, it can be said that the effect of damping is greatest in the medium-frequency region of the spectrum, both because the amplitude of the oscillatory component of the response is appreciable in this case and because the maximum deformation of the undamped system is usually attained near the end of the disturbance, after the system has undergone one or more cycles of oscillation. For this region, the greater the periodicity of the input motion, the greater is the reduction in the peak value of deformation achieved with a given amount of damping. Of course, the response of the elastic system increases with increasing periodicity of the input motion.

These conditions are illustrated in Figs. 2.44a through 2.44d where deformation spectra are presented for systems subjected to velocity functions composed of from one to four parabolic pulses of equal amplitude and duration, as shown in the inset diagrams. Note that, whereas the peak values of the undamped spectra increase in almost direct proportion to the number of velocity pulses in the input motion, the corresponding values of the spectra for highly damped systems remain virtually unchanged. Note, in particular, that the curves in Figs. 2.44b through 2.44d are almost identical to each other for values of $\beta \geq 0.5$. In these cases, the order of the controlling maximum is the same, and it corresponds to an early, usually the first or second, maximum. It should finally be noted that the peak values of U, V and A for elastic systems are in good agreement with the approximate rules that have been presented.

2.14 Deformation Spectra for a Combination of Simple Pulses

The information in this section is intended to illustrate the manner in which the deformation spectra for simple pulses presented in the preceding sections may be modified by the effect of high frequency oscillations that may be superimposed on the main pulses. The procedure used to arrive at this information is approximate, and the results are mainly of qualitative significance.

Let V_1 and V_2 denote the values of the pseudo-velocity corresponding to the primary and the secondary pulse of the ground motion, respectively, and let V be the corresponding value for the combined pulse. The value of V may then be determined by a procedure analogous to that presented in Section 2.8, or more simply by taking the sum of the maximum contributions of the component pulses, i.e.

$$V \leq V_1 + V_2 \tag{2.57}$$

That the value of V determined from this equation may be considerably greater than the actual value, may be appreciated by noting that Eq. 2.57 is independent of the relative position of the component pulses.

In Figs. 2.45a and 2.45b are given deformation spectra determined by application of Eq. 2.57 for a combination of two versed-sine velocity pulses, as shown in the inset diagrams. The quantities a_1 and a_2 in these figures denote the maximum accelerations of the primary and the secondary pulse, respectively, and v_1 and v_2 denote the corresponding maximum velocities. The total duration of the primary pulse is denoted by t_d, and that of the secondary pulse by t_2. In these figures, both the frequency parameter used as abscissa and the response quantities are normalized with respect to the relevant dimensions of the primary pulse.

Figure 2.44a illustrates the method of computation for a combination of versed-since pulses with $t_2/t_d = 0.1$ and $v_2/v_1 = 0.5$. The ratio of the maximum accelerations a_2/a_1 is given by the equation

$$\frac{a_2}{a_1} = \frac{v_2}{v_1} \frac{t_d}{t_2}$$

and corresponds to a value of 5. The dashed line curve on the left shows the spectrum V_1 for the primary pulse. The corresponding curve on the right is the same as V_1 but displaced along the frequency axis by the amount t_d/t_2. The solid line, representing the spectrum for the combined pulse, is obtained by adding to the ordinates of the V_1 curve the ordinates of the displaced curve multiplied by the ratio v_2/v_1.

The spectra in Fig. 2.45b are for pulses with different combinations of t_2/t_d and v_2/v_1, as shown in the figure. It can clearly be seen from these plots that the secondary pulse has practically no effect on the low-frequency region of the spectrum. This result might have been anticipated from the material presented previously, since the maximum displacement of the ground, which controls the maximum deformation of low-frequency systems, it practically unaffected by the high-amplitude high-frequency acceleration pulses considered in these examples.

On the other hand, the high-frequency region of the spectrum is influenced to a very significant degree by the superimposed oscillation, since the magnitude of the maximum input acceleration, which controls the response in this case, is increased significantly. In each case, the limiting value of A becomes equal to the maximum possible value of the acceleration of the combined pulse, $a_1 + a_2$.

In the intermediate range of frequencies, the spectrum for the primary pulse is modified in two significant respects: it becomes wider, and a second peak appears at the frequency corresponding to the peak of the displaced curve. In addition, the peak value of the spectrum for the main pulse is increased, but this change is relatively small for the range of parameters considered. From a consideration of the manner in which the spectrum for the combined pulse is obtained, it should be clear that, other things being equal, the distance between peaks will increase with decreasing value of t_2/t_d, the level of the second peak will increase with increasing value of v_2/v_1, and the increase in the peak value of the spectrum for the primary pulse will be less significant for highly peaked spectra than for spectra having a flat top.

The trend referred to in the last statement can be seen in Fig. 2.46 which includes deformation spectra for a combination of two full-cycle sinusoidal velocity pulses. Note that, for the same values of the parameters, the percentage increase in the peak value of the spectrum for the primary pulse is smaller in this figure than for the spectrum given in Fig. 2.45b. The one additional difference between the sets of curves given in the two figures is that in Fig. 2.46 the limiting value of A at high frequencies is $2(a_1 + a_2)$ instead of $a_1 + a_2$. This difference is a consequence of the discontinuities in the input acceleration function.

If the secondary component of the input function has several pulses of nearly equal amplitudes but different durations, the spectrum for the combined motion would exhibit several peaks corresponding to the peaks of the component spectra.

Finally, the general shape of the curves presented in Figs. 2.45b and 2.46 emphasize that the middle region of the spectrum for a combination of pulses cannot be determined on the basis of the maximum values alone of the input velocity and acceleration functions, but that proper regard should also be given to the detailed features of these functions. In particular, the results obtained may be quite conservative if the middle region of the spectrum is approximated by a horizontal line and a diagonal line of constant value of A, as was recommended for the case of simple pulses. However, such errors are likely to be important only when a_2/a_1 is large and v_2/v_1 is small, that is when the secondary acceleration pulses are of very high amplitude and short duration.

2.15 Deformation Spectra for Systems Subjected to Earthquake Motions

2.15.1 General. It is shown in this section that the significant features of the deformation spectra corresponding to ground motions even of the complexity of those induced by strong motion earthquakes can be estimated with reasonable accuracy from the information for simple pulses that has been presented in the preceding sections. To accomplish this, the acceleration, velocity and displacement of the ground must be known as a function of time.

The left-hand portion of the spectrum may then be estimated from the characteristics of the displacement function, the middle portion may be estimated from the characteristics of the velocity function, and the right-hand portion may be estimated from the characteristics of the acceleration function.

The input motions considered in this study include the nearly north-south component of the ground motion recorded during the Eureka, California earthquake of 21 December 1954, and the north-south component of the record obtained during the El Centro, California earthquake of May 18, 1940. The time histories of the acceleration, velocity, and displacement for these motions are shown in Figs. 2.47 and 2.48. The maximum absolute value of an input function will be identified with the subscript o, and the subscript o,p will be used for maximum value of the dominant wave in the primary component of that function. The recorded accelerograms were approximated by a series of straight line segments, and the velocity and displacement histories were determined by numerical integration. The base line of the accelerograms was adjusted so that the resulting velocity diagram oscillated about the zero line, and certain minor adjustments were made at the beginning of the records to account for uncertainties regarding the time of initiation of the shock.

2.15.2 Presentation of Data. The deformation spectra corresponding to these records are given in Figs. 2.49 and 2.51 for systems with coefficients of damping in the range between zero and 40 percent critical. In addition, the times of occurrence of these maxima, t_o, are plotted in Figs. 2.50 and 2.52 as a function of frequency. The data used to prepare these plots are tabulated in Appendix A along with the maximum values of the quantities $\dot{u}$, $\ddot{U}$, x, $\dot{x}$ and $\ddot{x}$, and their associated times of occurrence.

Each of the curves was established with 22 data points. For the El Centro record, some additional solutions were obtained for systems with

$\beta = 0.02$, to evaluate the detailed features of the spectra. These results are presented in Fig. 2.53, in which the solutions used earlier are represented by open circles and the additional solutions are shown in solid circles. It can be seen that the data points corresponding to the coarse frequency interval define with reasonable accuracy the salient features of the actual spectra. The accuracy should be still better for systems having more than 2 percent critical damping, since the irregularities of the spectra generally decrease with increasing damping.

There are striking similarities between the spectra presented in these figures and many of the spectra for simple pulses presented earlier. Specifically, at low frequencies, the deformation U approaches the maximum ground displacement, y_o; at high frequencies, the acceleration A approaches the maximum value of the ground acceleration, $\ddot{y}_o$; at the intermediate frequency range, the pseudo-velocity is nearly constant; and the maximum values of U and A occur to the left and to the right of this intermediate nearly flat region. (The magnitudes of these maxima will be considered later.) Furthermore, as would be expected from the data and the discussion presented earlier for simple pulse-like inputs with continuous acceleration diagrams, the effect of damping is most pronounced in the intermediate frequency range and is practically negligible at very low and at very high frequencies.

The one major difference between the results presented in this section and those given earlier concerns the reduction in the value of the maximum response obtained with 2 percent critical damping. Whereas the reduction achieved is practically negligible for the simple pulses, for the earthquake motions it is quite significant, particularly in the region of the spectrum where A attains its maximum value. This difference is due to the secondary, high frequency component of the earthquake motion, which, because of its nearly periodic character, produces an almost resonant condition. The effectiveness of damping under such a condition is known to be great and to increase with increasing duration of the excitation. The resulting reduction in response would be expected to be particularly pronounced in the case of the El Centro record which is of longer duration and for which the high frequency components have greater amplitudes and occupy a greater portion of the record than for the Eureka record. This prediction is substantiated by the curves presented in Fig. 2.50 and 2.52 which show that, for systems without damping, the maximum deformation occurs near the end of the record, while for systems with as little as 2 percent critical damping it occurs at a much earlier time. For example, for the El Centro record, the values of A and the associated times of occurrence for systems with f = 20 cps are as follows:

β	t_o sec.	$\frac{A}{\ddot{y}_o}$
0	24.5	2.73
0.02	9.6	1.29
0.05	9.6	1.11
0.1	2.1	1.03
0.2	2.1	1.03
0.4	2.1	1.02

Because of this difference in the response of completely undamped systems to the two forms of excitation, and in view of the fact that all physical systems have some amount of damping, the earthquake spectra for β = 0.02 will be used as a basis of comparison, and, unless otherwise noted, they will be considered to be comparable to the undamped spectra for simple pulses.

In the computation of the effects of the earthquake motions, if only the portion of the record between the origin and time t_o had been considered, the computed value of the maximum response would obviously have been the same.

It is important to note that, even for systems having as little as 2 percent critical damping, the portion of the record which controls the maximum deformation is a small fraction of the total duration of the record.

2.15.3 Relationship Between Characteristics of Input Motions and Response Spectra. For the ground record corresponding to the Eureka quake, it can be clearly seen from Fig. 2.47 that the most significant portion of the motion extends from about 2.5 to 7 seconds, and this portion may be expected to control the response of systems with damping. That this is indeed the case can be seen from Fig. 2.50 which shows that, with minor exceptions, the maximum deformation of systems with as little as 2 percent critical damping generally occurs at less than 7 seconds.

The dominant portions of the velocity and displacement diagrams for the Eureka shock are reproduced in Fig. 2.54a in solid lines. Superimposed on these as dashed line curves are what are considered to be the primary components of the motions.

In the displacement trace, the dominant wave is a skewed half-sine pulse with an amplitude, $y_{o,p}$, slightly less than the maximum ground displacement, y_o, and a duration of about 3.3 seconds, as shown in the figure. On the basis of this information, the left-hand region of the spectrum would be expected to be similar to the corresponding regions of the spectra shown in Fig. 2.26, with the value of U being equal to the maximum ground displacement for frequencies determined from the equation

$$t_d f < \frac{1}{2\pi}\frac{y_o}{y_{av}} \qquad \text{(see Eq. 2.46)}$$

and with the frequency corresponding to the maximum value of U determined from Eq. 2.48x. These frequencies are

$$f < \frac{1}{3.3}\,\frac{1}{2\pi}\,\frac{\pi}{2} = 0.076 \text{ cps}$$

and

$$f = \frac{1}{3.3}\,(0.4)\,\frac{\pi}{2} = 0.19 \text{ cps}$$

respectively, and agree well with the actual data given in Fig. 2.49.

For $\beta = 0.02$, the absolute value of U is $U_o = 1.32\ y_o$, which considering that $y_{o,p} \doteq 0.9\ y_o$, becomes

$$U_o = 1.5\ y_{o,p}$$

This value coincides with the value obtained from the design spectrum presented in Fig. 2.28.

The primary component of the velocity trace is a full-cycle pulse with an amplitude of about $0.7\ \dot{y}_o$ and an average duration of about 1.8 secs. for each half-cycle. The total duration of the three major pulses in the superimposed secondary component is about 1.8 secs., as shown in the figure. The middle region of the spectrum would therefore be expected to exhibit two major peaks. The one corresponding to the primary pulse would be expected approximately at a frequency determined from Eq. 2.49, or at

$$f \doteq 0.6/1.8 = 0.33 \text{ cps},$$

and the second peak would be expected at a frequency

$$f \doteq 0.6/0.6 = 1 \text{ cps}$$

These results are also in good agreement with the data given in Fig. 2.49, where it is worth noting that the absolute maximum value of V for $\beta = 0.02$ occurs at 0.08 cps, and not at the frequency for which the curve for $\beta = 0$ attains its maximum value.

The magnitude of the maximum amplification factor for V in the middle region of the spectrum is somewhat smaller than the value of 3 which one might be temped to assume on the basis of the full-cycle velocity pulse that dominates the ground motion. This apparent discrepancy is due to the fact that V has been normalized with respect to $\dot{y}_o$ instead of the maximum value of the primary component of the velocity, $\dot{y}_{o,p} \doteq 0.7\ \dot{y}_o$.

Because of the nearly erratic character of the ground acceleration diagram, the magnitude of the maximum value of A cannot be estimated reliably. However, the significant features of the high-frequency portion of the spectrum, including the location of the maximum value of A, can still be related to the dominant features of the input acceleration.

For example, considering that the average duration of the most intense pulses in the acceleration trace of the motion is of the order of 0.3 secs., the peak value of A would be estimated to occur at a frequency

$$f \doteq 0.6/0.3 = 2 \text{ cps.}$$

Furthermore, since the rise time for the pulse corresponding to the maximum input acceleration is less than one-half its duration, the frequency beyond which A may be considered to be equal to the maximum ground acceleration is estimated from Eq. 2.30 to be greater than

$$f \doteq 1.25/0.15 = 8.3 \text{ cps} .$$

These results are again in good agreement with the actual data for systems with $\beta = 0.02$.

Referring now to the El Centro earthquake records given in Fig. 2.48, one observes that the most intense waves which can be expected to control the response are concentrated in the first 6 seconds of the acceleration and velocity records, and in the first 10 seconds of the displacement record. The primary components of the waves in the early portions of the velocity and displacement records are shown in dashed lines in Fig. 2.54b.

The dominant displacement wave is approximately a half-sine pulse with an amplitude nearly equal to the maximum ground displacement and an effective duration of about 6.1 seconds. Superimposed on this, there is a secondary full-cycle wave of smaller amplitude and duration of about 2.2 sec., as shown in the diagram.

Considering only the contribution of the primary wave, the value of U would be expected to be equal to the maximum ground displacement for a range of frequencies determined from Eq. 2.46, i.e.,

$$f < \frac{1}{6.1} \frac{1}{2\pi} \frac{\pi}{2} = 0.04 \text{ cps}$$

and the maximum value of U would be expected to occur (see Eq. 2.48x) at

$$f \simeq \frac{1}{6.1} (0.4) \frac{\pi}{2} = 0.10 \text{ cps}$$

In addition, a second maximum, corresponding to the effect of the superimposed full-cycle wave and of the wave preceding the primary pulse would be expected roughly at the average frequency of these waves, or at

$$f \simeq 1/2.65 = 0.38 \text{ cps}$$

Excepting the fact that the computed value of U at $f = 0.04$ cps is 33 percent greater than the estimated value of y_o, these results are in excellent agreement with those given in Fig. 2.53.

Of the two peak values of U, the one corresponding to the lower frequency has a magnitude of 2.06 y_o, as shown in Fig. 2.53. The difference between this value and the value of 1.7 y_o reported earlier for a half-sine displacement pulse is due mainly to the neglected effects of the secondary wave and of the wave preceding the major pulse, both of which tend to increase the response. The fact that the maximum deformation occurs near the end of the record suggests further than the contribution of the waves following the main pulse is not entirely negligible, although it is expected to be quite small. The maximum possible contribution of the wave preceding the main pulse may be considered to be approximately equal to the amplitude of the residual oscillation induced by a full-cycle sinusoidal wave with a duration of 3.1 sec. and an amplitude of 3 inches. For a system with a natural frequency $f = 0.10$ cps, this amplitude is determined from Eq. 2.52 as 1.8 in., or $0.22\ y_o$.

The most significant part of the primary component of the ground velocity is shown approximately by the dashed line in the upper diagram of Fig. 2.54b. It consists of a sequence of five half-cycle waves the amplitudes and durations of which are as indicated. The amplitudes of the major waves in the nearly periodic, secondary component are from about 0.5 to 1.0 times the peak amplitude of the primary components and their average period is about 0.7 seconds.

The middle region of the deformation spectrum would, therefore, be expected to have the general appearance of the dashed-dotted line curve shown in Fig. 2.46 with the exception that the two peaks of this curve should be closer to each other. The peak associated with the primary waves would be expected to occur at a frequency determined from Eq. 2.49, with the quantity $t_{1,a}$ taken as the duration of the third wave which, because of its shape and amplitude, is believed to be the dominant one. This frequency is

$$f \simeq 0.6/1.5 = 0.4 \text{ cps} . \qquad (2.58)$$

Because of the nearly periodic character of the waves in the secondary component of the velocity diagram, the second peak would be expected at a frequency close to the average frequency of these waves, or at

$$f \simeq 1/0.7 = 1.4 \text{ cps} .$$

These results are in good agreement with the exact values shown in Fig. 2.53.

The magnitude of the first peak value of V would be estimated from the expression

$$V \simeq 1.5(25.1) + 0.7\ (10.6) = 45 \text{ in/sec.} \qquad (2.59)$$

where the first term on the right-hand member gives the contribution of the first four waves in accordance with Eq. 2.54, and the second term represents a liberal estimate of the contribution of the fifth wave. The amplification factor of 1.5 is not appropriate for the latter wave, because its duration is only 0.6/1.5 times that of the most dominant pulse. The factor 0.7 was determined from the spectrum for a versed-sine velocity pulse given in Fig. 2.17a by taking the ordinate of the curve at a value of $t_d f = 0.6(0.4) = 0.24$, where 0.4 cps represents the frequency determined in Eq. 2.58.

The value of V given in Eq. 2.59 may be expected to represent an upper bound to the effect of the primary velocity component for systems with $\beta = 0.02$. Because of the nearly periodic nature of the input velocity function, the possible reduction due to 2 percent critical damping, although small, is not entirely negligible in this case. On the other hand, this reduction will be partially compensated by the increase due to the effect of the secondary wave. Accordingly, the estimated value should be directly comparable to the first maximum value of V in Fig. 2.53, which is

$$V = 3.30(13.7) = 45.2 \text{ in/sec.}$$

Referring now to the ground acceleration diagram presented in Fig. 2.48 it is noted that the most intense pulses are concentrated in the region between two and three seconds and that there are four half-cycle pulses of nearly equal amplitude and an average duration of about 0.15 sec. each. This information suggests that the peak value of A would be controlled by this portion of the diagram and that it will occur approximately at a frequency of about 0.6/0.15 = 4 seconds. Furthermore, the presence of several high intensity acceleration pulses of both shorter and greater durations suggests that the value of A would be close to its maximum value for a fairly wide range of natural frequencies. These trends are substantiated by the actual data presented in Figs. 2.51 and 2.53. That the response of a system having a natural frequency of the order of 4 cps is indeed controlled by the high intensity portion of the ground acceleration diagram can clearly be seen from the dashed line curve in Fig. 2.53, which shows that the majority of the data points in the frequency range between 2 and 5 seconds correspond to a value of $t_o \doteq 2.7$ seconds.

Concerning the magnitude of the peak value of A, it can only be noted that the computed value for an undamped system is about $9.3\ \ddot{y}_o$, for a system with $\beta = 0.02$ it is $4.3\ \ddot{y}_o$, and for a system with $\beta = 0.40$ it is almost equal to $\ddot{y}_o$. It is particularly noteworthy that the entire right-hand portion of the spectrum for $\beta = 0.40$ is represented almost exactly by the diagonal line $A = \ddot{y}_o$. This is also true of the corresponding spectrum for the Eureka quake presented in Fig. 2.49.

Finally, noting that the shortest rise time for the four most intense pulses in the acceleration diagram of the ground is of the order of 0.05 seconds, it is concluded that, for damped systems, the response acceleration A should be of the order of $\ddot{y}_o$ for values of f greater than 20 cps. The fact that the value of A for f = 10 cps and $\beta = 0.02$ is almost twice as great as the maximum ground acceleration should not be surprising, therefore.

It should perhaps be emphasized that what has been referred to as "predicted" or "estimated" data was arrived at after the response spectra were evaluated. However, the degree of agreement achieved and the straightforwardness of the procedure used to arrive at these results illustrate clearly the intimate relationship that exists between the response spectra for simple pulses and those for complex earthquake motions, and should leave but little doubt about the possibility of determining these spectra with reasonable accuracy from the gross characteristics of the acceleration, velocity and displacement records of the motion.

In Figs. 2.55 and 2.56 are given response spectra for the maximum positive and the maximum negative deformations of systems with $\beta = 0.02$ and $\beta = 0.40$ for the two earthquake motions considered. In general, the two sets of curves are in good agreement between each other. The agreement is better in the case of the El Centro motion because the positive and negative parts of the acceleration and velocity records of this motion are more nearly balanced about the zero line than are those of the records for the Eureka earthquake.

2.16 Spectra for Other Response Quantities

2.16.1 Spectra for Relative Velocity. For the class of ground motions considered in this study, it has been shown in Section 2.4 that the relative velocity $\dot{u}$ due to an input velocity function $\dot{y}_2(t)$ is the same as the deformation u produced by a displacement function $y_1(t)$ of the same shape. Each response quantity is considered to be normalized with respect to the maximum value of the corresponding input function.

It is desirable to plot the spectral values of $\dot{u}$ on a four-way logarithmic plot similar to that used earlier, with the vertical and the diagonal scales representing the quantities $p\dot{U}/\dot{y}_o$, $\dot{U}/\dot{y}_o$ and $p^2\dot{U}/\dddot{y}_o$, as shown in part (b) of Fig. 2.57. The resulting spectrum for a prescribed velocity function will then be identical to the deformation spectrum corresponding to a displacement function of the same shape. The spectra for U can, therefore, be constructed approximately by application of the design rules presented earlier.

In particular, the $\dot{U}$ spectrum for a half-cycle acceleration pulse may be approximated by the diagram given in Fig. 2.23, and the corresponding spectra for a half-cycle velocity pulse and a half-cycle displacement pulse (i.e. full cycle velocity pulse) may be approximated by the diagrams given in Figs. 2.28 and 2.33, respectively.

In Figs. 2.58a and 2.58b the relative velocity spectra for the earthquake records are plotted in the form described above for systems with coefficients of damping between zero and 40 percent critical. The right-hand diagonal scale is not shown because the values of $\dddot{y}_o$ for the input motions, are not known. The spectra in Figs. 2.58a and 2.58b can also be interpreted as the deformation spectra for ground motions the velocity diagrams of which have the same shape as the acceleration diagrams of the Eureka and the El Centro earthquakes, respectively. It is of interest to note in passing that the effect of damping in the high-frequency regions of these spectra is considerably more pronounced than for the corresponding deformation spectra given in Figs. 2.49 and Fig. 2.51. This difference is due to the discontinuous nature of the $\dddot{y}$ diagrams and might have been anticipated from the data given earlier.

2.16.2 <u>Comparison of Pseudo-Velocity and True Relative Velocity</u>. In the field of earthquake engineering, the relative velocity $\dot{U}$ has sometimes used in lieu of the pseudo-velocity V. The degree of approximation involved in replacing one quantity by the other has been investigated recently by Hudson (Ref. 9) for three earthquake motions. The minimum value of natural frequency considered in this study was about 0.27 cps, and the maximum value ranged from about 5 cps to 12 cps for the three records. In general, the values of V and $\dot{U}$ were found to be in close agreement between each other, but in some cases the differences were of the order of 40 percent for systems with $\beta = 0.20$.

This problem was also investigated in the present study considering a wider range of natural frequencies than that used before. Both pulse-like excitations and earthquake motions were considered. Fig. 2.59 shows the results obtained for an undamped system subjected to a skewed versed-sine velocity pulse with $t_1/t_d = 1/4$. For values of $t_d f$ between 0.3 and 0.9 the two quantities are identical because they both attain their maximum values during free vibration when the system executes a simple harmonic motion and, consequently, $\dot{U} = pU = V$. In the low frequency region of the spectrum V is smaller than $\dot{U}$. As $t_d f \rightarrow 0$, the deformation $U \rightarrow y_o$, the relative velocity $\dot{U} \rightarrow \dot{y}_o$, but the pseudo-velocity $V = pU \rightarrow 0$ by virtue of the fact that $p \rightarrow 0$. In the high frequency region of the spectrum, V is greater than $\dot{U}$, the difference between the two quantities increasing with increasing frequency.

Similar plots are given in Figs. 2.60 for systems with 2 percent critical damping subjected to the Eureka and El Centro earthquake motions. The striking similarities between these plots and those presented in Fig. 2.59 are further evidence of the intimate relationship that exists between the spectra for pulse-like excitations and earthquake motions. The agreement between the two quantities at high frequencies is better for the El Centro motion because the dominant waves for this motion are of shorter duration, t_1, than for the Eureka record, with the result that the frequency parameter $t_1 f$ is comparatively closer to the region where U and V may be considered to be the same.

The effect of damping on the relationship between V and U is illustrated in Figs. 2.61a and 2.61b which should be self-explanatory.

2.16.3 <u>Spectra for Absolute Acceleration</u>. The absolute acceleration $\ddot{X}$ for a system without damping is equal to the pseudo-acceleration A. Accordingly, it may be determined directly from the right-hand diagonal scale of the deformation spectrum.

As a measure of the error involved in taking $\ddot{X} = A$ when $\beta \neq 0$, in Fig. 2.62a the response spectra for these two quantities are compared for systems with $\beta = 0.20$ and $\beta = 1.00$, and in Fig. 2.62b the ratio $A/\ddot{X}$ is plotted as a function of frequency for different values of damping. The ground motion in this comparison is a versed-sine velocity pulse. In Fig. 2.63 are given similar results for systems subjected to the ground motions of the Eureka and the El Centro earthquakes. As before, the salient features of the curves for the simple pulses and the earthquake motions are the same. Even for large amounts of damping, the quantities $\ddot{X}$ and A are very nearly the same at high frequencies. However, in the low frequency region, the differences between the two quantities are appreciable for large values of β.

2.16.4 Spectra for Absolute Velocity and Absolute Displacement. It is convenient to plot the spectra for these quantities on the four-way logarithmic grid used previously, with the diagonal and vertical scales normalized as shown in parts (c) and (d) of Fig. 2.57. In this figure the quantities $\left[\int y(\tau)d\tau\right]_o$ and $\left[\int(\int y(\tau)d\tau)dt\right]_o$ represent, respectively, the maximum values of the first and the second integrals of the ground displacement function.

By virtue of Eq. 2.11, the spectrum of $\dot{X}$ corresponding to a ground velocity function can also be interpreted as the acceleration spectrum for an input acceleration having the shape of the prescribed velocity function. Similarly, the spectrum of X for a given displacement function can be interpreted as that of $\ddot{X}$ due to an input acceleration of the shape of the prescribed displacement function. Recalling now that, for lightly damped systems, $\ddot{X}$ may be replaced by the pseudo-acceleration A, it is concluded that the velocity spectrum $\dot{X}$ for a prescribed velocity disturbance may be considered to be the same as the deformation spectrum for an acceleration disturbance of the same shape. A similar statement can also be made for the displacement spectrum X.

The deformation spectra can, of course, be approximated by application of the appropriate design rules presented in this report. It must be emphasized that this approach is valid only for lightly damped systems.

As an illustration, in Fig. 2.64 are given velocity spectra, plotted in the form described above, for systems subjected to the full-cycle velocity pulse considered previously in Fig. 2.37. As would be expected from the preceding discussion, the spectrum for $\beta = 0$ is similar to the deformation spectrum corresponding to a full-cycle acceleration pulse (half-cycle displacement pulse), such as that considered in Fig. 2.34. It is important to note that the trends of corresponding curves in Figs. 2.34 and 2.64, while similar to each other for medium-frequency and high-frequency systems, differ significantly for low-frequency systems with values of β on the order of 0.2 or more. These differences are analogous to those between the quantities A and $\ddot{X}$ considered in Figs. 2.62.

In Figs. 2.65a and 2.65b are given similar spectra for the Eureka and El Centro Earthquake records. For values of β less than about 0.10, these spectra can also be interpreted as deformation spectra for ground motions the acceleration diagrams of which have the shapes of the velocity diagrams of the earthquake motions considered.

SECTION 3

RESPONSE OF INELASTIC SYSTEMS

3.1 General

This chapter is concerned with the response of inelastic systems having a single degree of freedom. Primary attention is given to elastoplastic systems and, in an exploratory way, to bilinear systems of the softening type. Only the maximum deformations of the systems are investigated.

Figure 3.1a shows the resistance-deformation relationship for a bilinear system. The symbols k_1 and k_2 denote the slopes of the first and second portions of the diagram as indicated. For a bilinear system of the softening type, $k_2 < k_1$, and for an elastoplastic system, $k_2 = 0$. The yield levels in the two directions of deformation are considered to be the same, and unloading from a point of maximum deformation is assumed to take place along a line parallel to the initial elastic portion of the curve. A typical cycle of loading, unloading and reloading is shown in the figure. The yield point deformation is denoted by u_y, and the absolute maximum deformation, without regards to sign, is denoted by u_m. In an analogous manner, the yield point resistance is designated by Q_y, and the maximum spring force by Q_m. For an elastoplastic system, the force Q_m is, of course, equal to Q_y for deformations in excess of the yield point deformation.

The ground motions considered include five pulse-like excitations and the two earthquake records used in the study of elastic systems. In addition, the effects of certain limiting forms of excitation are studied. The acceleration, velocity and displacement diagrams for the simple pulses are shown in Fig. 3.2. The acceleration diagrams consist of straight line segments and, except for one pulse, they are discontinuous at the beginning and the end of the diagram. The velocity diagrams have from one to four parabolic half-cycles of oscillation. The initial values of the velocity and displacement diagrams are zero in all cases.

3.2 Definitions and Fundamental Relations

It is convenient and instructive to relate the maximum response of the inelastic system to that of an elastic system having the same stiffness as the initial stiffness of the inelastic system. Let u_o be the absolute* value of the maximum deformation of the associated elastic system, and Q_o be the corresponding spring force, as shown in Fig. 3.1b. The yield resistance of the inelastic system, Q_y, may then be expressed as a fraction of the resistance Q_o required for elastic behavior. The ratio Q_y/Q_o, which is also equal to u_y/u_o, will be referred to as the reduction factor and will be denoted by the symbol c. That is,

$$c = \frac{Q_y}{Q_o} = \frac{u_y}{u_o} \tag{3.1}$$

For an elastic system, the quantities Q_y and u_y may be considered to be equal to Q_o and u_o, respectively. Accordingly, the reduction factor is equal to unity in this case. For a system that deforms in the inelastic range, c is evidently smaller than unity.

The reciprocal of the reduction factor, $1/c$, expresses the intensity of the ground motion in terms of that which the system can withstand elastically, and will be referred to as the overload factor.

The maximum deformation of the inelastic system, u_m, can conveniently be expressed in terms of its yield point deformation, u_y. The dimensionless ratio

$$\mu = \frac{u_m}{u_y} \tag{3.2}$$

*This notation is not consistent with that used in Section 2, where the subscript o referred to the maximum value of the quantity taken with its appropriate sign.

will be referred to as the ductility factor. With this notation, the maximum inelastic deformation of the system is $(\mu - 1)u_y$.

With the values of c and μ known, the ratios u_m/u_o and Q_m/Q_o can be determined from the following equations

$$\frac{u_m}{u_o} = \mu c \tag{3.3}$$

and

$$\frac{Q_m}{Q_o} = \left[1 + \frac{k_2}{k_1}(\mu - 1)\right] c \tag{3.4}$$

Equation 3.3 follows directly from Eqs. 3.1 and 3.2, whereas Eq. 3.4 can readily be derived by reference to Fig. 3.1b.

For a system without damping, the spring force is proportional to the acceleration of the mass, and consequently

$$\frac{Q_m}{Q_o} = \frac{\ddot{x}_m}{\ddot{x}_o} . \tag{3.5}$$

The symbols $\ddot{x}_m$ and $\ddot{x}_o$ denote the absolute maximum accelerations of the inelastic and the elastic systems, respectively.

3.3 Response to Limiting Forms of Ground Excitation

With a view of establishing certain guide lines for the interpretation of the results to be presented later, we consider first the relationships between the maximum deformation of the elastoplastic system and the associated elastic system for certain limiting forms of ground excitation. These include

(a) an instantaneous displacement change,

(b) an instantaneous velocity change, and

(c) an instantaneous acceleration change.

The system is presumed to be undamped and initially at rest.

3.3.1 Instantaneous Displacement Change. For a system subjected to an instantaneous displacement change, $-y_o$, the "initial" value of the resulting deformation will be y_o, irrespective of whether the system behaves elastically or deforms in the plastic range. Furthermore, since there is no additional energy imparted to the system after the displacement change has taken place, the extremum values of deformation for the ensuing motion will be numerically equal to or less than y_o, and the initial deformation will be the absolute maximum deformation. In other words,

$$u_m = u_o = y_o$$

and the reduction factor for the inelastic system, determined from Eq. 3.3, is

$$c = \frac{1}{\mu} \tag{3.6}$$

Note that this expression is independent of the ratio k_2/k_1.

As an illustration, Fig. 3.1c shows the resistance-deformation diagram of an elastoplastic system with $u_y < y_o$. The abscissas of points b and c define, respectively, the deformations of the inelastic and the associated elastic systems immediately after the initial displacement change. The extremum values of deformation for the ensuing motion will correspond to points c and c' of this figure for the elastic system, and to points b and b' for the elastoplastic system.

Although the initial deformations for the inelastic and elastic systems are the same under the conditions assumed, the energies imparted to these two systems are different. The energy imparted to an elastic system is

$$E_o = \frac{1}{2} Q_o u_o, \tag{3.7a}$$

and that imparted to an inelastic system is given by the equation

$$E_m = Q_y(u_m - \frac{1}{2} u_y) + \frac{1}{2} k_2(u_m - u_y)^2 .$$

The latter equation can also be written in the form

$$E_m = \frac{1}{2} Q_y u_y \left[(2\mu - 1) + \frac{k_2}{k_1} (\mu - 1)^2\right] \tag{3.7b}$$

Utilizing the fact that when $u_m = u_o$, $Q_y/Q_o = u_y/u_o = 1/\mu$, one obtains the following expression for the ratio of the two energies.

$$\frac{E_m}{E_o} = \frac{(2\mu - 1) + \frac{k_2}{k_1} (\mu - 1)^2}{\mu^2} \; . \tag{3.8}$$

It may be noted in passing that for an elastoplastic system with $\mu = 5$ the ratio $E_m/E_o = 0.36$.

3.3.2 Instantaneous Velocity Change. The energy imparted to a system by an instantaneous velocity change, v_o, is $\frac{1}{2} mv_o^2$, irrespective of whether the system remains elastic or not. Consequently, the energies absorbed by an elastic and an inelastic system up to the point of their respective maximum deformation will also be the same. This equality is expressed by the equation

$$\frac{1}{2} k_1 u_o^2 = \frac{1}{2} Q_y u_y \left[(2\mu - 1) + \frac{k_2}{k_1} (\mu - 1)^2\right] \tag{3.9}$$

whence

$$u_o^2 = u_y^2 \left[(2\mu - 1) + \frac{k_2}{k_1} (\mu - 1)^2\right]$$

and the reduction factor becomes

$$c = \frac{1}{\sqrt{(2\mu - 1) + \frac{k_2}{k_1} (\mu - 1)^2}} \tag{3.10}$$

Consider now an elastic and an inelastic system subjected to a prescribed motion of arbitrary shape, but assume that the conditions are such that (a) the absolute maximum deformations of both systems occur during free vibration and (b) no yielding occurs during forced vibration.

By virtue of the second restriction, the energy of the two systems at the beginning of free vibration will be the same, and, from the material just presented, it follows that the maximum deformations of these systems will also be governed by Eq. 3.9, with the reduction factor c given by Eq. 3.10.

3.3.3 Instantaneous Acceleration Change. The effect of a ground acceleration, $\ddot{y}(t)$, can most conveniently be analyzed by considering the equivalent problem of a force $-m\ddot{y}(t)$ applied to a fixed-base structure.

For an acceleration step of infinite duration, $\ddot{y}_o$, the work performed by the external force up to the point of maximum deformation is $[-m\, \ddot{y}_o u_m]$, and the energy absorbed by the structure is given by the right-hand member of Eq. 3.9. By equating these two quantities, one obtains

$$-\frac{p^2 u_y}{\ddot{y}_o} = \frac{2\mu}{(2\mu - 1) + \frac{k_2}{k_1}(\mu - 1)^2} \tag{3.11a}$$

where $p = \sqrt{k_1/m}$. For an elastic system, $u_y = u_o$ and $\mu = 1$, and Eq. 3.11a reduces to

$$-\frac{p^2 u_o}{\ddot{y}_o} = 2 \tag{3.11b}$$

The reduction factor c is obtained as the ratio of Eqs. 3.11a and 3.11b, yielding

$$c = \frac{\mu}{(2\mu - 1) + \frac{k_2}{k_1}(\mu - 1)^2} \tag{3.12}$$

In this case, it is instructive to consider also the ratio of the maximum forces developed in the inelastic and the associated elastic systems. From Eqs. 3.4 and 3.12, one obtains

$$\frac{Q_m}{Q_o} = \frac{\mu\left[1 + \frac{k_2}{k_1}(\mu - 1)\right]}{(2\mu - 1) + \frac{k_2}{k_1}(\mu - 1)^2} \tag{3.13}$$

This ratio is evaluated in the following table for several values of k_2/k_1 and μ.

μ	Values of Q_m/Q_o as Given by Eq. 3.13				
	$k_2/k_1 = 0$	$k_2/k_1 = 0.1$	$k_2/k_1 = 0.2$	$k_2/k_1 = 0.5$	$k_2/k_1 = 1.0$
1	1.0	1.0	1.0	1.0	1.0
2	0.67	0.71	0.75	0.94	1.0
5	0.56	0.66	0.74	0.88	1.0
10	0.53	0.70	0.80	0.92	1.0
∞	0.50	1.0	1.0	1.0	1.0

It can be seen that the possible range of variation of Q_m/Q_o is from 1.0 to 0.5.

3.3.4 Discussion. The results presented in the preceding paragraphs can be summarized as follows:

(a) For a system subjected to an instantaneous displacement change, the maximum deformations of the elastic and inelastic systems are the same.

(b) For a system subjected to an instantaneous velocity change, the energy absorbed by the system up to the point of maximum deformation for the elastic case is the same as that for the inelastic case, and the reduction factor is given by Eq. 3.10. This relationship is also valid for an arbitrary ground motion, provided the elastic and the inelastic systems both reach their absolute maximum deformation during free vibration, and the inelastic system behaves elastically during forced vibration. The latter condition requires that the yield level of the system be equal to or greater than the maximum deformation attained by the elastic system during forced vibration.

(c) For a system subjected to an instantaneous acceleration change, the maximum spring force for the inelastic system can be no less than 50 percent of that for the associated elastic system, the actual magnitude of the reduction being a function of the ratio k_2/k_1 and of the amount of inelastic deformation that can be tolerated (See Eq. 3.13).

It would be expected that the first relationship involving conservation of maximum deformations would also be applicable to systems subjected to ground displacements for which the rise time is small in comparison to the natural period of the system. The second relationship, involving conservation of energies, would be expected to apply also to quarter-cycle velocity pulses for which the rise time is small in comparison to the natural period of the system (i.e., half-cycle acceleration pulses of short duration), and, possibly, to half-cycle velocity pulses of short rise time and sufficiently long decay time such that, at the time of the first maximum deformation, the value of the ground velocity is close to its maximum value. Finally, the third relationship would also be expected to be valid for acceleration pulses with a sharp rise and long duration in comparison to the natural period of the system.

In the following table are listed the values of the reduction factor, c, and of the ratio u_m/u_o corresponding to different values of the ductility ratio for the three limiting forms of excitation investigated. The systems considered are of the elastoplastic type, i.e. $k_2/k_1 = 0$.

μ	Reduction Factor, $c = Q_y/Q_o = u_y/u_o$			Values of $u_m/u_o = \mu c$		
	Displac. Change, Eq. 3.6	Velocity Change, Eq. 3.10	Acceler. Change, Eq. 3.12	Displac. Change	Velocity Change	Acceler. Change
1	1.00	1.00	1.00	1.00	1.00	1.00
1.25	0.80	0.82	0.83	1.00	1.02	1.04
1.5	0.67	0.71	0.75	1.00	1.06	1.12
2	0.50	0.58	0.67	1.00	1.16	1.33
3	0.33	0.45	0.60	1.00	1.34	1.80
5	0.20	0.33	0.56	1.00	1.67	2.78
10	0.10	0.23	0.53	1.00	2.29	5.26

It can be seen from this table that even a relatively small amount of inelastic deformation, as represented by a value of μ on the order of 1.5, produces a significant reduction in the value of the required yield resistance and a relatively small increase in the value of the maximum deformation. Considering that for values of μ less than 2 the values of the reduction factor and of the ratio u_m/u_o for the three forms of excitation differ by less than 33 percent, it is concluded that the form of excitation is not a very significant parameter as long as the magnitude of inelastic deformation involved is small. However, for the greater values of μ, the differences between the three sets of results are quite important, especially when the effects of a displacement change and an acceleration change are compared. The results for a velocity change are intermediate between those for a displacement change and an acceleration change.

3.4 Relations Between Response of Elastic and Inelastic Systems

In Fig. 3.3 the response spectrum for the absolute maximum deformation u_m of elastoplastic systems with $c = 0.25$ is compared with the spectrum for the corresponding deformation u_o of the associated elastic systems. The inelastic spectrum is applicable to systems for which the yield level is one fourth of that required for elastic behavior, or alternatively, to ground motions that are four times as intense as those which the systems can withstand elastically. The ground motion is the parabolic velocity pulse considered earlier in Figs. 2.19 and 2.20. The systems are considered to have no damping and to be initially at rest. The natural frequency of the inelastic system is determined from the slope of the initial elastic portion of the resistance-deformation diagram.

It can be seen from this figure that the same percentage reduction in the yield level of the system (or equivalently, the same overload) has quite different effects on systems with different natural frequencies. For flexible systems, u_m and u_o are equal; for high-frequency systems, u_m is significantly greater than u_o; and for medium-frequency systems, u_m is smaller than u_o.

Figures 3.4 show the effect of progressively reducing the yield level of the elastoplastic system. The reduction factor, c, is plotted as a function of the ductility ratio, μ, for fixed values of the frequency parameter.

It is noted that for values of $t_d f \leq 0.2$, the reduction factor is represented almost exactly by the expression $1/\mu$ (Eq. 3.6); in other words, the maximum deformations of the inelastic and elastic systems are the same, irrespective of the yield level involved.

For values of $t_d f > 1$, the reduction factors are generally greater than those obtained by the relation $1/\mu$; furthermore, they are quite sensitive to variations in the value of $t_d f$, as can readily be seen from Fig. 3.4b. As $t_d f$ approaches infinity, the input function approaches an acceleration step of long duration and, as would be expected from the discussion in Section 3.3, the results approach the relation $c = \mu/(2\mu - 1)$, which is a specialized form of Eq. 3.12.

For the particular case investigated, the expression $c = 1/\sqrt{2\mu - 1}$, which defines the effect of an instantaneous velocity change, may be considered to be valid for a value of $t_d f$ of about 1.25. It can be shown that for this value of $t_d f$, the ratio $V/\dot{y}_o$ for the associated elastic system is slightly smaller than unity.

Figure 3.4c, which refers to a value of $t_d f = 0.5$, is typical of the results obtained for medium-frequency systems. In this case, the absolute maximum deformation of the elastic system, u_o, occurs during free vibration and corresponds to the second extremum value, while the first extremum occurs during forced vibration and is equal to 0.57 u_o. The curve abc in this figure refers to the first extremum, and the curve de refers to the second. For values of $0.57u_o < u_y < u_o$, yielding initiates after termination of the pulse, and, as would be anticipated from the material presented in Section 3.3.2, the equation of line ab is $c = 1/\sqrt{2\mu - 1}$.

We digress now from the discussion of Fig. 3.4c and refer to Fig. 3.5 to define the conditions under which the expression $c = 1/\sqrt{2\mu - 1}$ is applicable. The dashed curve in this figure gives the maximum deformation of an elastic system during forced vibration, and the solid curve, discontinued in the regions where it lies below the dashed curve, gives the corresponding deformation during free vibration. It follows that the equation $c = 1/\sqrt{2\mu - 1}$ is valid in the regions of the spectrum where the absolute maximum deformation occurs during free vibration, provided the yield level of the system u_y is greater than the value of deformation obtained from the dashed curve. For example, for $t_d f = 0.68$, where the difference between the ordinates of the two curves is greatest, the expression $1/\sqrt{2\mu - 1}$ will be valid for values of

$$\frac{u_y}{u_o} > \frac{0.87}{1.73} \doteq 0.5 \ .$$

From the table given on p. 3-8, it can be seen that these values of u_y/u_o correspond to values of $\mu \leq 2.5$.

Returning now to Fig. 3.4c, we observe that as u_y is decreased below $0.57\ u_o$, yielding initiates during forced vibration, and the expression $c = 1/\sqrt{2\mu - 1}$ is no longer applicable. In fact, the magnitude of the second extremum decreases sharply, as can be seen from the break of the curve abc at point b. However, for values of u_y between those corresponding to points d and f, the absolute maximum deformation still corresponds to the second extremum. Finally, as u_y approaches its limiting value of zero, the value of the maximum deformation u_m will approach the maximum displacement of the ground, y_o.

It should be noted that there can be more than one yield level corresponding to a given value of μ. Fig. 3.4c shows three yield levels corresponding to a value of $\mu = 2$. Although all three of these solutions are distinct and "stable", from a design standpoint it is desirable to consider only the one corresponding to the highest yield level. This amounts to replacing the portion of bc which curves to the left by the vertical line shown dotted.

Figures 3.6a and 3.6b show the relationship between the reduction factor and the ductility factor for elastoplastic systems subjected to parabolic velocity pulses with one and two cycles of oscillation, as shown in the inset diagrams. In each case, the frequency parameter $t_1 f = 0.50$.

For the conditions considered in Fig. 3.6a, the maximum deformation of the elastic system, u_o, corresponds to the third extremum, and, as a consequence, the resulting plot consists of three branches with two transition curves. As an illustration of the significance of the various branches, we note that, for values of $0.056\ u_o < u_y < 0.28\ u_o$, yielding initiates at the instant that the associated elastic system attains its first extremum, but that the absolute maximum deformation is obtained at the second extremum instead of at the first. Under the conditions considered in Fig. 3.6b, the peak deformation u_o corresponds to the fifth extremum, and, consequently, the graph consists of five branches with four transition curves.

It must be emphasized that the discontinuities in the plots presented in Figs. 3.4c and 3.6 for medium-frequency systems do not, in general, occur for systems with low and high frequencies, because the order of the controlling maximum in the latter cases is usually the same for the elastic and the inelastic systems.

In Figs. 3.7a through 3.8c is presented information on the response of elastoplastic systems subjected to the two earthquake motions considered in the study of elastic systems. The plots in Figs. 3.7 are analogous to those given in Fig. 3.3, and the plots in Figs. 3.8 are analogous to those given in Figs. 3.4 and 3.6. The similarities between these curves and the corresponding curves for the simple pulses are indeed most impressive.

In Fig. 3.9 the spectra of maximum deformation for elastoplastic and elastic systems subjected to the Eureka earthquake are compared with those for bilinear systems with $k_2/k_1 = 0.2$. The yield level of each inelastic system is considered to be one fourth of that required for elastic behavior. It can be seen that, in the low-frequency region of the spectrum, the maximum deformations for all systems are for all practical purposes identical. In the medium-frequency region, the differences in the results are greater but still insignificant. The major differences occur in the high-frequency region, where the results appear to be quite sensitive to the ratio of k_2/k_1. These general trends are in agreement with those obtained on the basis of the limiting forms of excitation considered in Section 3.3.

The effect of the parameter k_2/k_1 on the maximum response of the system is illustrated in Fig. 3.10. Results are presented for selected natural frequencies and yield levels. The systems are assumed to have a damping factor of 2 percent. It can be seen that the effect of k_2/k_1 is important only for high-frequency systems.

3.5 Deformation Spectra for Elastoplastic Systems

3.5.1 General. The design of an elastoplastic system involves essentially the determination of the yield strength (or yield point deformation) necessary to limit the maximum inelastic deformation of the system to a prescribed value. It is desirable, therefore, to define the deformation spectra for inelastic systems in such a manner that this information can readily be determined.

With this in mind, the relative pseudo-velocity for an elastoplastic system is defined, as in Ref. 10, by the expression

$$V = pu_y \tag{3.14}$$

where p denotes the undamped circular natural frequency of the system corresponding to the initial elastic range of its load-deformation diagram, and u_y is the yield point deformation, (not the maximum deformation). Thus, the quantity

$$\frac{1}{2} mV^2 = \frac{1}{2} ku_y^2$$

represents the maximum strain energy that the system must be capable of developing without yielding. The relative pseudo-velocity spectrum is considered to be a plot of V against frequency for fixed values of the ductility ratio, μ.

The pseudo-acceleration, A, is defined as

$$A = p^2 u_y. \tag{3.15}$$

The yield force, Q_y, may then be determined from the expression

$$Q_y = C\ W \tag{3.16}$$

where the lateral force coefficient, C, as in the case of an elastic system, is equal to the value of A expressed in units of gravity. For an elastoplastic system without damping, Eq. 3.15 is also equal to the maximum value of the acceleration of the mass. It should be noted that these definitions of V and A for the inelastic system are consistent with those used for the elastic system, since, as previously noted, the yield deformation of the elastic system may be considered to be equal to its maximum deformation.

On a logarithmic plot of V against frequency similar to that used for elastic systems, the set of diagonal lines extending in the north-east direction represents values of constant yield deformation, u_y, and the set of lines extending in the north-west direction represents values of constant pseudo-acceleration. Thus the values of u_y, V and A corresponding to a prescribed ductility ratio can be read directly. The value of the maximum deformation can then be determined from the expression $u_m = \mu\ u_y$.

3.5.2 Spectra for a Half-Cycle Acceleration Pulse. In Fig. 3.11 are given deformation spectra, as defined above, for undamped systems with ductility ratios in the range between 1 (elastic case) and 10. The ground motion is an initially peaked triangular acceleration pulse. As in previous plots, the spectral quantities are normalized with respect to the maximum value of the corresponding ground motion. Note that the ratio of the ordinates of the curves for an inelastic and elastic system, represents the reduction factor, c, for the particular condition considered.

In the low-frequency region of the spectrum, the relationship between the curves for the inelastic and the elastic systems is represented almost exactly by the expression $c = 1/\sqrt{2\mu - 1}$ obtained from Eq. 3.10. This result was anticipated in Section 3.3.4, since as $t_d f$ approaches zero, the ground velocity approaches a step function for which Eq. 3.10 applies exactly. In a similar manner, the limiting values of the pseudo-acceleration A at high frequencies are as defined by the right-hand member of Eq. 3.11a. The transition curves between these limiting values are smooth in this case, because the absolute maximum deformations for both the elastic and the inelastic systems correspond to the first extremum.

3.5.3 Spectra for Half-Cycle Velocity and Displacement Pulses. In Figs. 3.12a through 3.14b are given deformation spectra for elastoplastic systems with zero and 10 percent critical damping subjected to three different forms of ground motion, as shown in the inset diagrams.

The data used to prepare these plots were obtained on the ILLIAC, the digital computer of the University of Illinois as follows. First, the maximum deformation of the elastic system was computed for selected values of the frequency parameter. Then, the maximum response of the inelastic systems was evaluated for a range of yield levels, and the results for each value of the frequency parameter were plotted in the form presented in Figs. 3.4 and 3.6. The values of u_y corresponding to the selected ductility ratios were finally determined from these plots.

The salient features of these curves are as follows:

(a) At low frequencies, the relationship $c = 1/\mu$ is applicable to all cases considered, with the result that both u_o and u_m are equal to the maximum ground displacement, y_o. Furthermore, as the yield level of the system is decreased (or as μ increases), the expression $u_m \doteq y_o$ is valid for a wider range of natural frequencies than for the corresponding elastic system. This trend is particularly noticeable in the case of the half-cycle displacement pulses considered in Figs. 3.14. Note that the break in the left-hand portion of the curves in this figure shifts to the right with increasing values of μ.

(b) At high frequencies, the limiting values of A are essentially as defined by Eq. 3.11a* for the ground motions which have discontinuous accelerations. On the other hand, for the continuous acceleration functions, the limiting value of A may be considered to be the same for both the elastic and the inelastic systems.

(c) In the intermediate range of frequencies, the relationship between the elastic and inelastic systems is in general complex. It can broadly be said, however, that the reduction factors corresponding to a half-cycle displacement pulse are greater than those for a half-cycle velocity pulse.

<u>Design Rules.</u> For design purposes, the relationship between the deformation spectra for elastoplastic and elastic systems may be expressed approximately as follows:

(1) For the low-frequency range of the spectrum for which the maximum deformation of an elastic system may be considered to be equal to the maximum ground displacement, y_o, the maximum deformations of the inelastic and

*Equation 3.11a is strictly applicable to an undamped system only.

elastic systems are for all practical purposes the same, i.e., $u_m \approx u_o \approx y_o$. Consequently, the reduction factor is given by the expression $c \approx 1/\mu$.

(2) For the intermediate range of frequencies, the following relationships are applicable:

(a) When U_o is equal to or smaller than the maximum input displacement and V_o is of the order of 1.5 times the maximum input velocity, as is the case with half-cycle velocity pulses, the maximum deformation of the inelastic and the elastic systems may be considered to be the same up to a value of $t_d f$ slightly greater than the value corresponding to the peak of the elastic spectrum. In this case, the reduction factor is again given by the expression $c \approx 1/\mu$.

(b) On the other hand, when U_o is greater than y_o, and V_o is of the order of 2 to 3 times $\dot{y}_o$, as is the case with half-cycle displacement pulses, the maximum deformation of the inelastic system is generally <u>less</u> than that of the corresponding elastic system. In general, the greater amplification factors of U_o and V_o for the elastic spectrum, the more conservative are the results obtained by application of the relation $c = 1/\mu$. The differences are particularly noticeable for the larger values of μ. However, if the degree of conservatism implied by the use of this relationship can be tolerated, then the expression $c \approx 1/\mu$ can be considered to be valid up to a value of $t_d f$ located approximately at one-third the distance between the value of $t_d f$ corresponding to the peak of the elastic spectrum and the value of $t_d f$ beyond the peak for which the amplification factor for V is one.

(3) For the high-frequency range of the spectrum where the pseudo-acceleration A may be considered to be constant, the relationship between the inelastic and the elastic spectra may be stated in terms of the magnitude of the amplification factor for the elastic system. When the amplification factor

is of the order of two, as would be the case for an input acceleration with a discontinuity equal to the maximum input acceleration, the reduction factor may be approximated by the expression $c = \mu/(2\mu - 1)$. On the other hand, if the amplification factor of A for the elastic system is one, as would be the case for an acceleration function without any discontinuities, the reduction factor may be considered to be unity. In other words, the maximum forces for the elastoplastic and the elastic systems may be considered to be equal.

(4) For the range of frequencies between those covered under (2) and (3), the reduction factor is sensitive to changes in the value of the natural frequency. However, the spectrum curves for this range can usually be determined by drawing smooth transition curves between the curves applicable to the ranges considered in items (2) and (3). When this cannot be done readily, the equations

$$c = \frac{1}{\sqrt{2\mu - 1}} \qquad \text{and} \qquad c = \frac{\mu}{2\mu - 1}$$

may be used as guide posts. The first equation may be considered to be valid at a frequency for which the relative pseudo-velocity of the associated elastic system is from about 1.0 to 0.8 times the maximum ground velocity, and the second equation may be considered to be valid at a frequency for which the pseudo-acceleration of the associated elastic system is of the order of 2.

It follows that the response spectrum for an elastoplastic system corresponding to a specified value of the ductility ratio can be obtained from the spectrum applicable to the associated elastic system simply by dividing the ordinates of the elastic spectrum by a factor which depends on the value of the ductility ratio but which is different for the different frequency ranges. For convenience of reference, these factors are summarized in Fig. 3.15 for a representative spectrum corresponding to a half-cycle pulse of ground velocity.

In the application of these rules to design, it is suggested that the spectrum for the elastic system be represented by a smooth curve, without the undulations that are characteristic of response spectra.

These rules are proposed for systems with moderate amounts of damping (of the order of 10 percent critical or less) and ground motions for which the primary or dominant component may be represented either by a half-cycle velocity pulse or by a half-cycle displacement pulse. The reader is cautioned against using these rules for displacement pulses with two or more half-cycles of nearly equal amplitudes and durations.

Relative Effects of Damping and Inelastic Action. On comparing the inelastic spectra presented in this section with the corresponding spectra for damped elastic systems given in Section 2, it can be seen that the relative effects of damping and inelastic action in reducing the magnitude of the required resistance are quite different in the various regions of the spectrum. In particular, in the low-frequency range of the spectrum for which the effect of damping may be considered to be negligible, the effect of inelastic action is extremely important. These results show clearly that, in general, the effect of inelastic action cannot be considered in terms of a fixed amount of "equivalent damping".

3.5.4 Spectra for Multiple-Cycle Velocity Pulses. For half-cycle displacement pulses, it has been noted that in the regions of the spectrum where U and V attain their maximum values, the absolute maximum deformation of the elastoplastic system is generally smaller than that for the corresponding elastic system and that the results obtained from the expression $c = 1/\mu$ may be fairly conservative for large values of μ.

This effect is exaggerated under more nearly periodic excitations, as can be seen from the spectra presented in Figs. 3.16 and 3.17. These spectra are for velocity functions composed of three and four parabolic half-cycles, respectively. The details of the input functions are shown in the inset diagrams. Only the middle regions of the spectra are presented, since at the regions of low and high frequency the results are identical to those presented in Figs. 3.14.

It is of some interest to note that, if only the peak values of the curves corresponding to the inelastic and elastic curves are compared, the expression $1/\mu$ leads to reasonably accurate results.

3.5.5 Spectra for Earthquake Motions. The deformation spectra presented in Figs. 3.18 and 3.19 are self-explanatory. They refer to systems with 2 percent critical damping subjected to the Eureka and El Centro earthquake records considered earlier, and are directly comparable to the corresponding spectra presented for simple ground motions. It is important to note that, even for these complex input motions, the average relationships between the inelastic spectra and the corresponding elastic spectra for the various ranges of frequency are in very good agreement with the approximate rules presented for pulse-type of excitations.

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TABLE 1

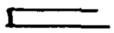
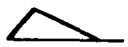
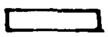
COORDINATES OF MAXIMUM VALUE OF A FOR HALF-CYCLE ACCELERATION PULSES

Pulse Type	$\frac{t_{r,a}}{t_d}$	$\frac{\bar{t}_{r,a}}{t_d}$	Coordinates of Max. Values of A			
			$\frac{A_o}{\ddot{y}_o}$	$t_d f$	$\frac{A_r}{\ddot{y}_o}$	$t_d f$
Triangular	1	1	1.26	0.67	1.26	0.67
Triangular	0.5	0.5	1.57	0.95	1.45	0.75
Versed Sine	0.5	0.32	1.72	1.0	1.63	0.8
Half-sine	0.5	0.32	1.77	0.85	1.73	0.7
Skewed Versed Sine	0.25	0.16	1.83	1.4	1.59	0.8
	0.125	0.08	1.90	2.0	1.53	0.8
Triangular	0	0	2.00	∞	1.26	0.67
Rectangular	0	0	2.00	0.5 or greater	2.00	0.5

TABLE 2

COORDINATES OF MAXIMUM VALUE OF V FOR HALF-CYCLE VELOCITY PULSES

Results may not be accurate to the number of significant figures recorded

Pulse No.	Acceler. Pulse	Velocity Pulse	$\frac{t_{r,v}}{t_d}$	$\frac{\bar{t}_{r,v}}{t_d}$	Coordinates of Max. Value of V $\frac{V_o}{\dot{y}_o}$	$t_d f$	$t_{o,v} f$
1			0	0	1.0	--	--
2a			0	0	1.26	0.66	0.66
2b			0.125	0.125	1.33	0.67	0.67
2c			0.25	0.25	1.40	0.73	0.73
2d			0.5	0.5	1.45	0.75	0.75
3a			0.125	0.08	1.54	0.84	0.84
3b			0.25	0.16	1.60	0.84	0.84
3c			0.5	0.32	1.65	0.84	0.84
4a			0.125		1.44	1.9	0.84
4b			0.25		1.53	1.1	0.84
5a			0.333	0.21	1.70	0.67	0.85
5b			0.5	0.32	1.72	0.68	0.86
6a			0.5	0.32	1.34	0.95	0.69
7a			0.5	0.25			
7b			0.5	0.25	1.72	0.85	0.85
7c			0.5	0.25	1.75	0.67	0.89
8			0	0	2.0	0.50	1.00

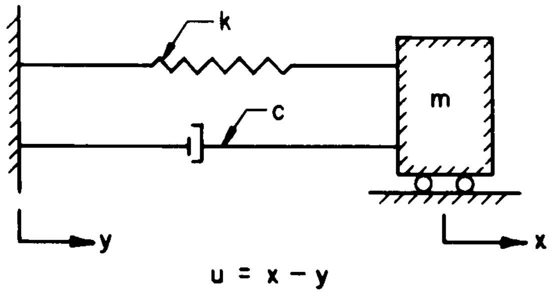
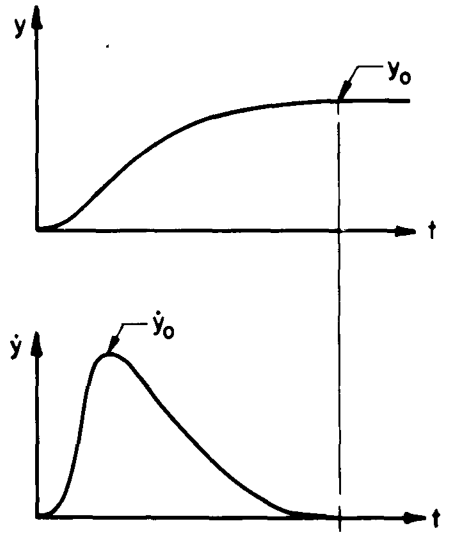
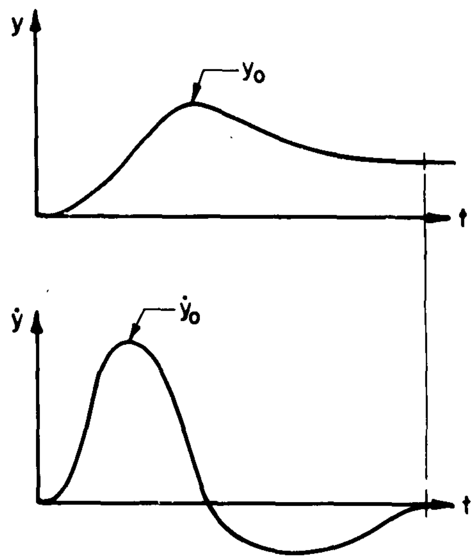


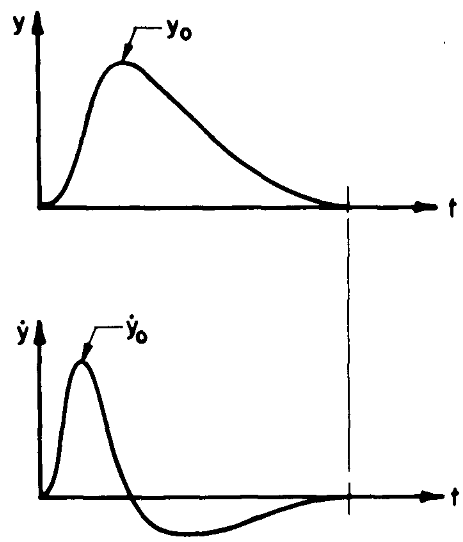
FIG. 2.1 SYSTEM CONSIDERED



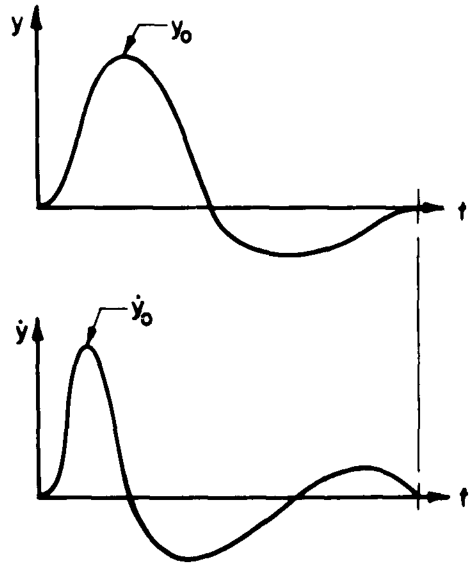
(a) Half-Cycle Velocity Pulse.



(b) Displacement Pulse With Partial Recovery.



(c) Half-Cycle Displacement Pulse.



(d) Full-Cycle Displacement Pulse.

FIG. 2.2 GROUND MOTIONS OF INTEREST

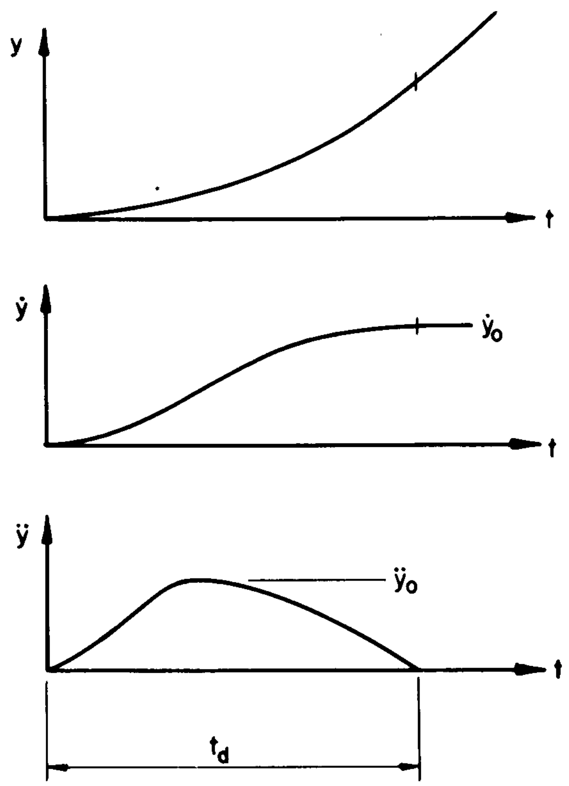


FIG. 2.3 HALF-CYCLE ACCELERATION PULSE AND ASSOCIATED VELOCITY AND DISPLACEMENT DIAGRAMS

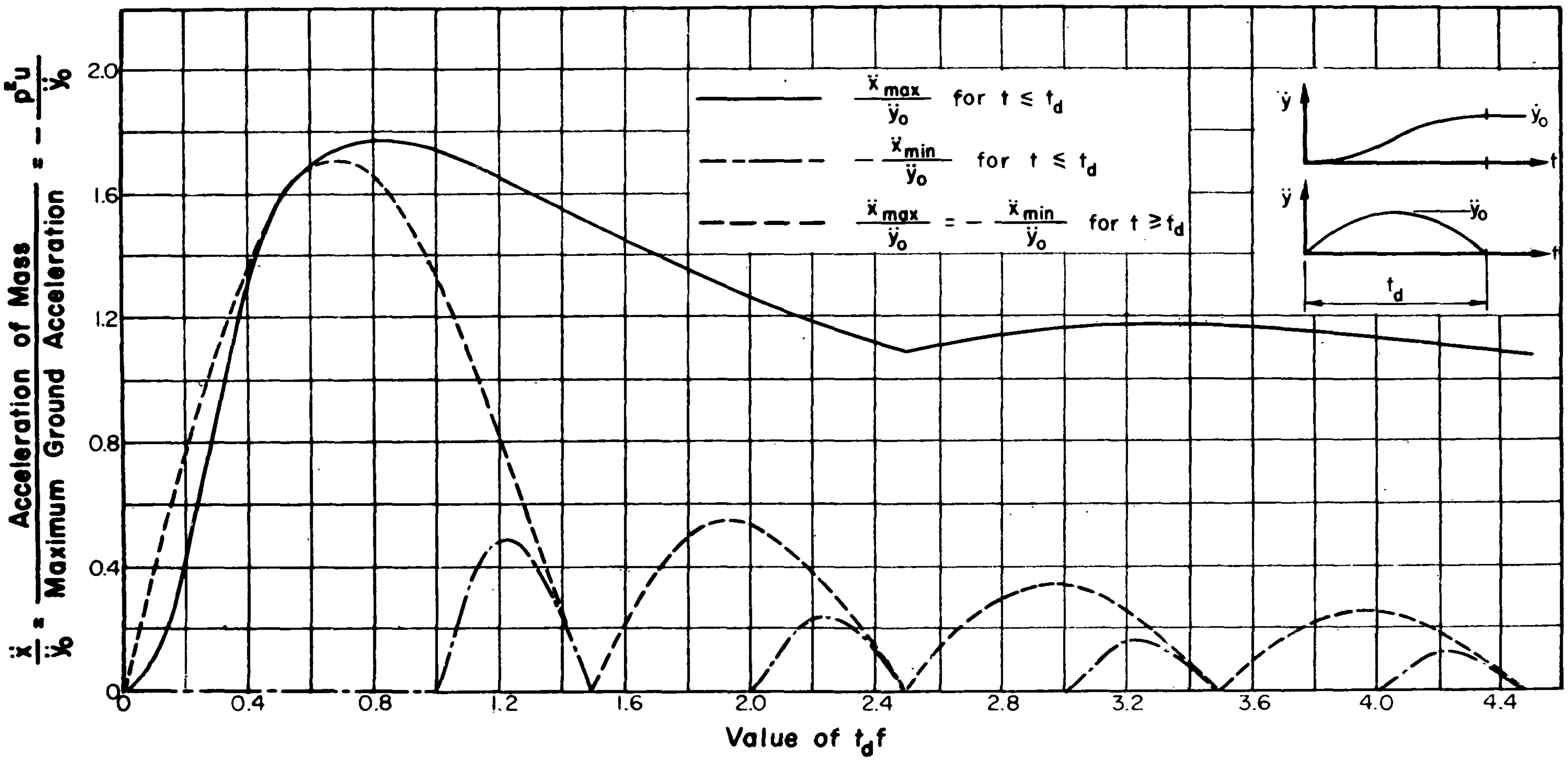


FIG. 2.4 SPECTRA FOR MAXIMUM AND MINIMUM ACCELERATIONS OF THE MASS
Undamped Elastic Systems Subjected to a Half-Sine Acceleration Pulse

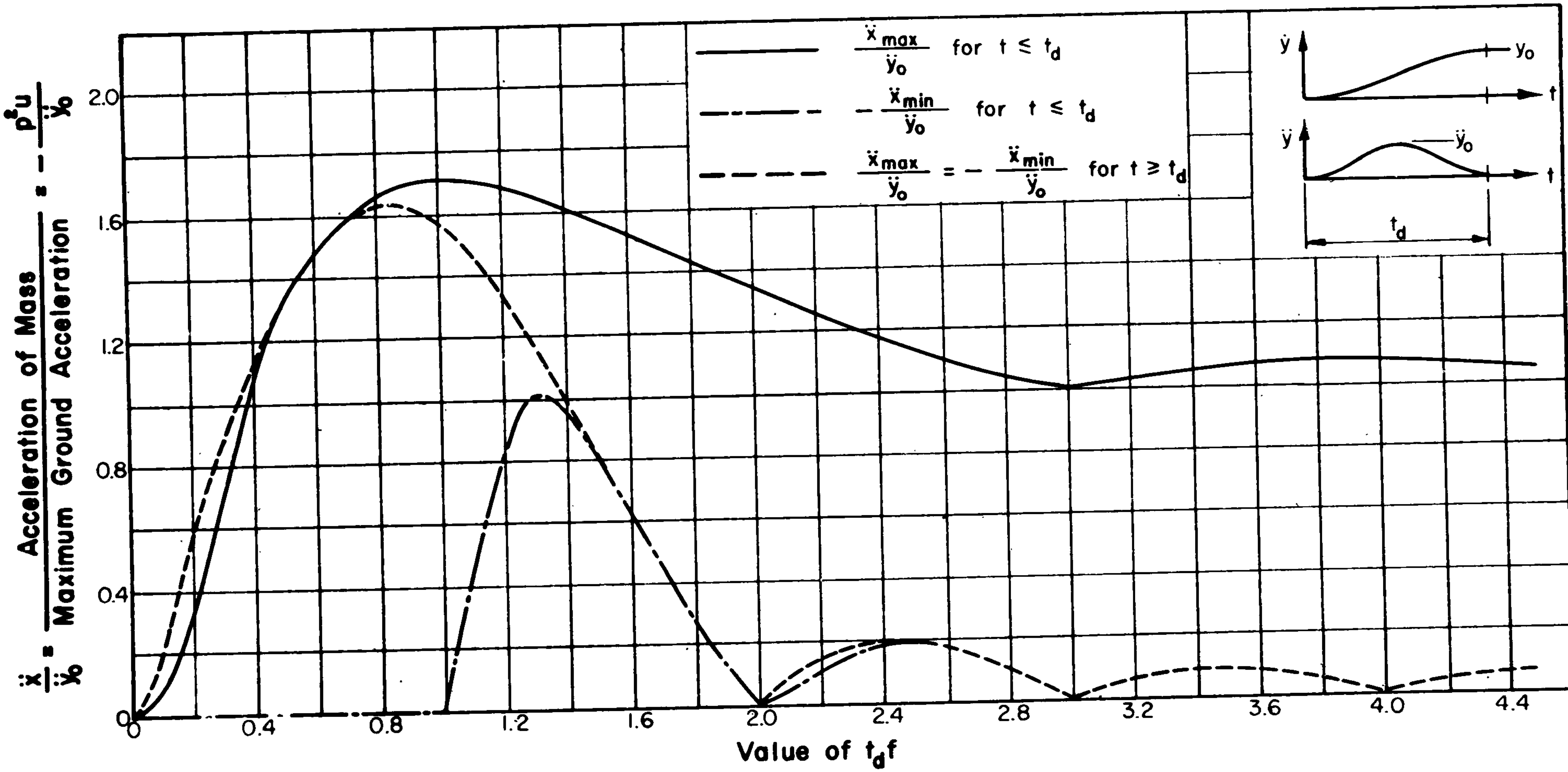


FIG. 2.5 SPECTRA FOR MAXIMUM AND MINIMUM ACCELERATIONS OF THE MASS
Undamped Elastic Systems Subjected to a Versed-Sine Acceleration Pulse

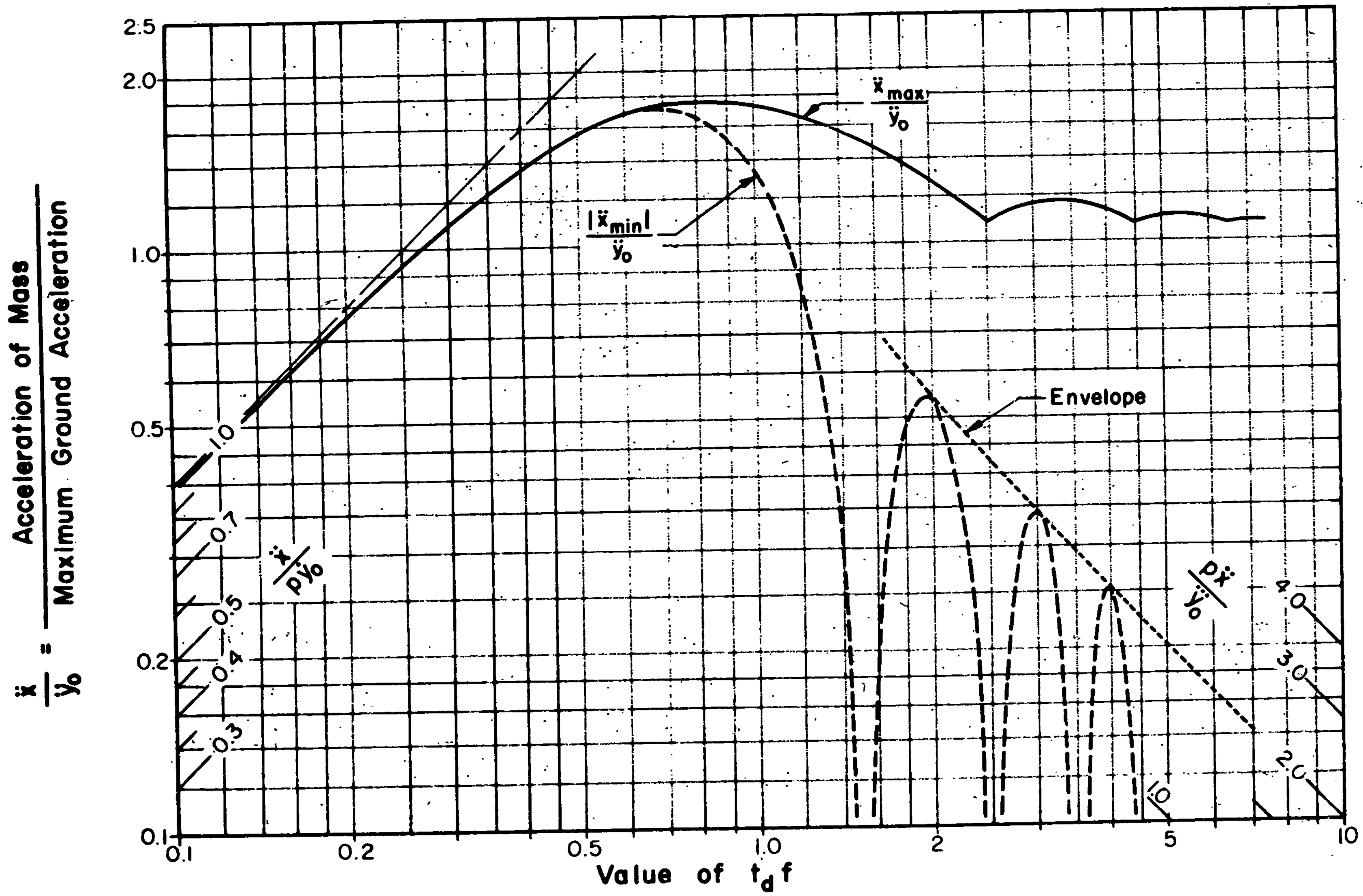


FIG. 2.6 SPECTRA FOR ABSOLUTE MAXIMUM AND ABSOLUTE MINIMUM ACCELERATIONS OF THE MASS
Undamped Elastic Systems Subjected to a Half-Sine Acceleration Pulse

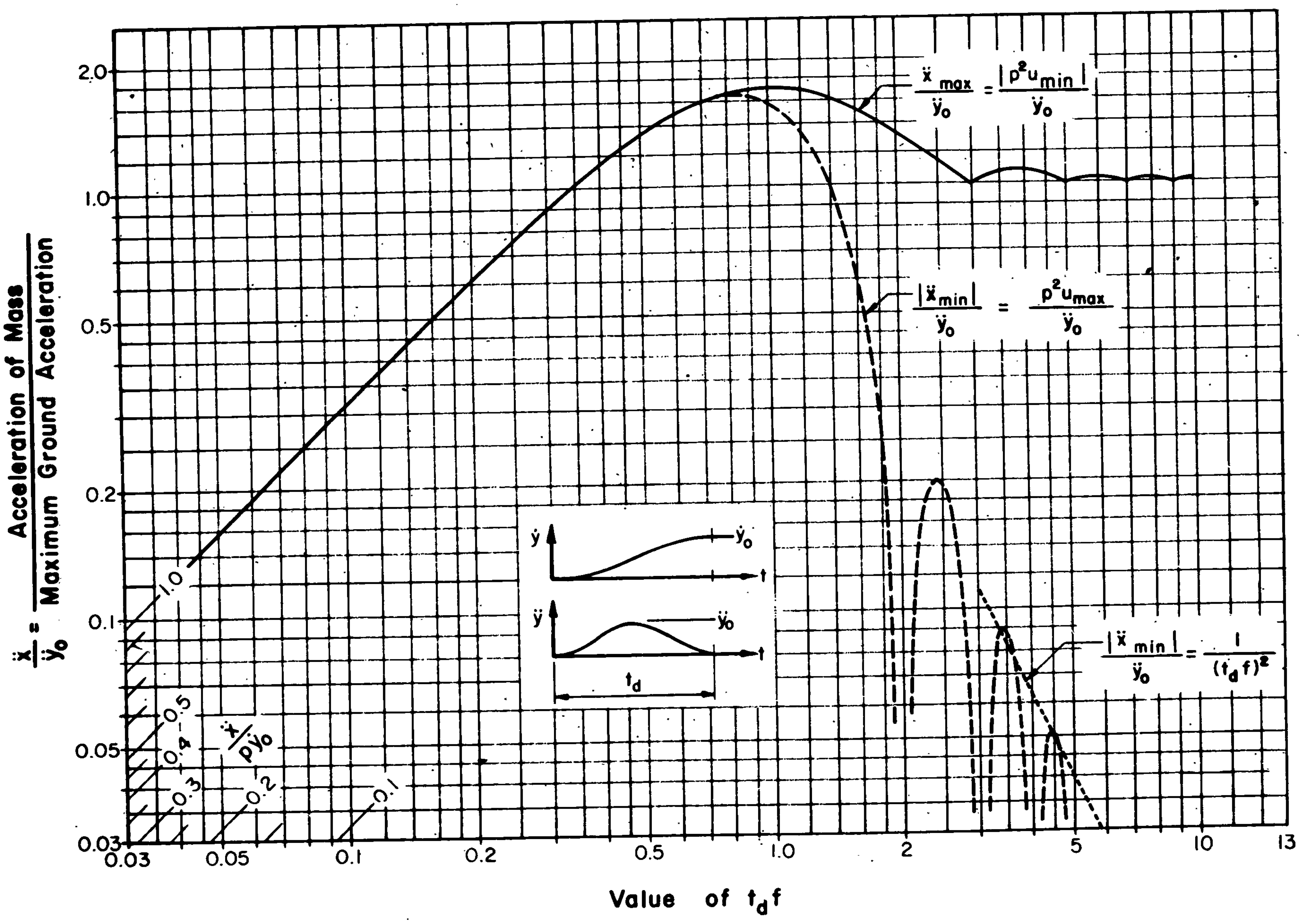


FIG. 2.7 SPECTRA FOR ABSOLUTE MAXIMUM AND ABSOLUTE MINIMUM ACCELERATIONS OF THE MASS
Undamped Elastic Systems Subjected to a Versed-Sine Acceleration Pulse

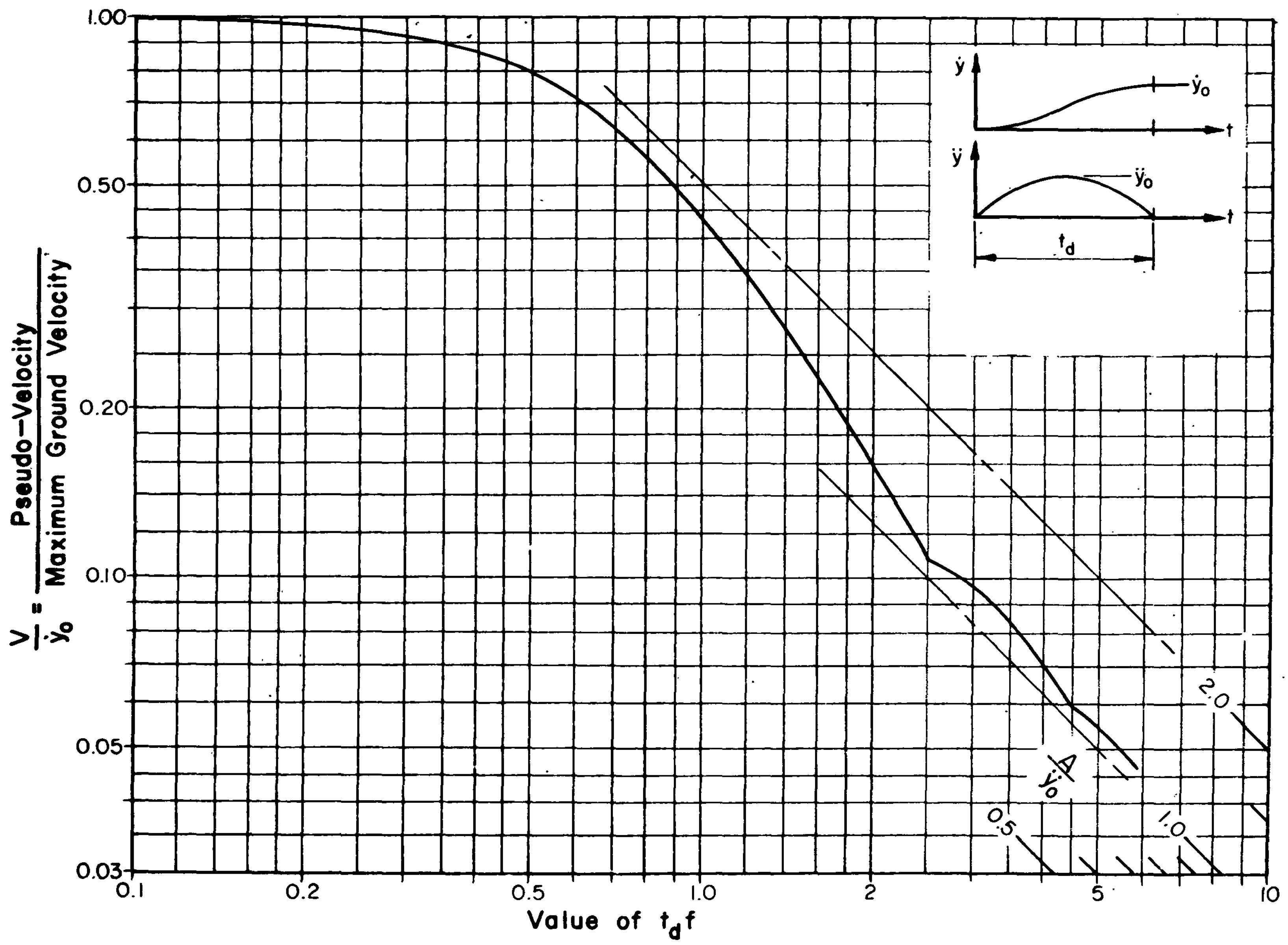


FIG. 2.8 DEFORMATION SPECTRUM FOR UNDAMPED ELASTIC SYSTEMS SUBJECTED TO A HALF-SINE ACCELERATION PULSE

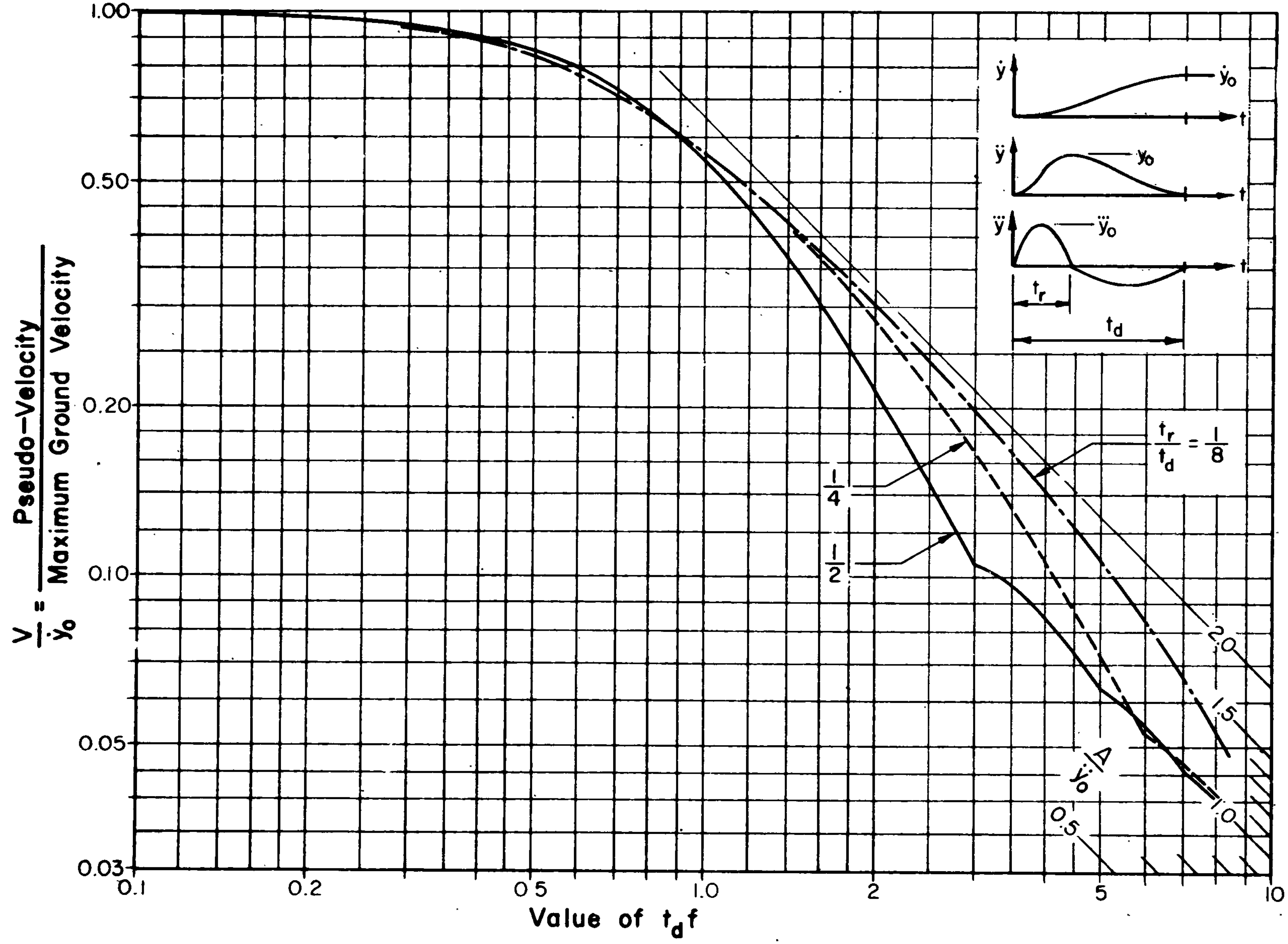


FIG. 2.9 DEFORMATION SPECTRA FOR UNDAMPED ELASTIC SYSTEMS SUBJECTED TO SKEWED VERSED-SINE ACCELERATION PULSES

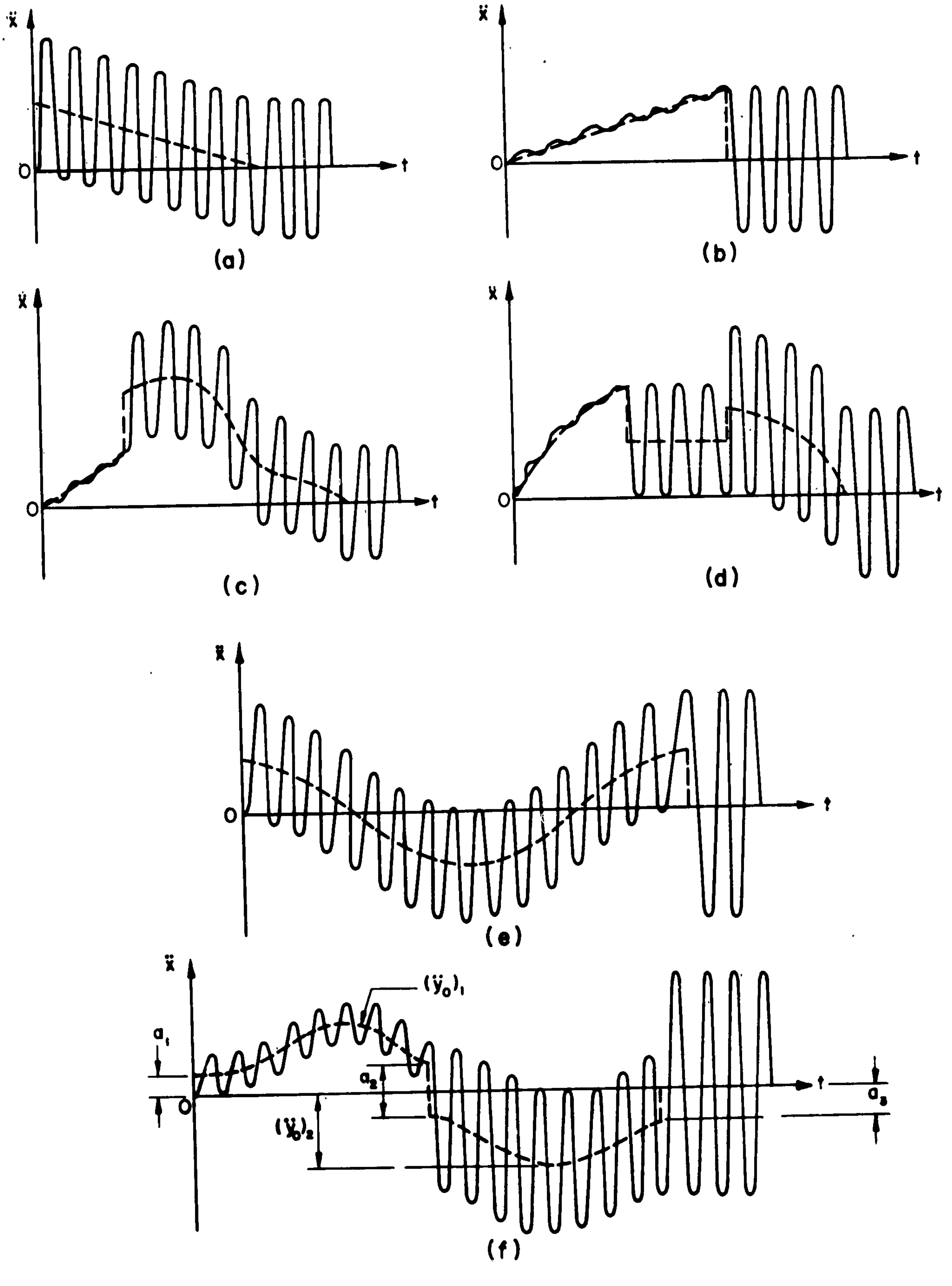


FIG. 2.10 RESPONSE HISTORIES OF HIGH-FREQUENCY, ELASTIC, UNDAMPED SYSTEMS SUBJECTED TO DISCONTINUOUS GROUND ACCELERATION FUNCTIONS

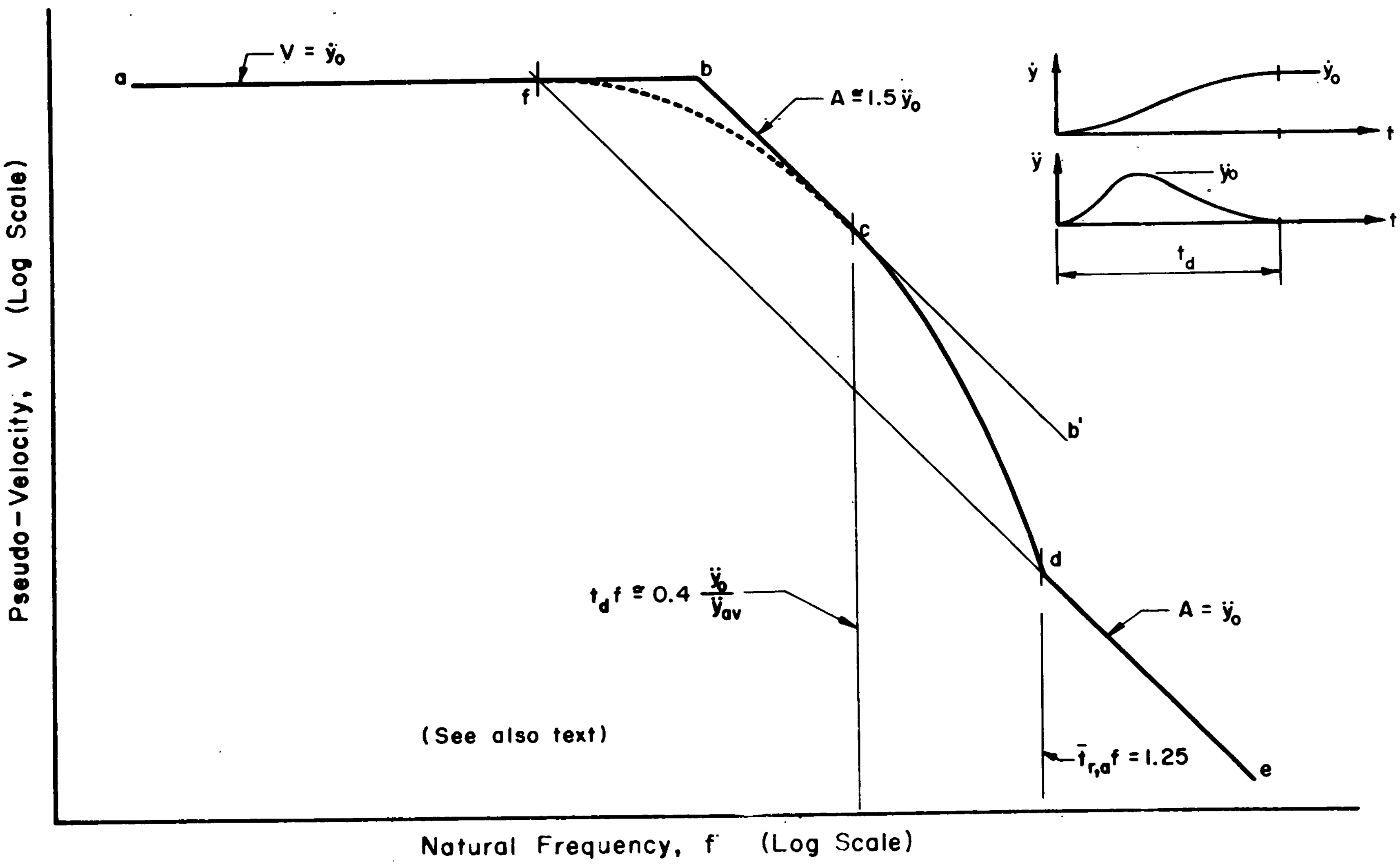


FIG. 2.11 DESIGN SPECTRUM FOR THE ABSOLUTE MAXIMUM DEFORMATION OF SYSTEMS SUBJECTED TO A HALF-CYCLE ACCELERATION PULSE
Undamped Elastic Systems; Continuous Input Acceleration Functions

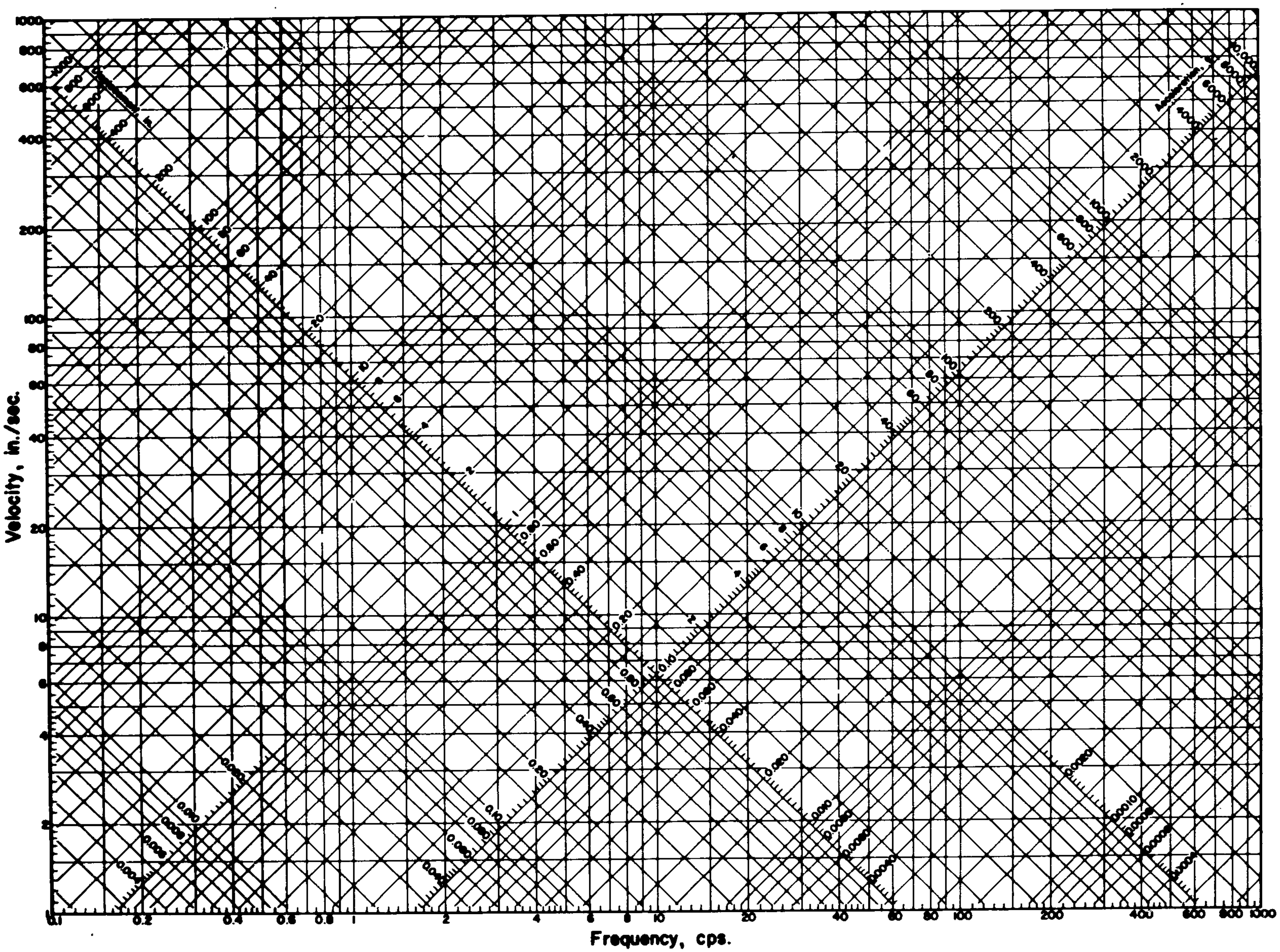
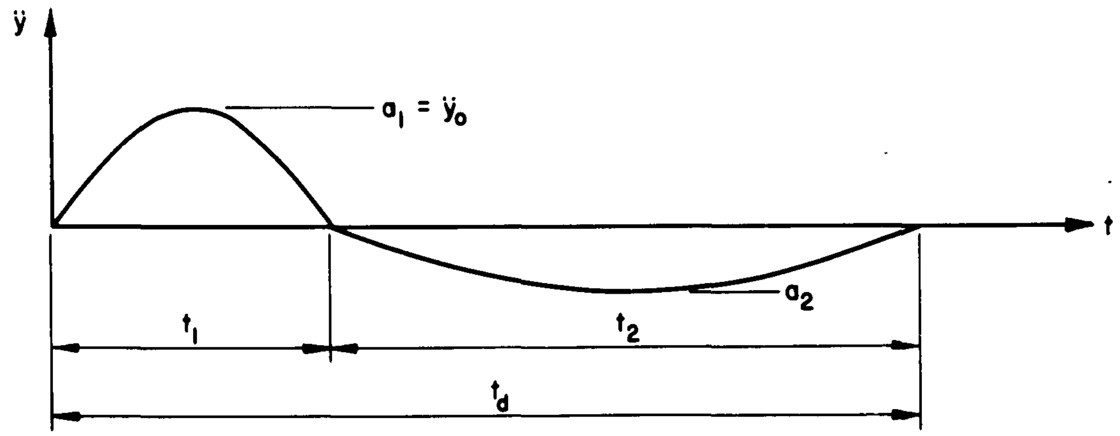
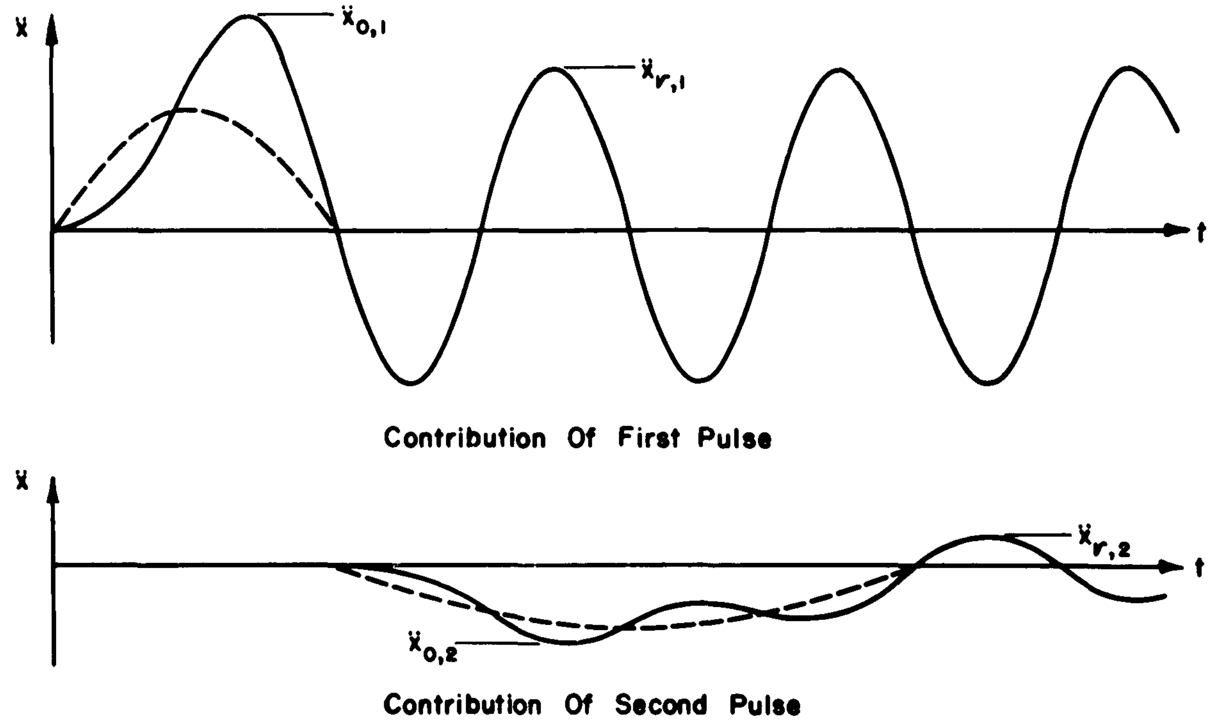


FIG. 2.12 FOUR-WAY LOGARITHMIC PLOT



(a) Ground Acceleration



(b) Response Due To Component Pulses

FIG. 2.13 RESPONSE OF AN UNDAMPED ELASTIC SYSTEM SUBJECTED TO A FULL-CYCLE ACCELERATION PULSE

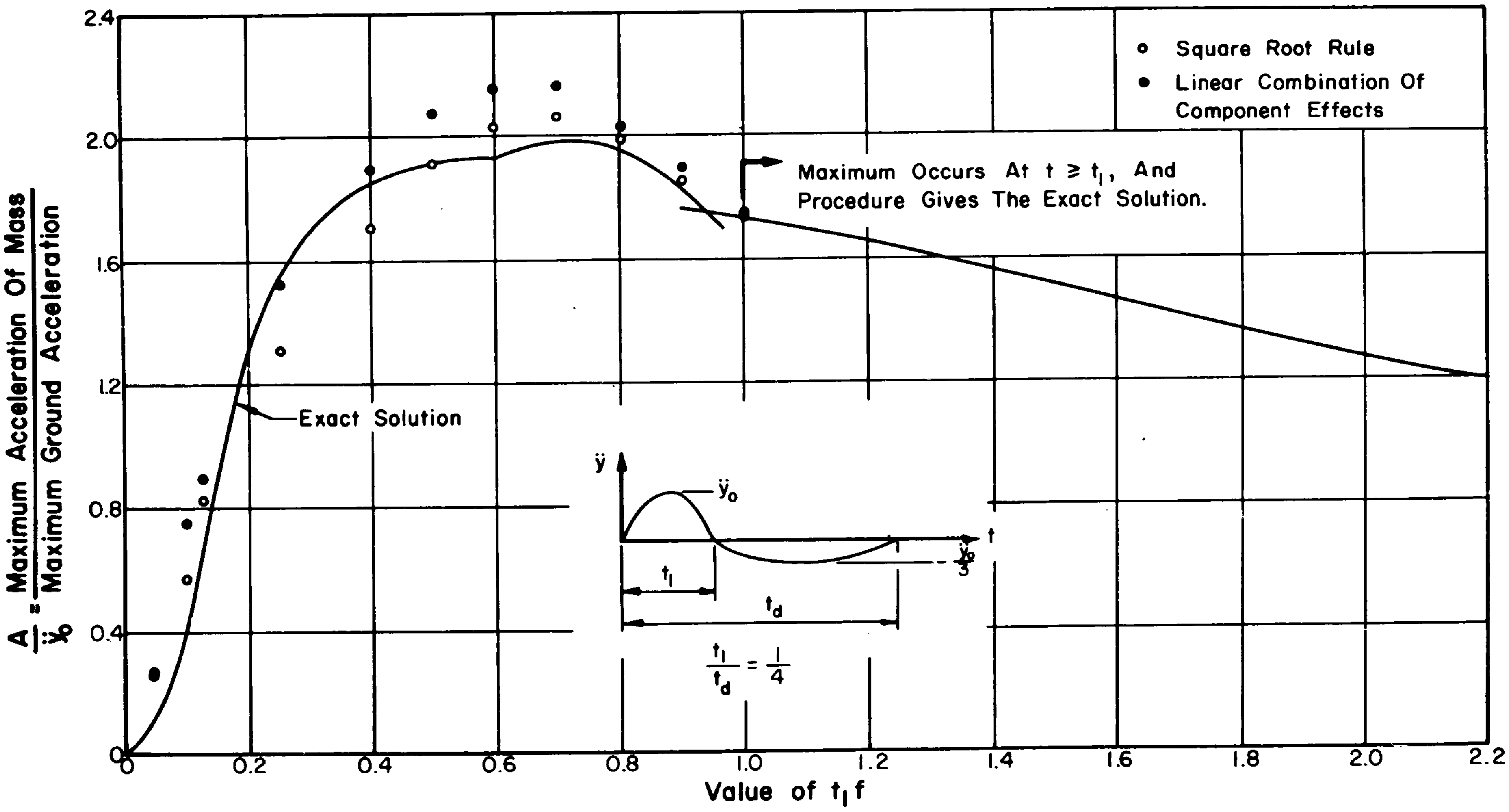


FIG. 2.14a COMPARISON OF SYNTHESIZED AND ACTUAL SPECTRA CORRESPONDING TO A FULL-CYCLE ACCELERATION PULSE
Undamped Elastic Systems; Acceleration Function Composed of Two Half-Sine Pulses

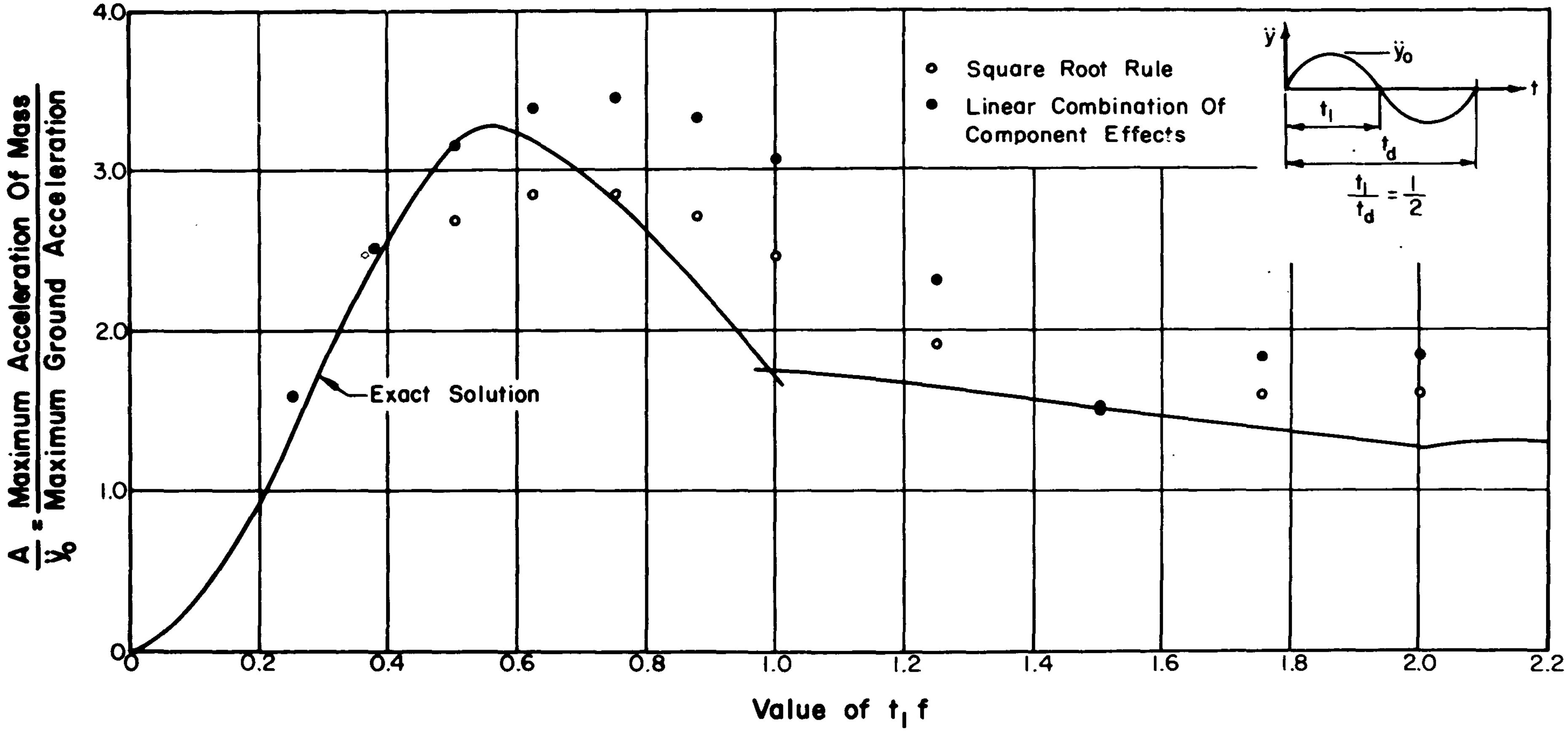


FIG. 2.14b COMPARISON OF SYNTHESIZED AND ACTUAL SPECTRA CORRESPONDING TO A FULL-CYCLE ACCELERATION PULSE--Undamped Elastic Systems; Sinusoidal Pulse

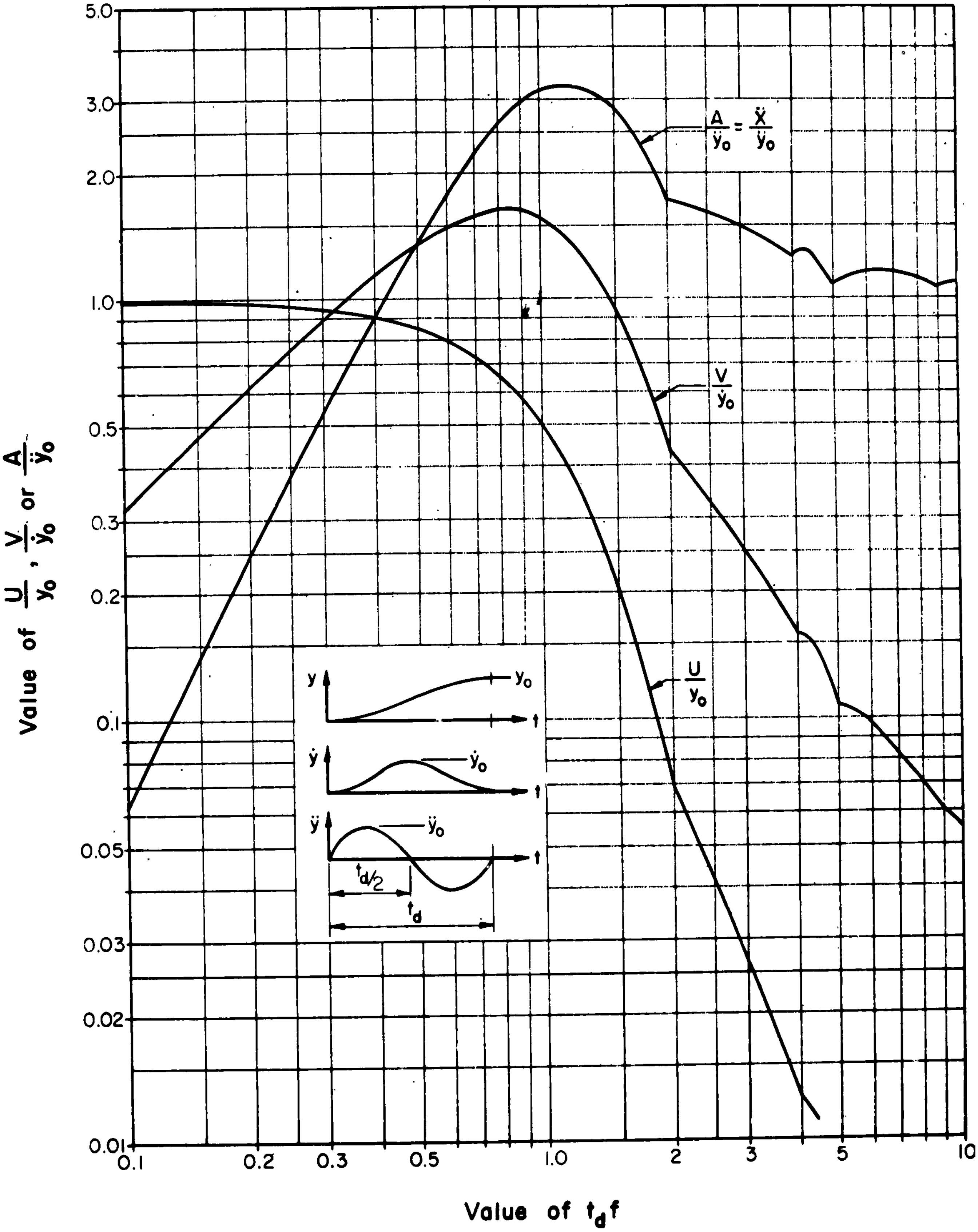


FIG. 2.15 DEFORMATION SPECTRA FOR UNDAMPED ELASTIC SYSTEMS SUBJECTED TO A VERSED-SINE VELOCITY PULSE

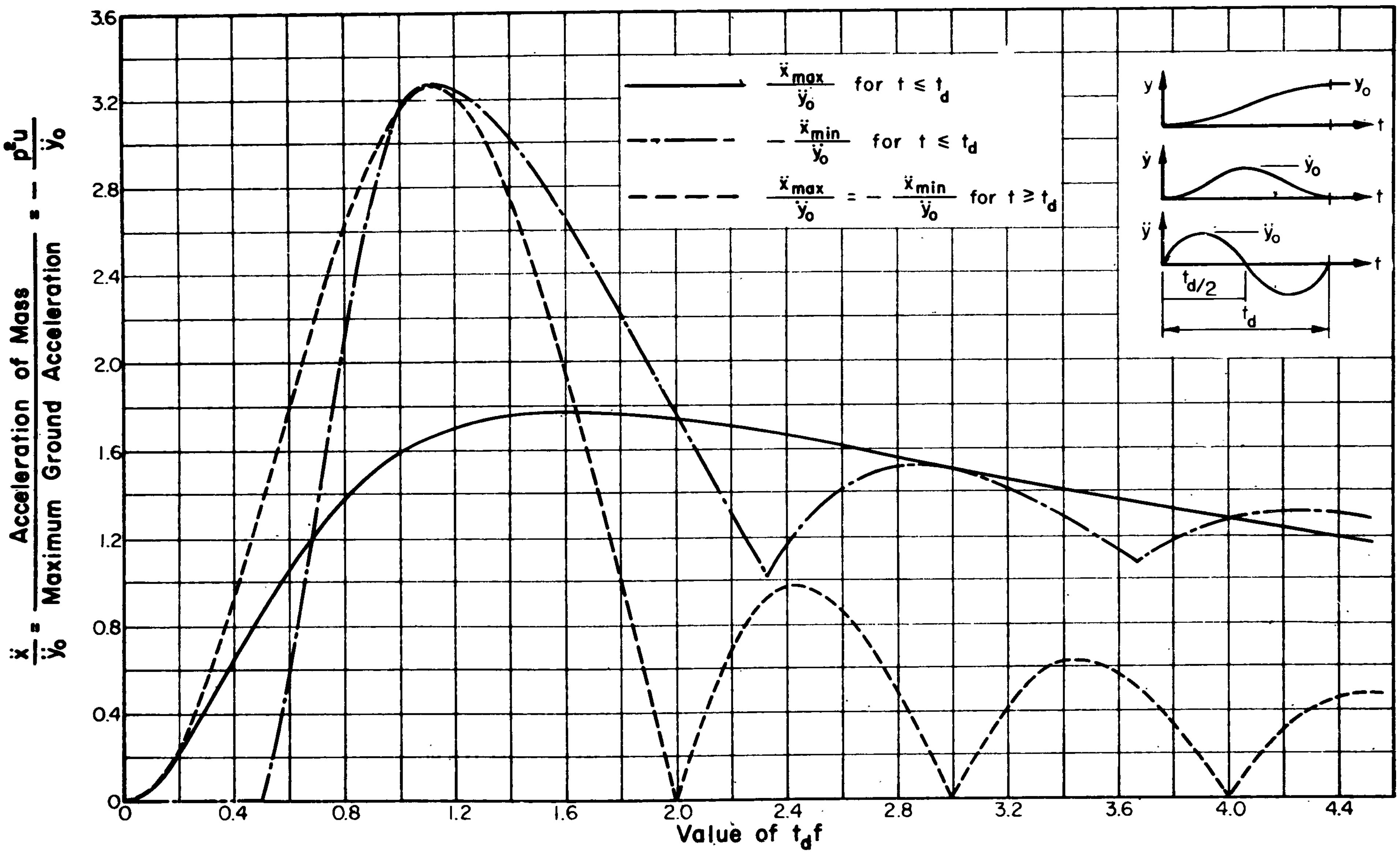


FIG. 2.16 SPECTRA FOR MAXIMUM AND MINIMUM ACCELERATIONS OF THE MASS
Undamped Elastic Systems Subjected to a Versed-Sine Velocity Pulse

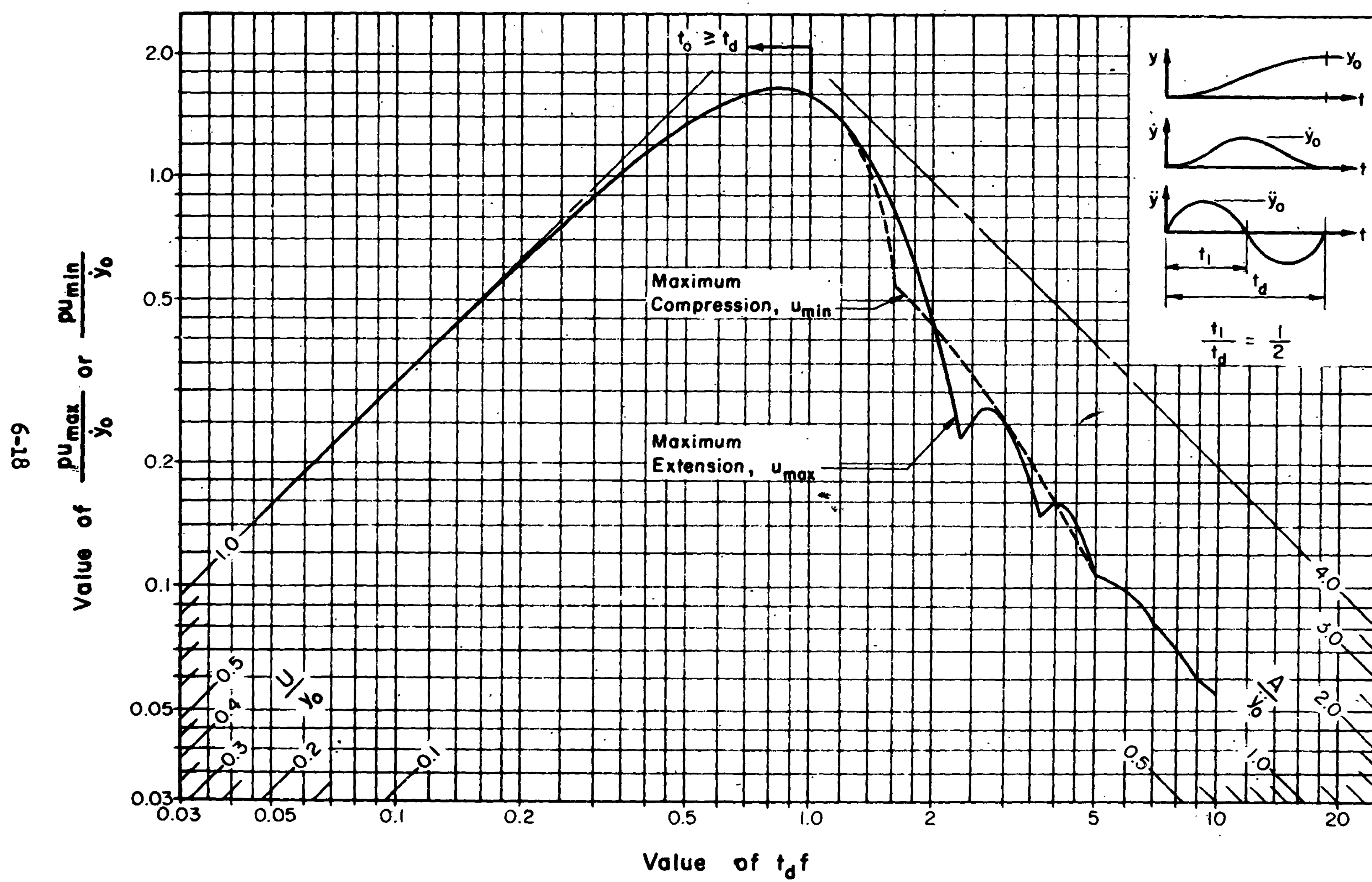


FIG. 2.17a SPECTRA FOR ABSOLUTE MAXIMUM AND ABSOLUTE MINIMUM DEFORMATIONS
Undamped Elastic Systems Subjected to a Versed-Sine Velocity Pulse

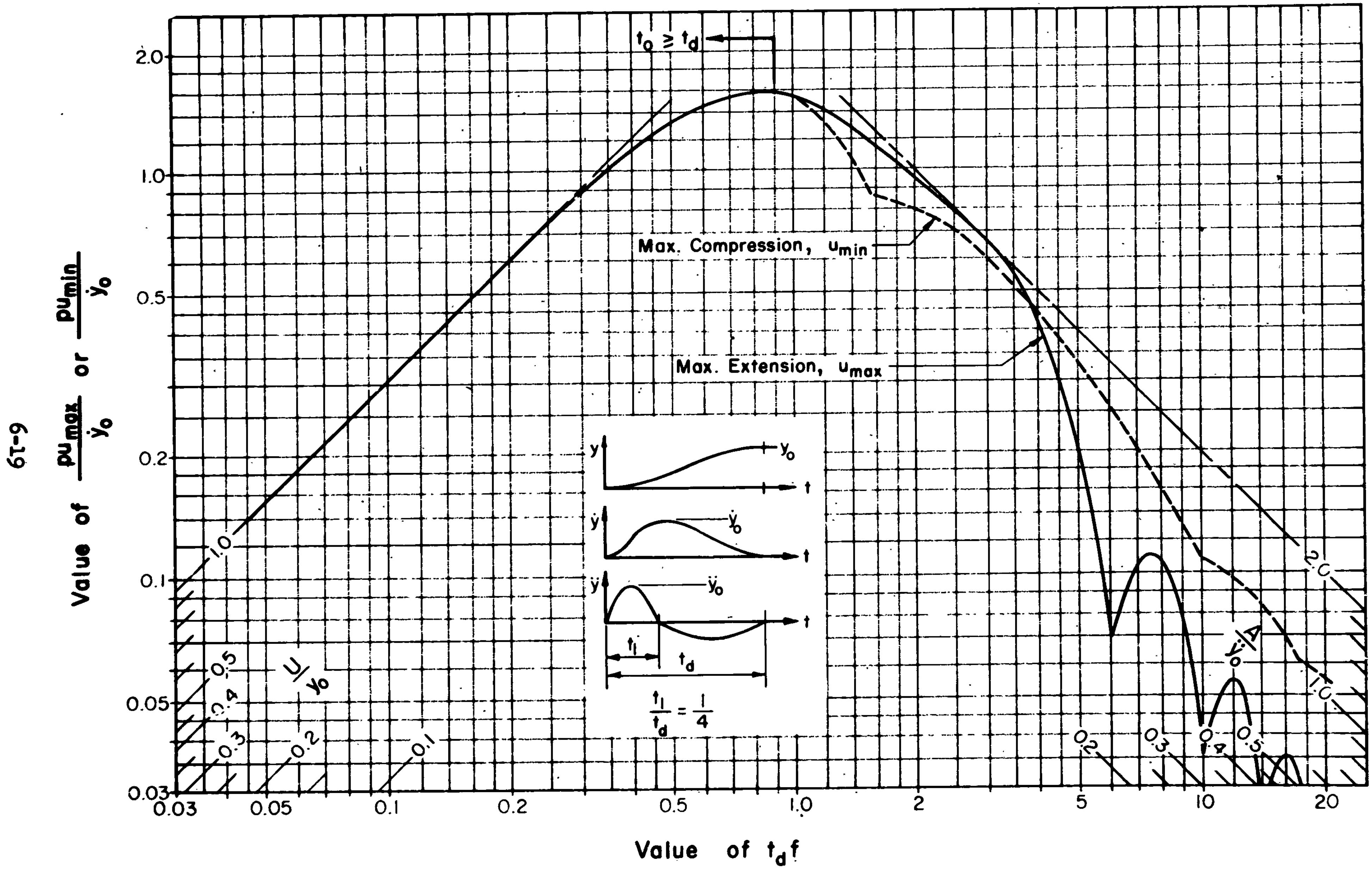


FIG. 2.17b SPECTRA FOR ABSOLUTE MAXIMUM AND ABSOLUTE MINIMUM DEFORMATIONS
Undamped Elastic Systems Subjected to a Skewed Versed-Sine Velocity Pulse

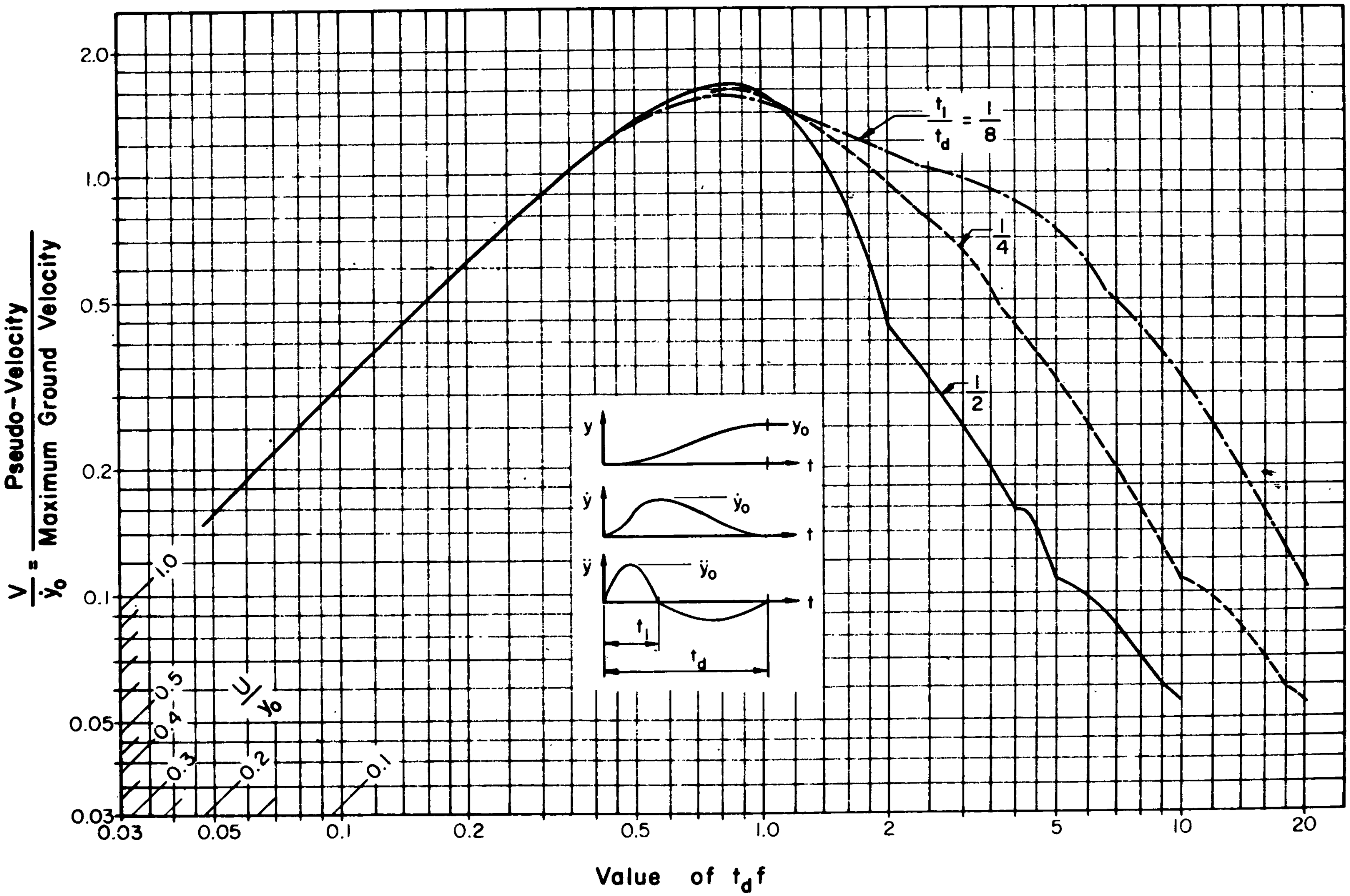


FIG. 2.18 DEFORMATION SPECTRA FOR UNDAMPED ELASTIC SYSTEMS SUBJECTED TO SKEWED VERSED-SINE VELOCITY PULSES

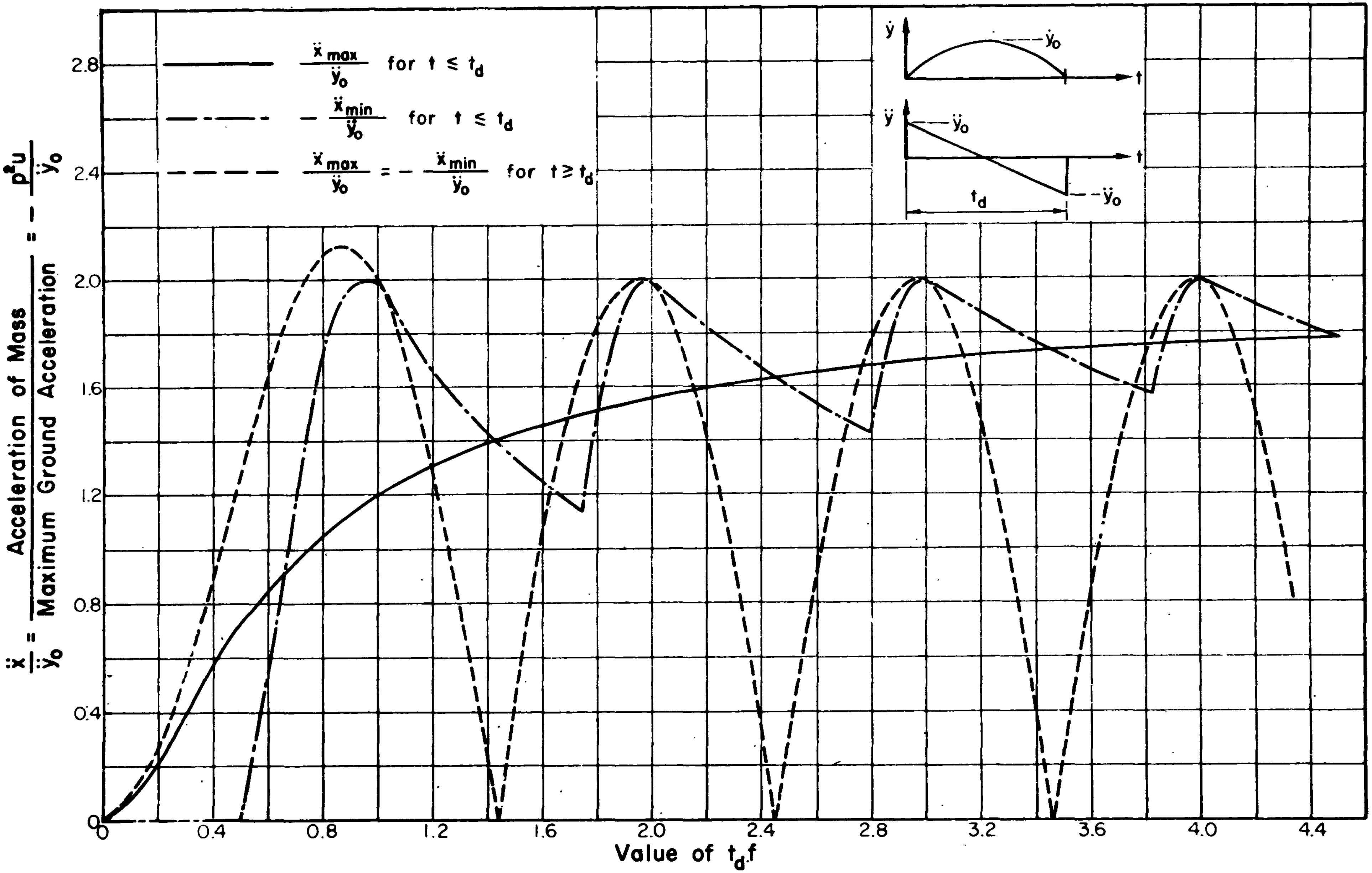


FIG. 2.19 SPECTRA FOR MAXIMUM AND MINIMUM ACCELERATIONS OF THE MASS
Undamped Elastic Systems Subjected to a Parabolic Velocity Pulse

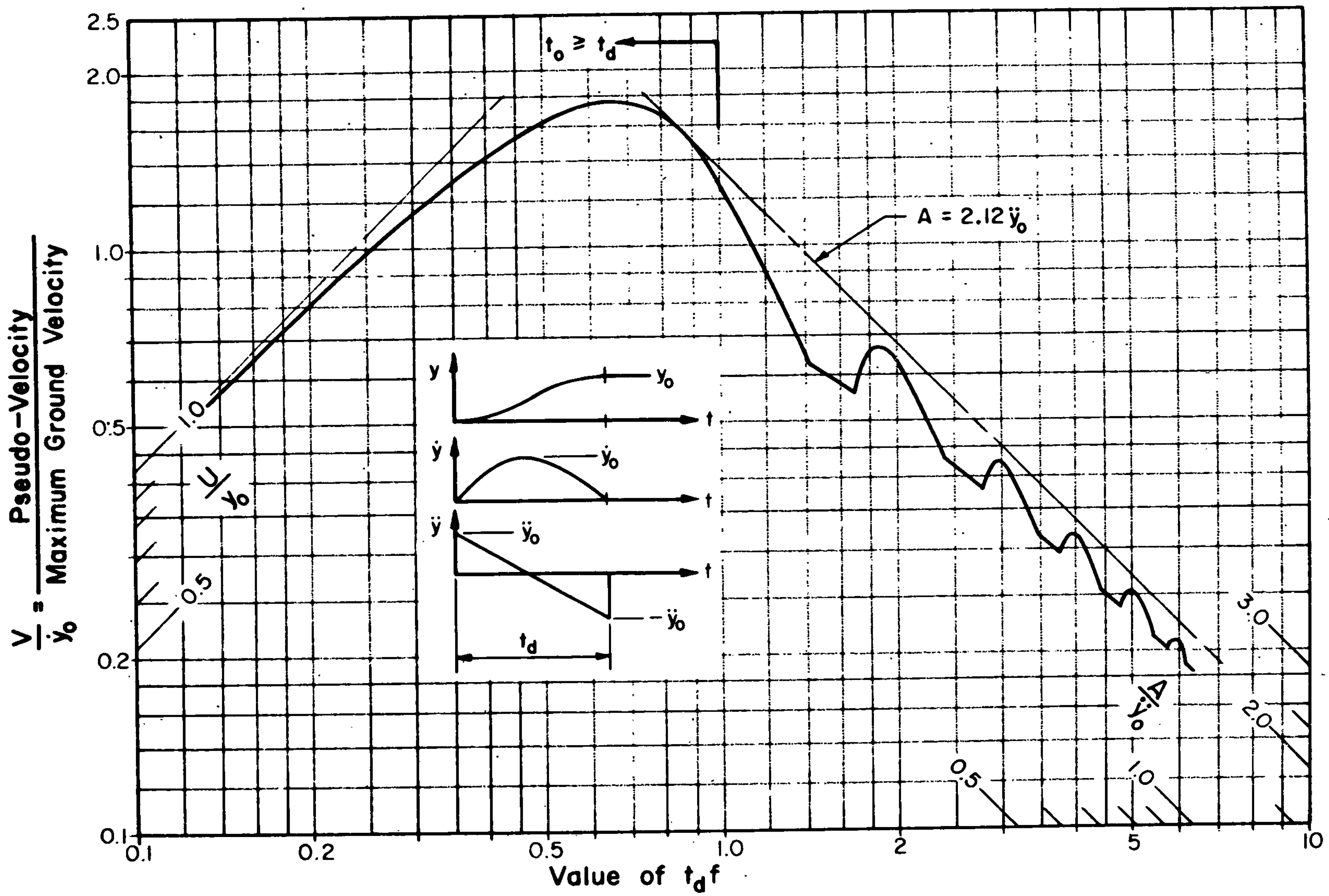


FIG. 2.20 DEFORMATION SPECTRUM FOR UNDAMPED ELASTIC SYSTEMS SUBJECTED TO A PARABOLIC VELOCITY PULSE

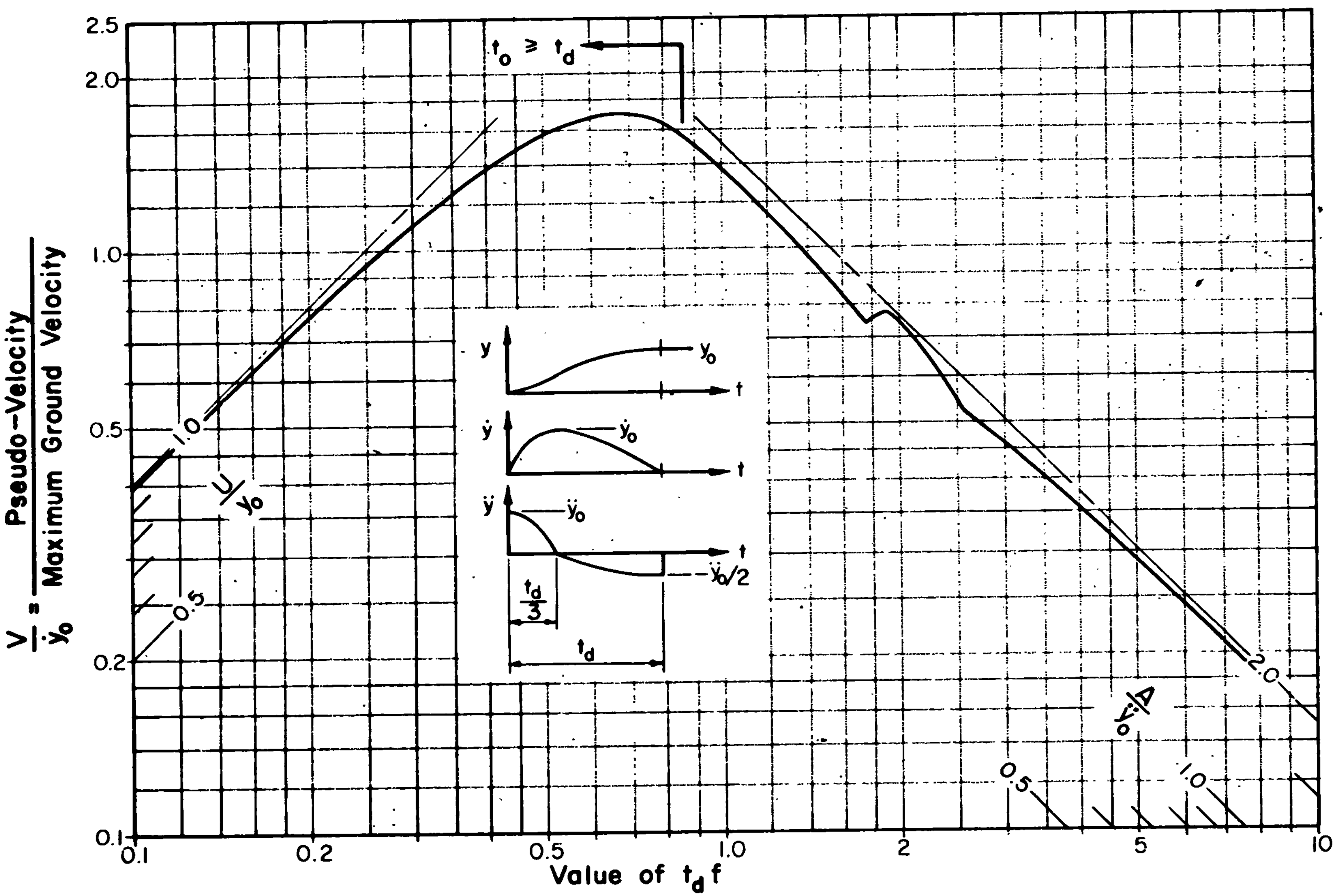


FIG. 2.21 DEFORMATION SPECTRUM FOR UNDAMPED ELASTIC SYSTEMS SUBJECTED TO A SKEWED SINUSOIDAL VELOCITY PULSE

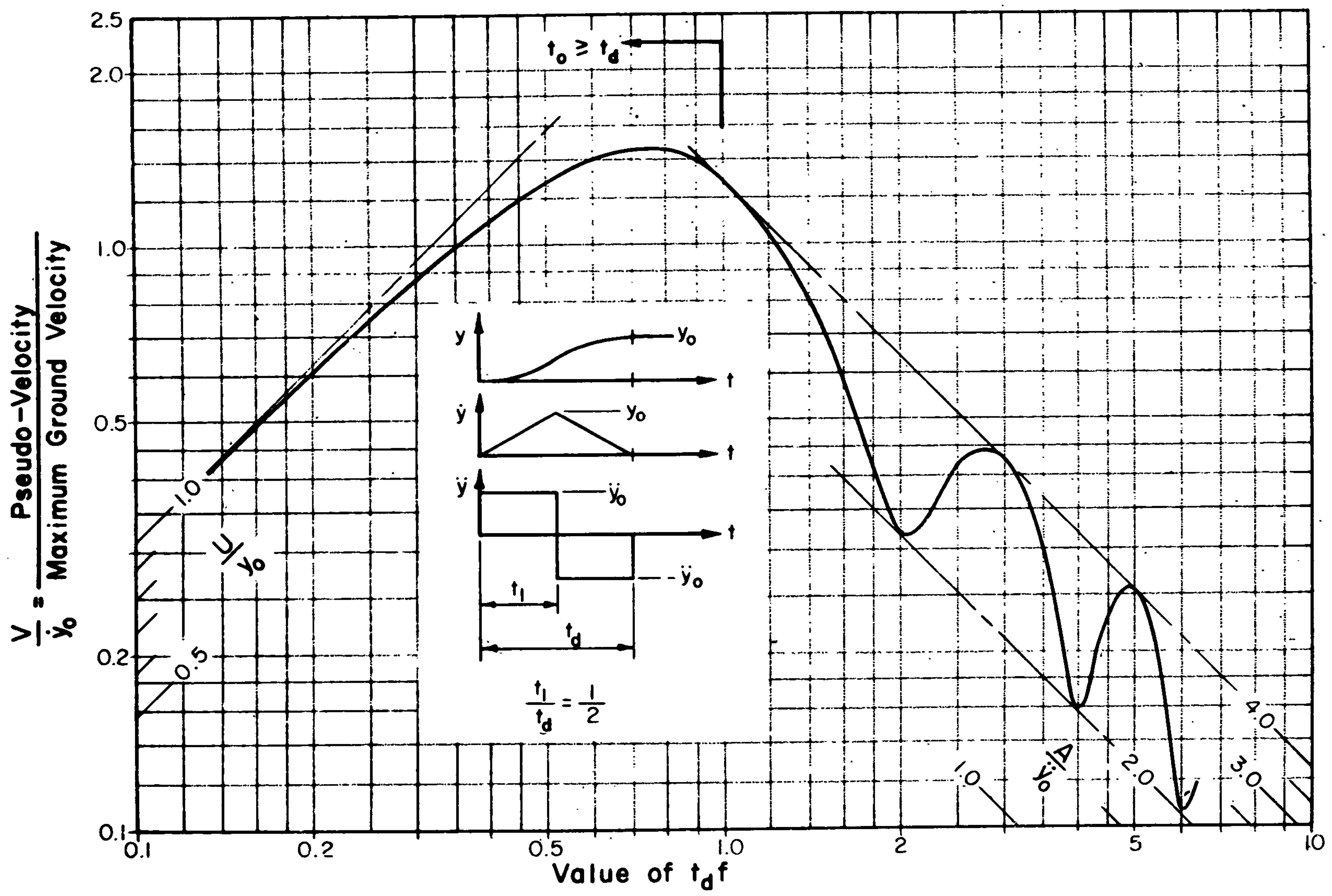


FIG. 2.22a DEFORMATION SPECTRUM FOR UNDAMPED ELASTIC SYSTEMS SUBJECTED TO A TRIANGULAR VELOCITY PULSE

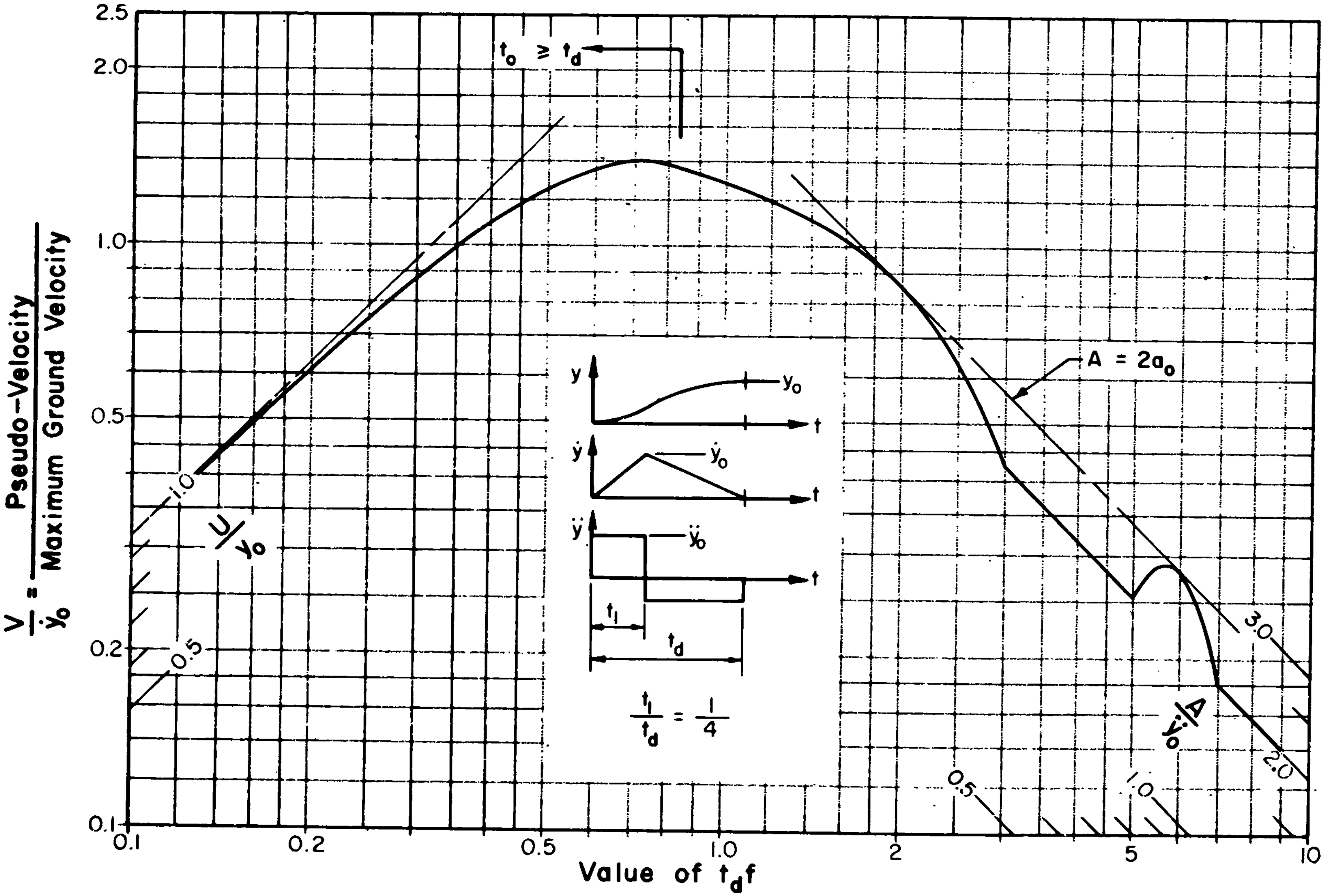


FIG. 2.22b DEFORMATION SPECTRUM FOR UNDAMPED ELASTIC SYSTEMS SUBJECTED TO A TRIANGULAR VELOCITY PULSE

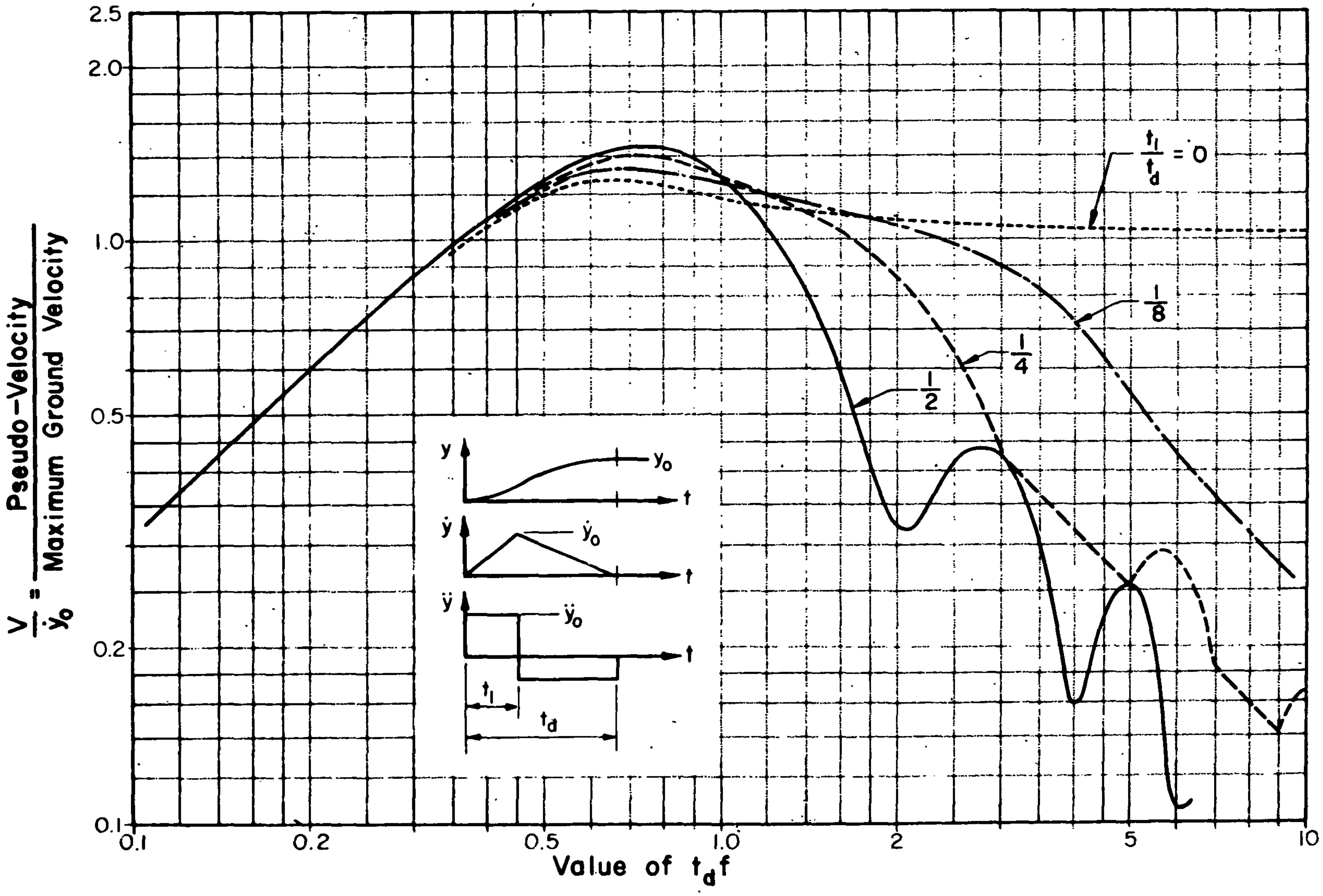


FIG. 2.22c DEFORMATION SPECTRA FOR UNDAMPED ELASTIC SYSTEMS SUBJECTED TO TRIANGULAR VELOCITY PULSES

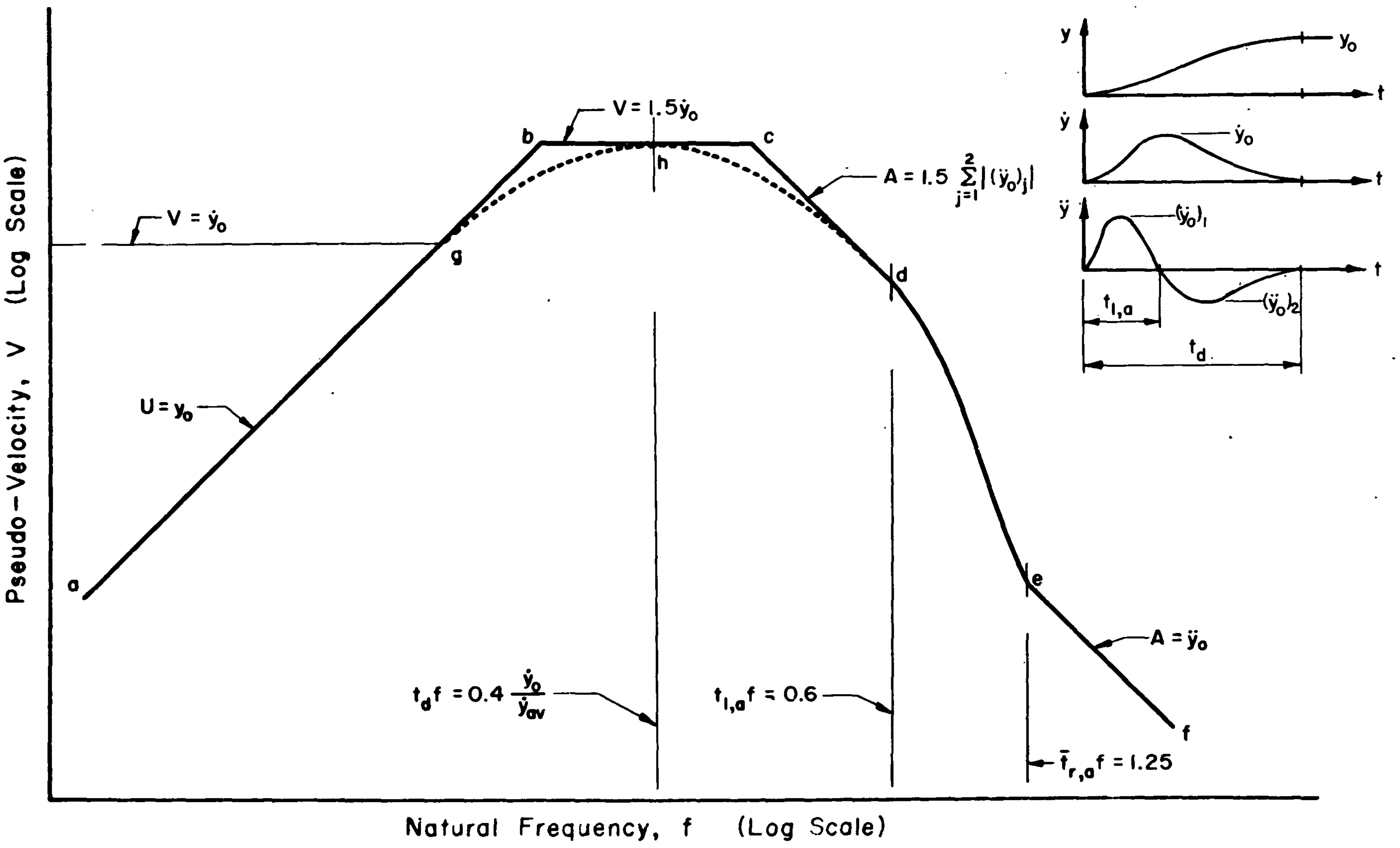


FIG. 2.23 DESIGN SPECTRUM FOR THE ABSOLUTE MAXIMUM DEFORMATION OF SYSTEMS SUBJECTED TO A HALF-CYCLE VELOCITY PULSE--Undamped Elastic Systems; Continuous Input Acceleration Functions

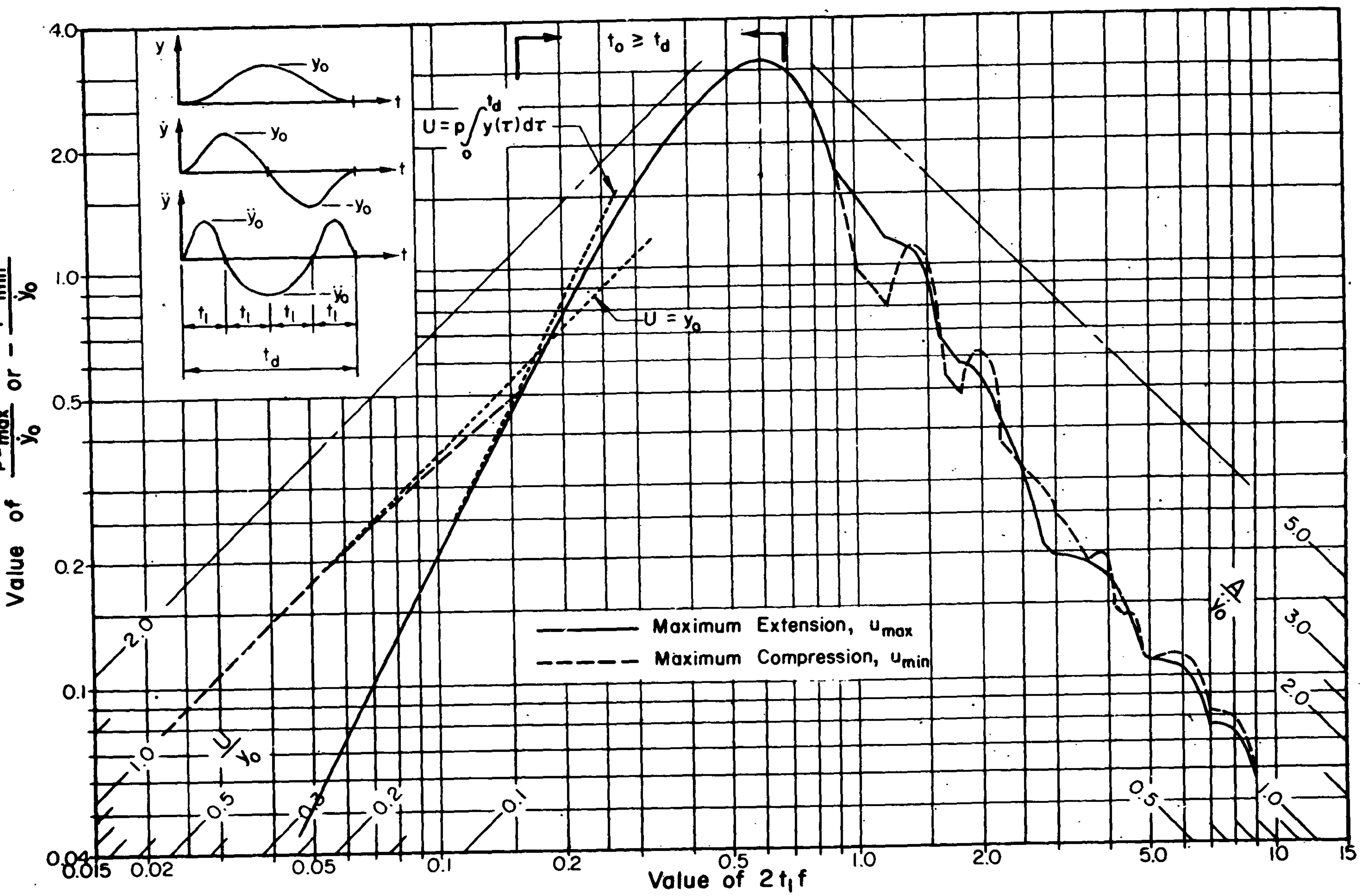


FIG. 2.24 SPECTRA FOR ABSOLUTE MAXIMUM AND ABSOLUTE MINIMUM DEFORMATIONS--Undamped Elastic Systems Subjected to a Half-Cycle Displacement Pulse; Acceleration Diagram Consists of Three Half-Sine Waves as Shown

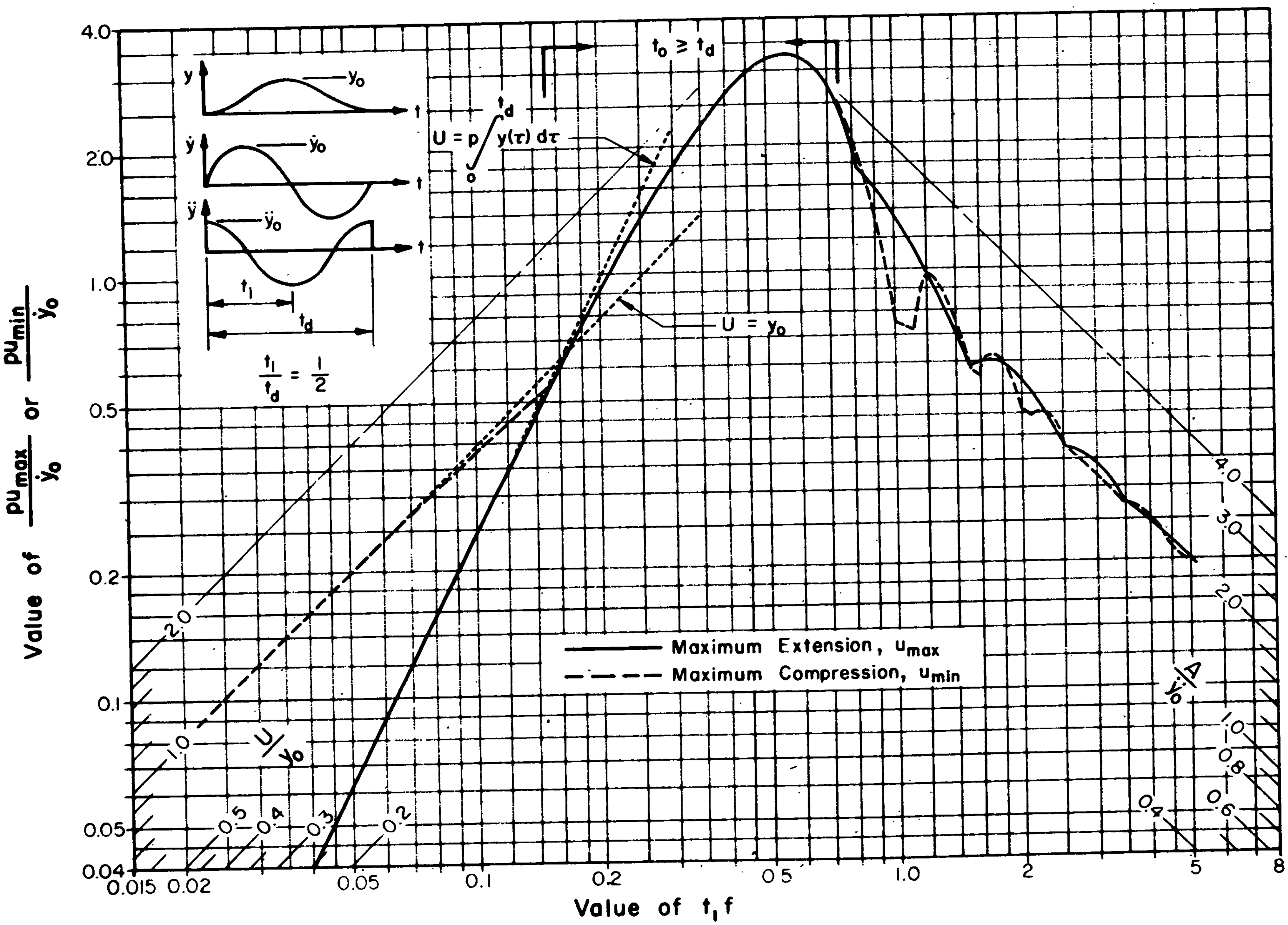


FIG. 2.25a SPECTRA FOR ABSOLUTE MAXIMUM AND ABSOLUTE MINIMUM DEFORMATIONS
Undamped Elastic Systems Subjected to a Versed-Sine Displacement Pulse

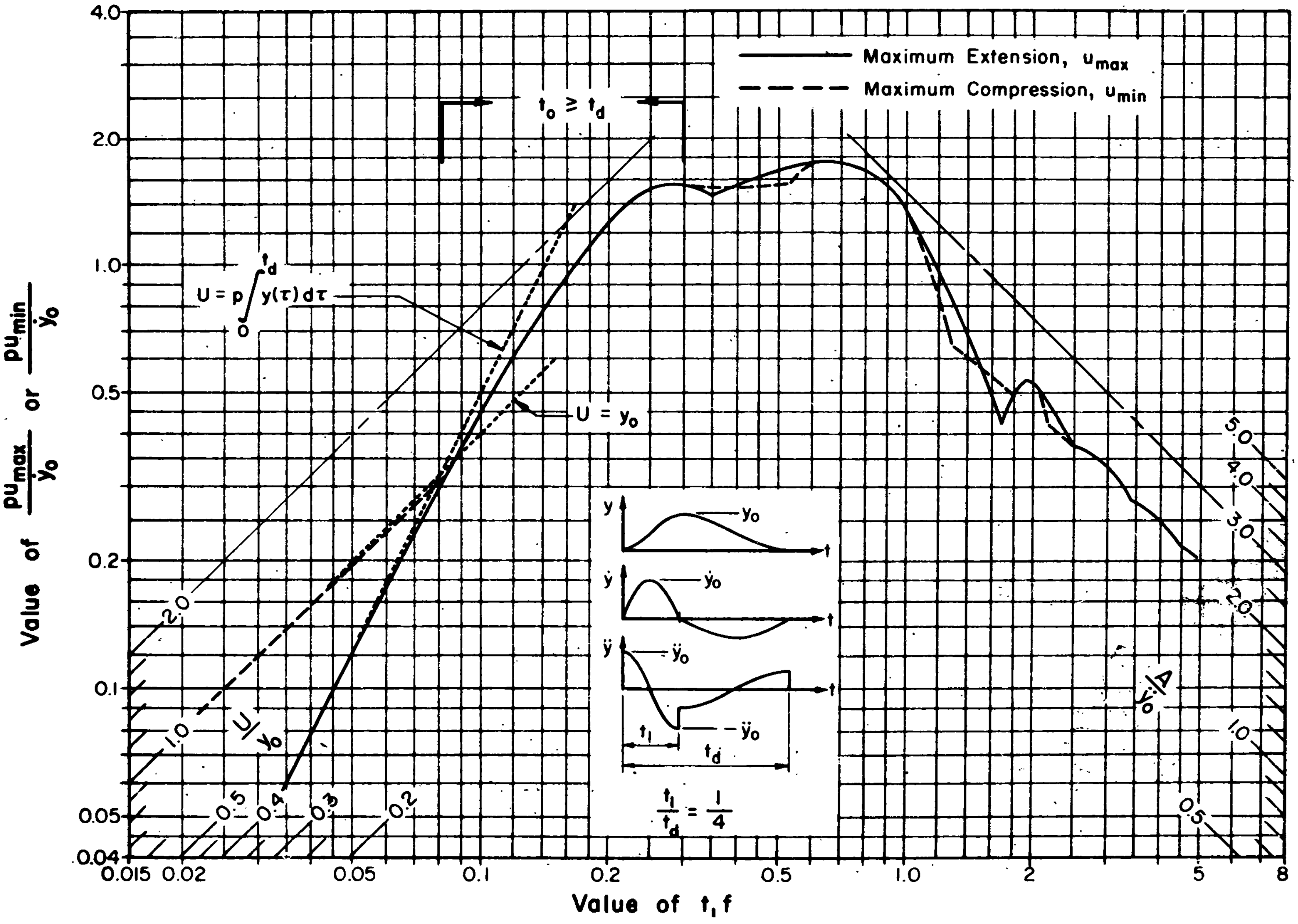


FIG. 2.25b SPECTRA FOR ABSOLUTE MAXIMUM AND ABSOLUTE MINIMUM DEFORMATIONS
Undamped Elastic Systems Subjected to a Skewed Versed-Sine Displacement Pulse

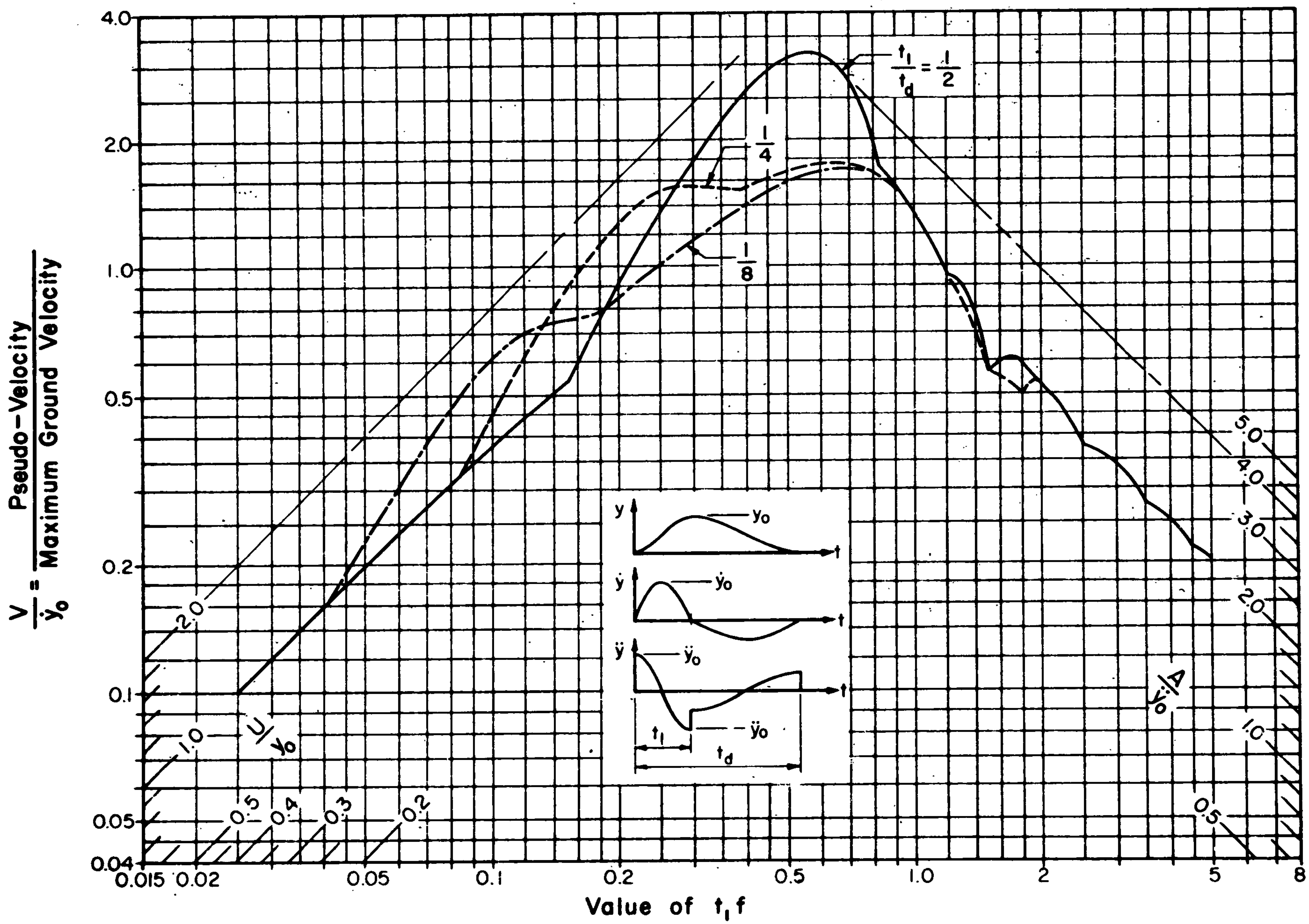


FIG. 2.26 DEFORMATION SPECTRA FOR UNDAMPED ELASTIC SYSTEMS SUBJECTED TO SKEWED VERSED-SINE DISPLACEMENT PULSES

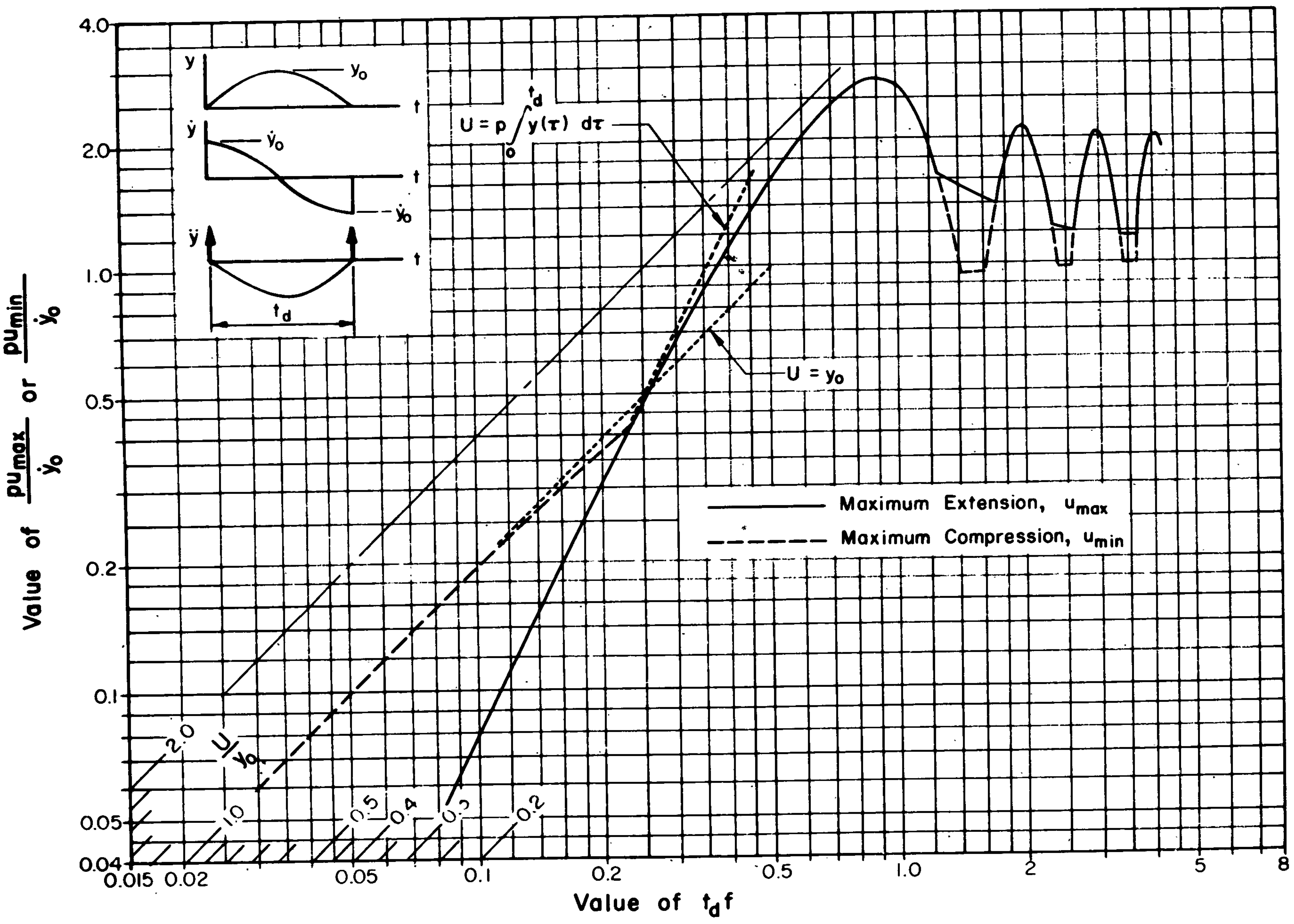


FIG. 2.27 SPECTRA FOR ABSOLUTE MAXIMUM AND ABSOLUTE MINIMUM DEFORMATIONS
Undamped Elastic Systems Subjected to a Half-Sine Displacement Pulse

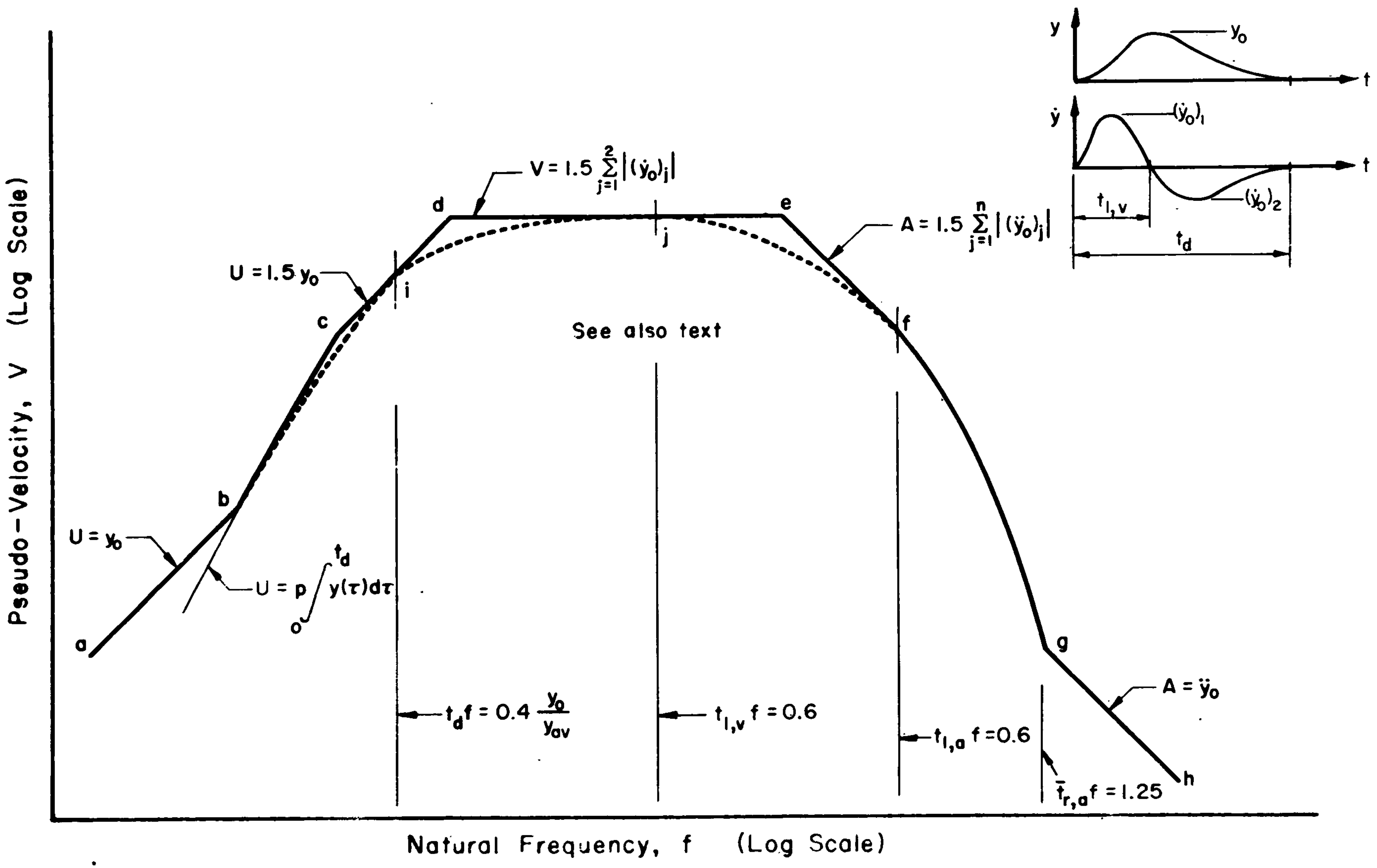


FIG. 2.28 DESIGN SPECTRUM FOR THE ABSOLUTE MAXIMUM DEFORMATION OF SYSTEMS SUBJECTED TO A HALF-CYCLE DISPLACEMENT PULSE
Undamped Elastic Systems; Continuous Input Acceleration Functions

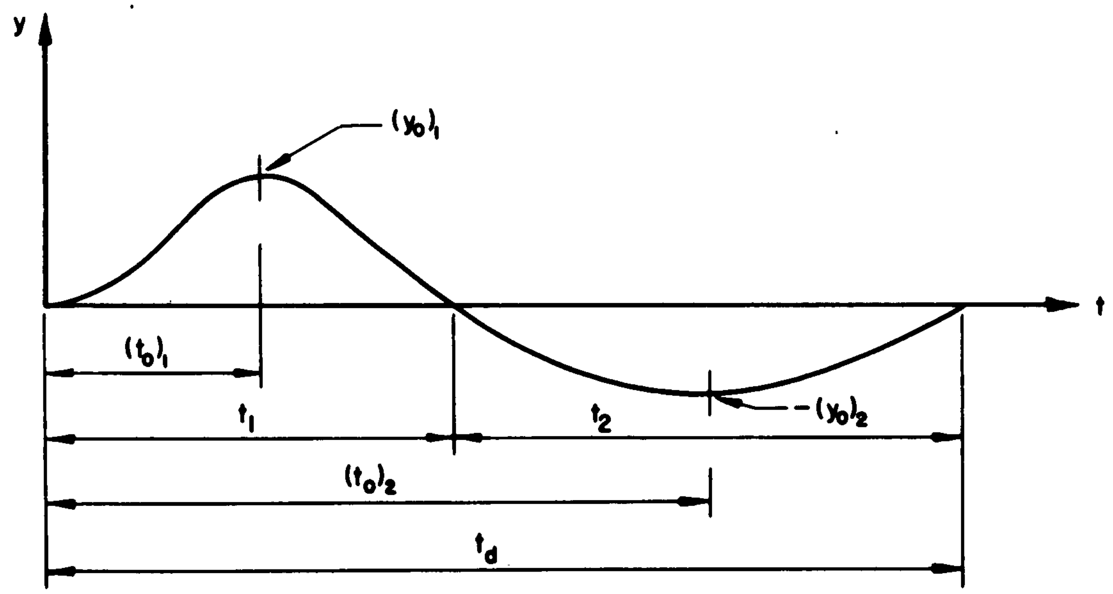


FIG. 2.29 TYPICAL FULL-CYCLE DISPLACEMENT PULSE

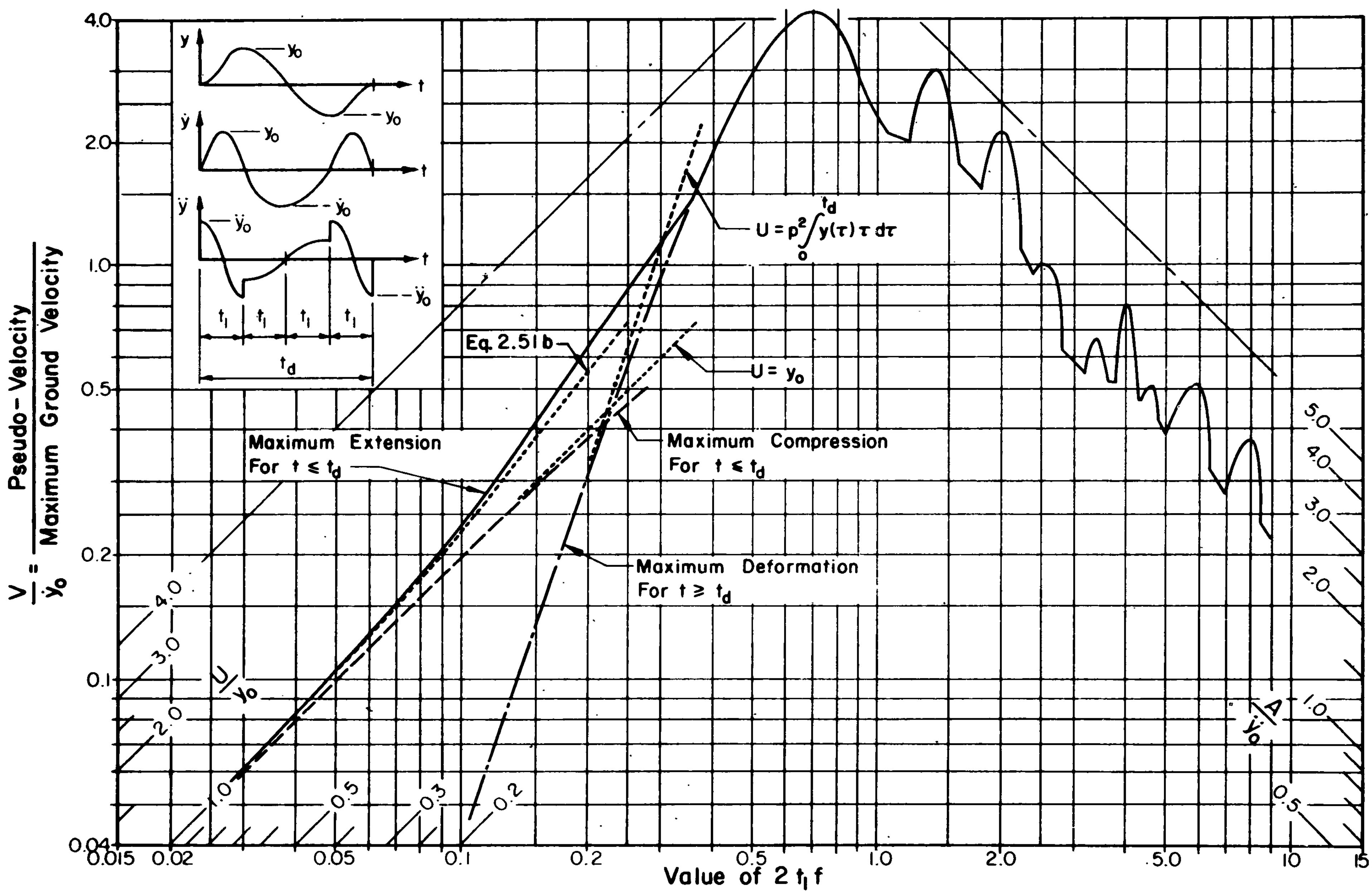


FIG. 2.30 SPECTRA FOR MAXIMUM AND MINIMUM DEFORMATIONS--Undamped Elastic Systems Subjected to a Full-Cycle Displacement Pulse; Velocity Diagram consists of Three Half-Sine Waves as Shown

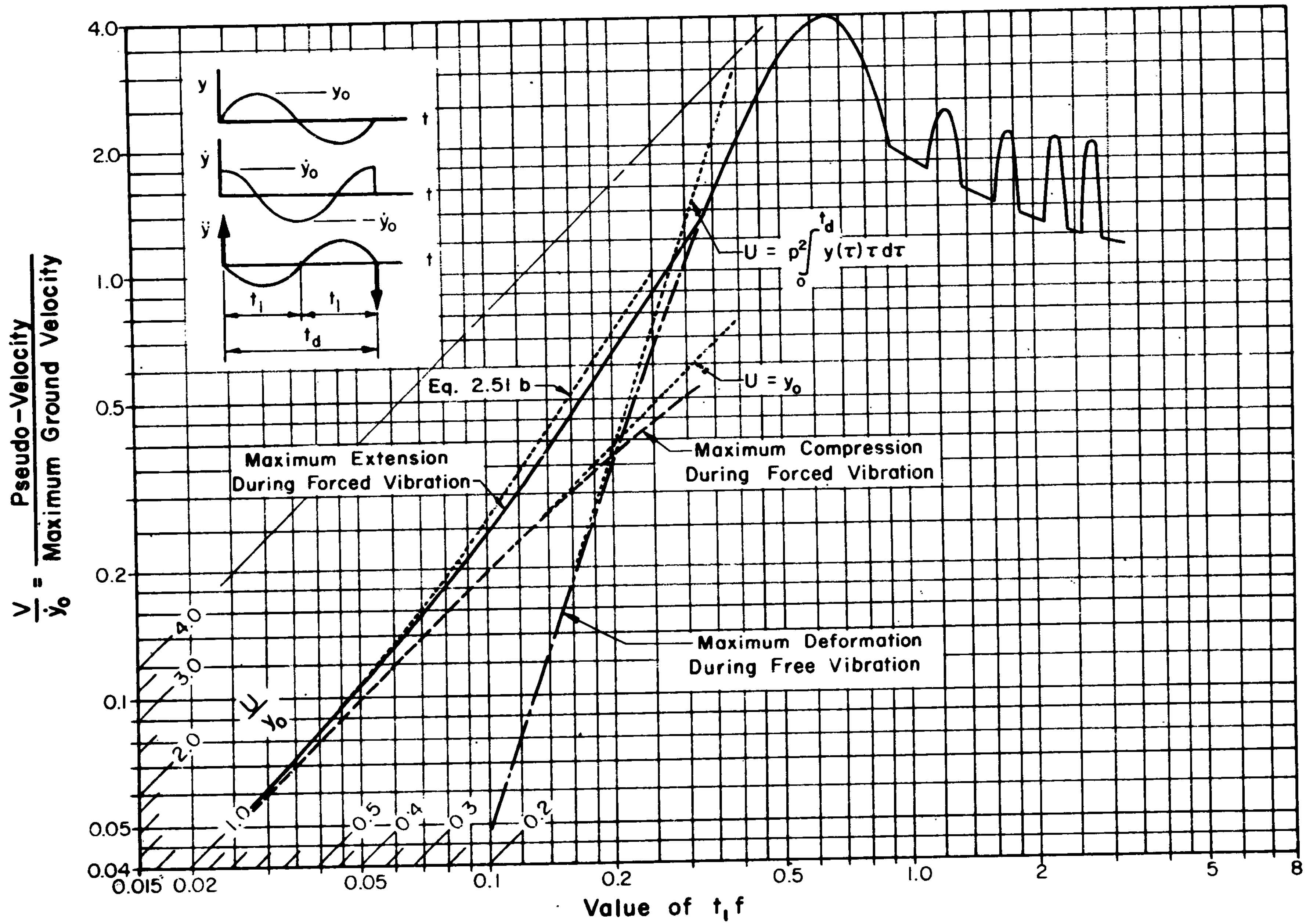


FIG. 2.31a SPECTRA FOR MAXIMUM AND MINIMUM DEFORMATIONS--Undamped Elastic Systems Subjected to a Sinusoidal Displacement Pulse

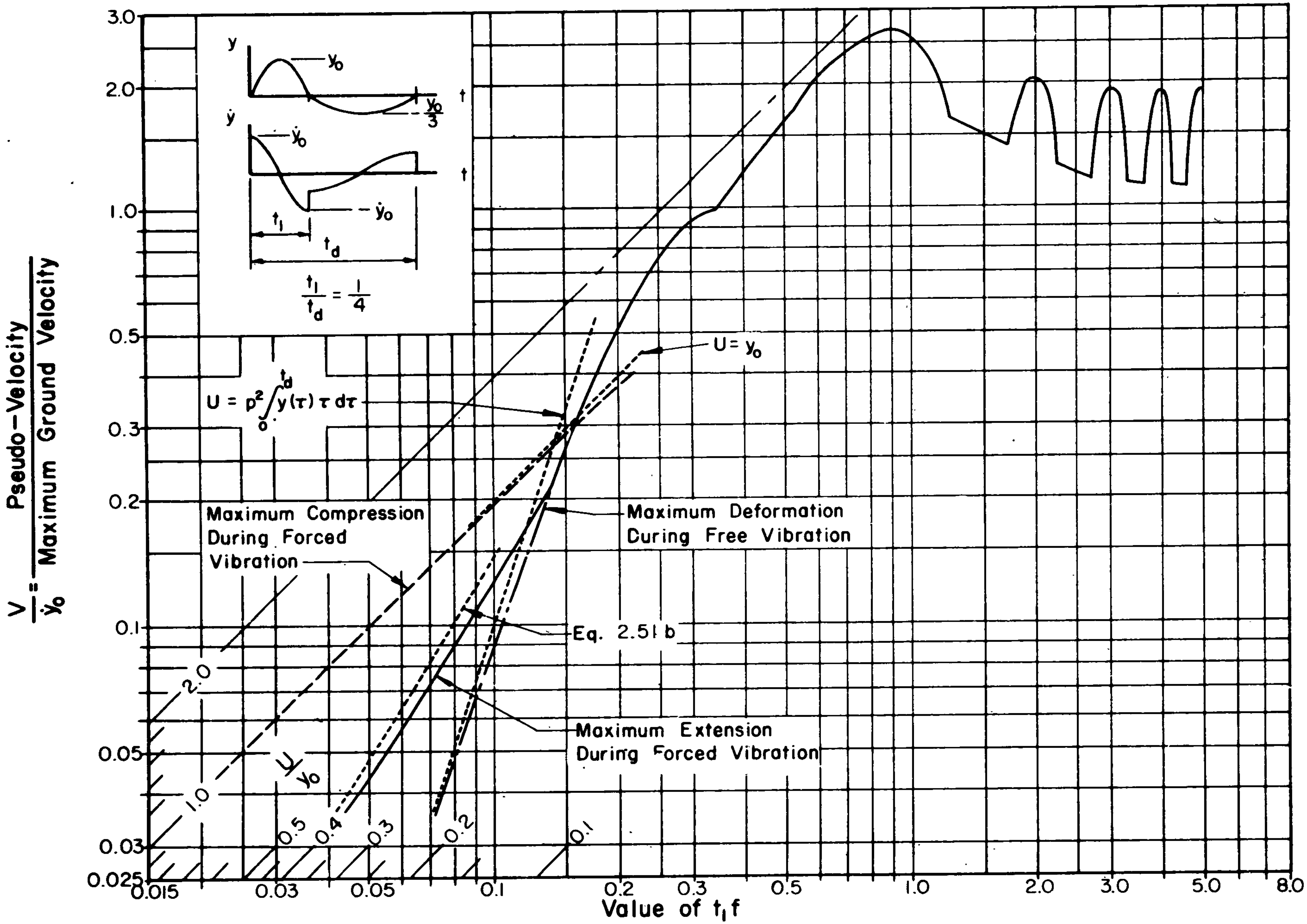


FIG. 2.31b SPECTRA FOR MAXIMUM AND MINIMUM DEFORMATIONS--Undamped Elastic Systems Subjected to a Displacement Pulse of Two Half-Sine Waves

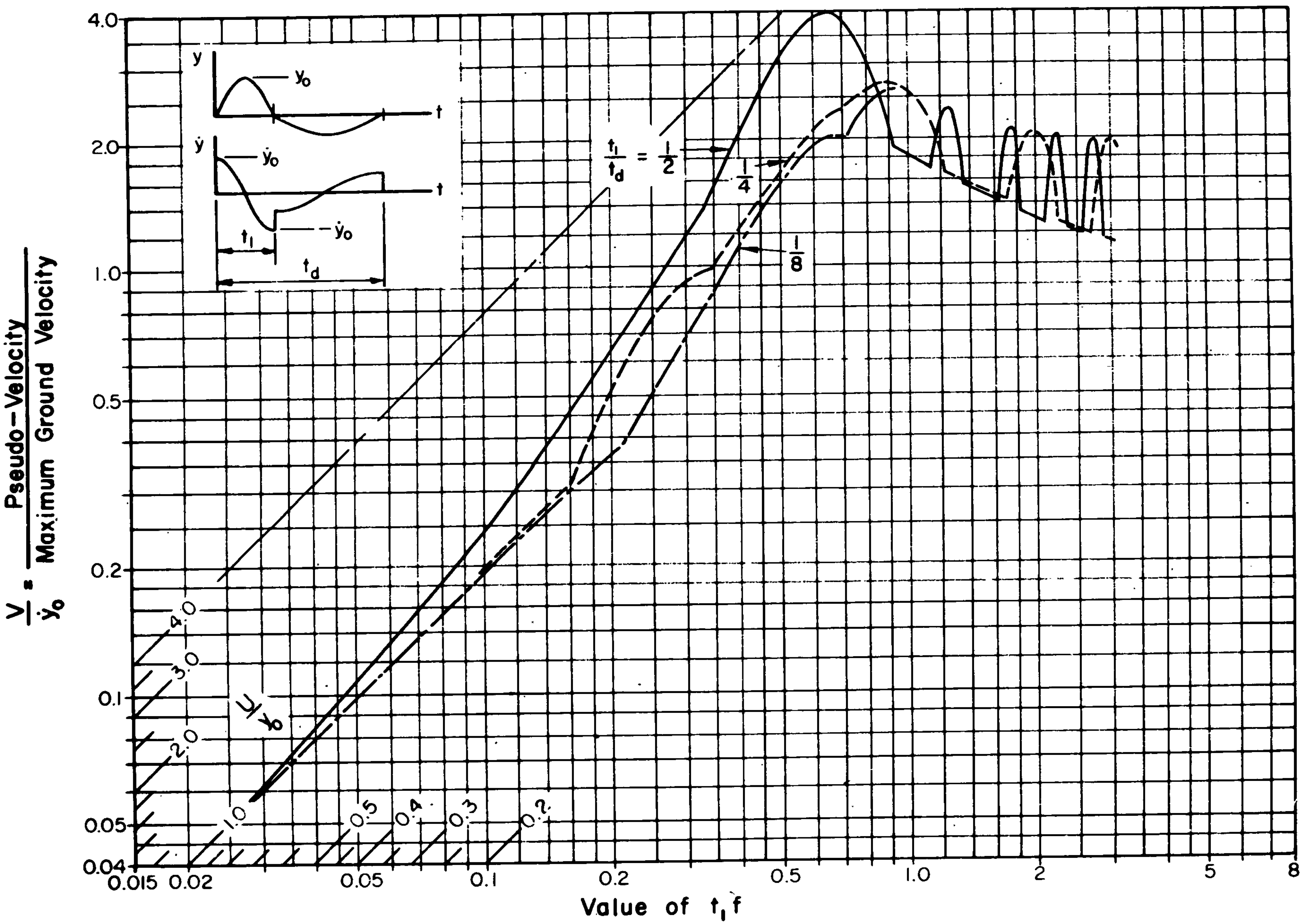


FIG. 2.32 DEFORMATION SPECTRA FOR UNDAMPED ELASTIC SYSTEMS SUBJECTED TO DISPLACEMENT PULSES COMPOSED OF TWO HALF-SINE WAVES

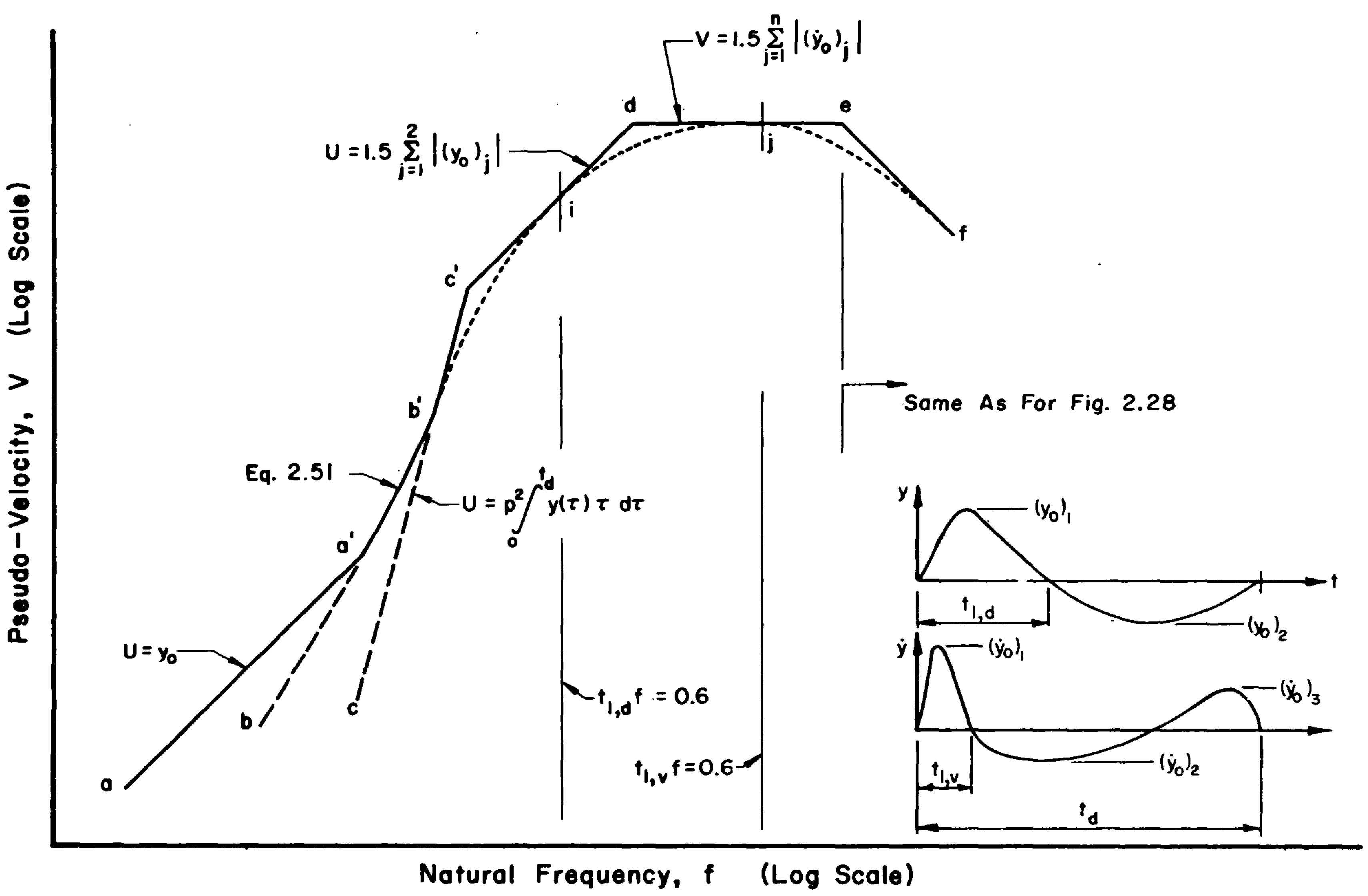


FIG. 2.33 DESIGN SPECTRUM FOR THE ABSOLUTE MAXIMUM DEFORMATION OF SYSTEMS SUBJECTED TO A FULL-CYCLE DISPLACEMENT PULSE
Undamped Elastic Systems; Continuous Input Acceleration Pulses

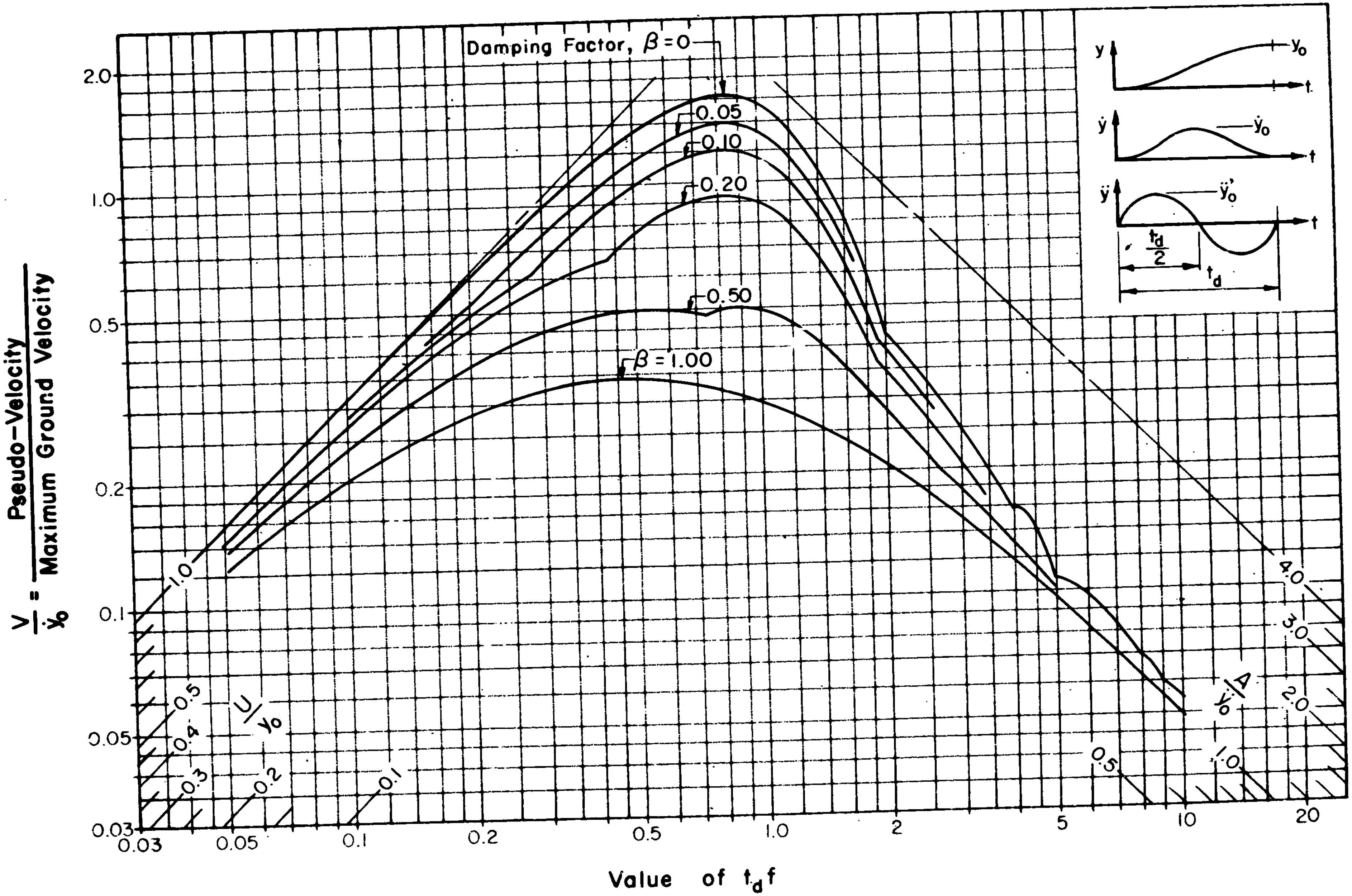


FIG. 2.34 DEFORMATION SPECTRA FOR DAMPED ELASTIC SYSTEMS SUBJECTED TO A VERSED-SINE VELOCITY PULSE

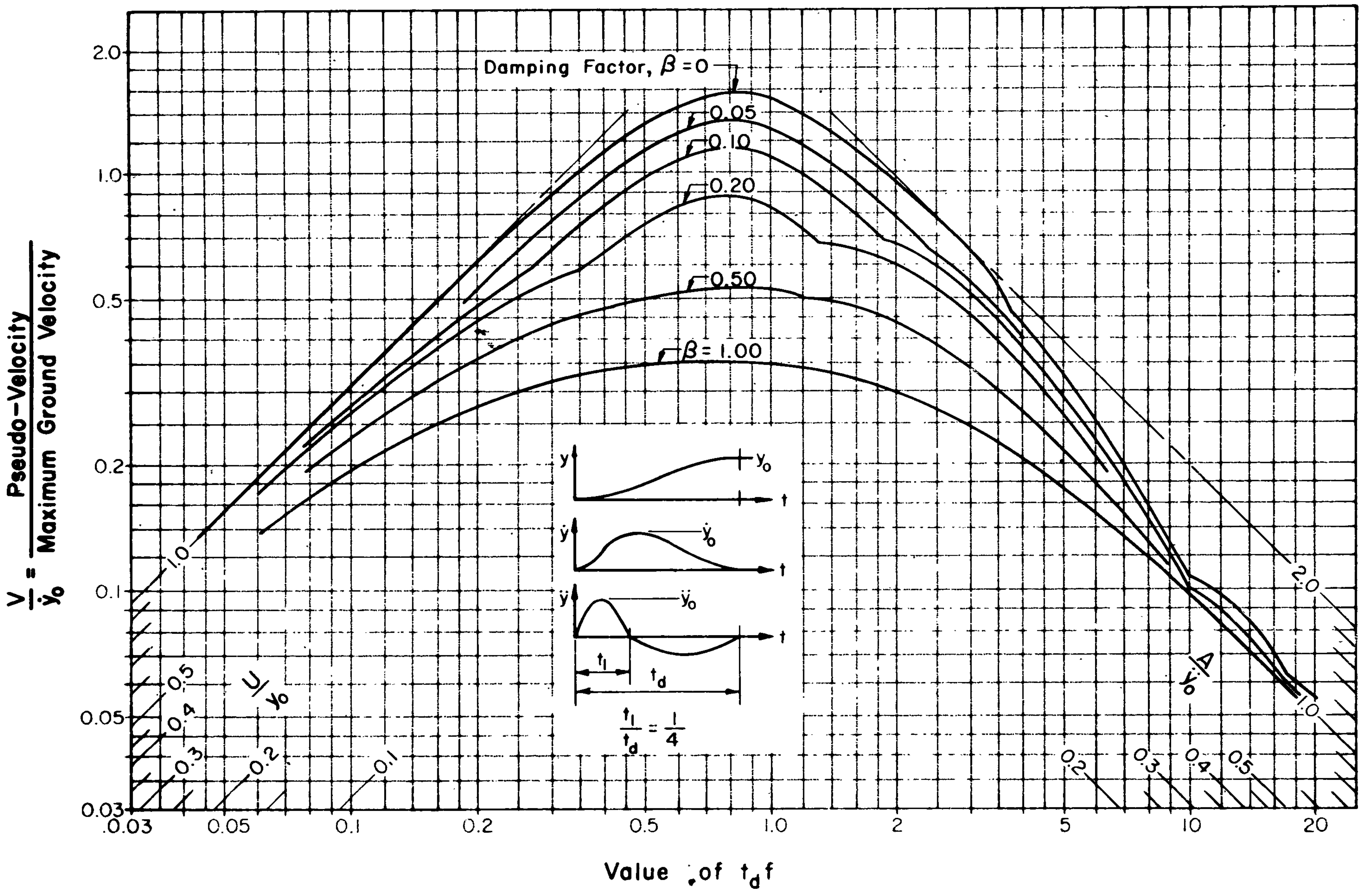


FIG. 2.35a DEFORMATION SPECTRA FOR DAMPED ELASTIC SYSTEMS SUBJECTED TO A SKEWED VERSED-SINE VELOCITY PULSE

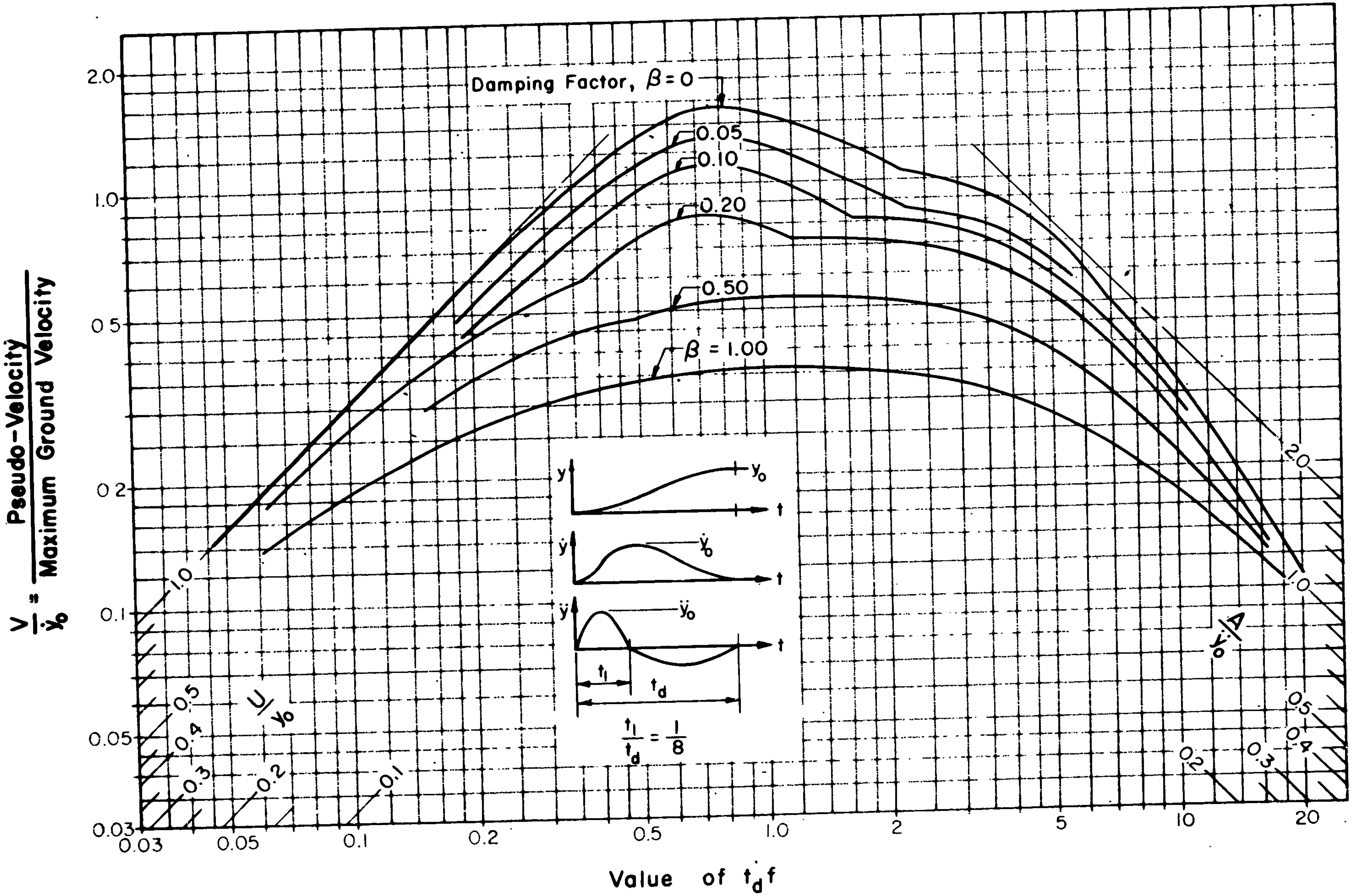


FIG. 2.35b DEFORMATION SPECTRA FOR DAMPED ELASTIC SYSTEMS SUBJECTED TO A SKEWED VERSED-SINE VELOCITY PULSE

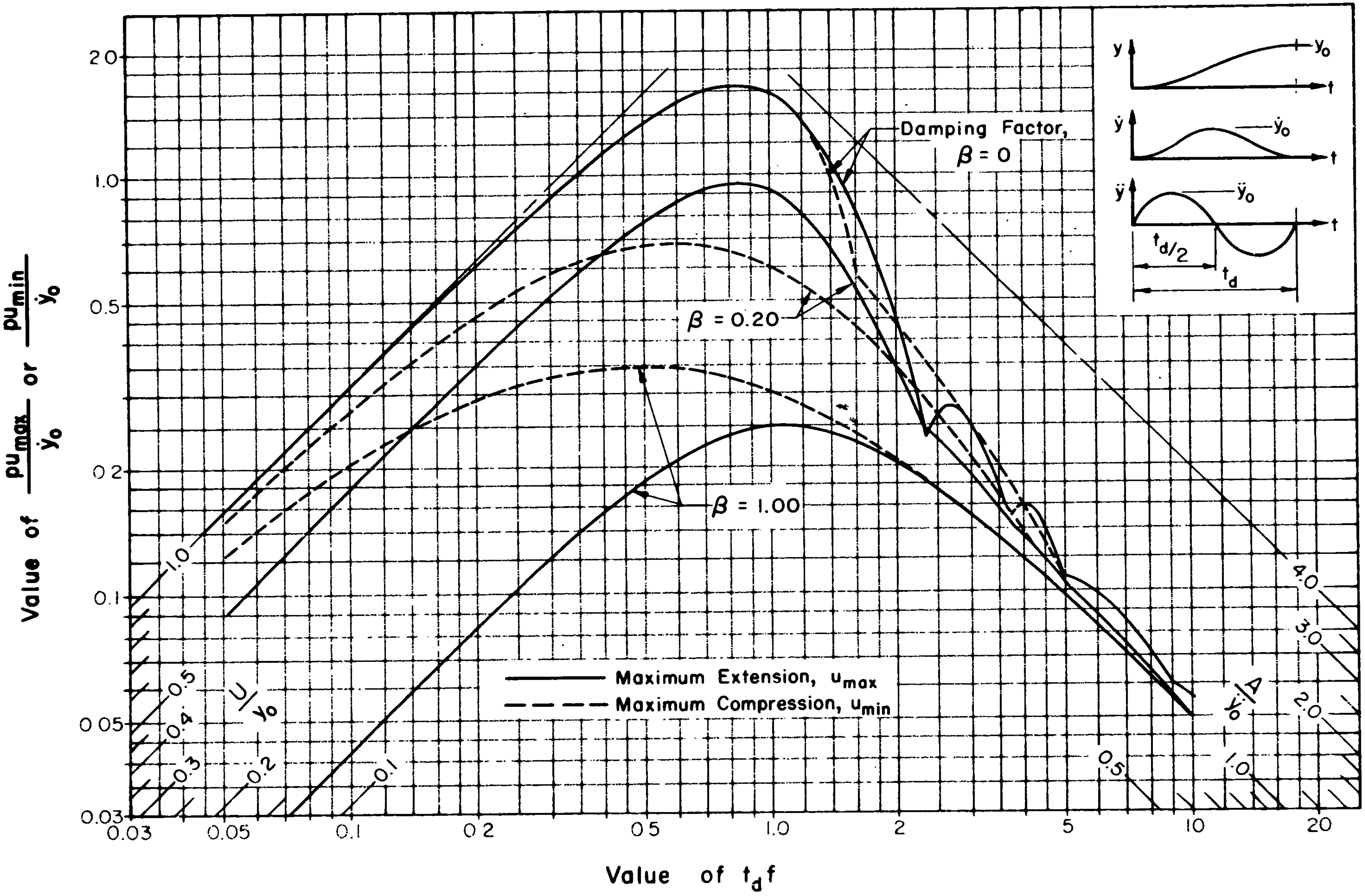


FIG. 2.36 COMPARISON OF SPECTRA FOR ABSOLUTE MAXIMUM AND ABSOLUTE MINIMUM DEFORMATIONS
Damped Elastic Systems Subjected to a Versed-Sine Velocity Pulse

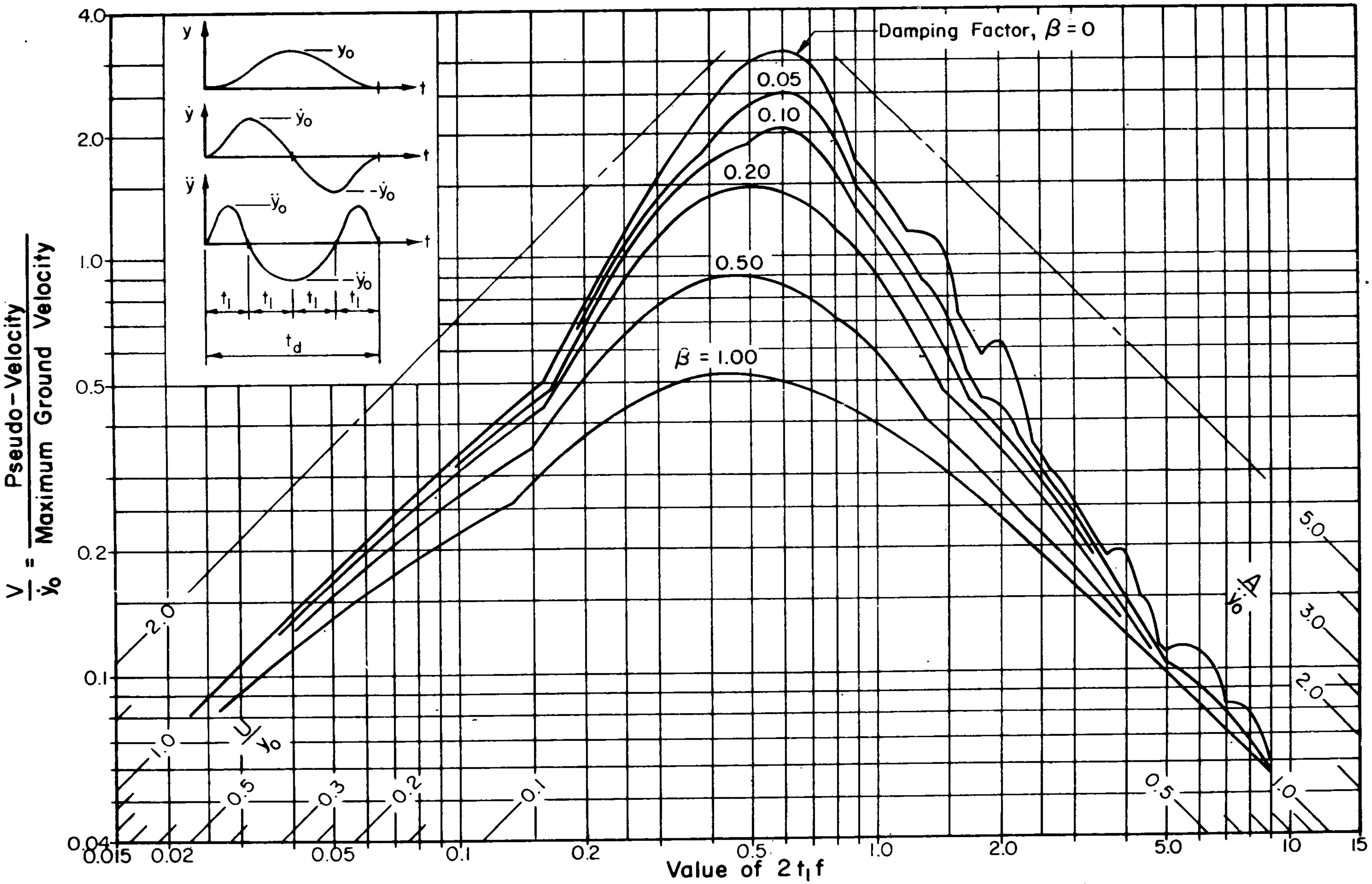


FIG. 2.37 DEFORMATION SPECTRA FOR DAMPED ELASTIC SYSTEMS SUBJECTED TO A HALF-CYCLE DISPLACEMENT PULSE
Acceleration Diagram Consists of Three Half-Sine Waves as Shown

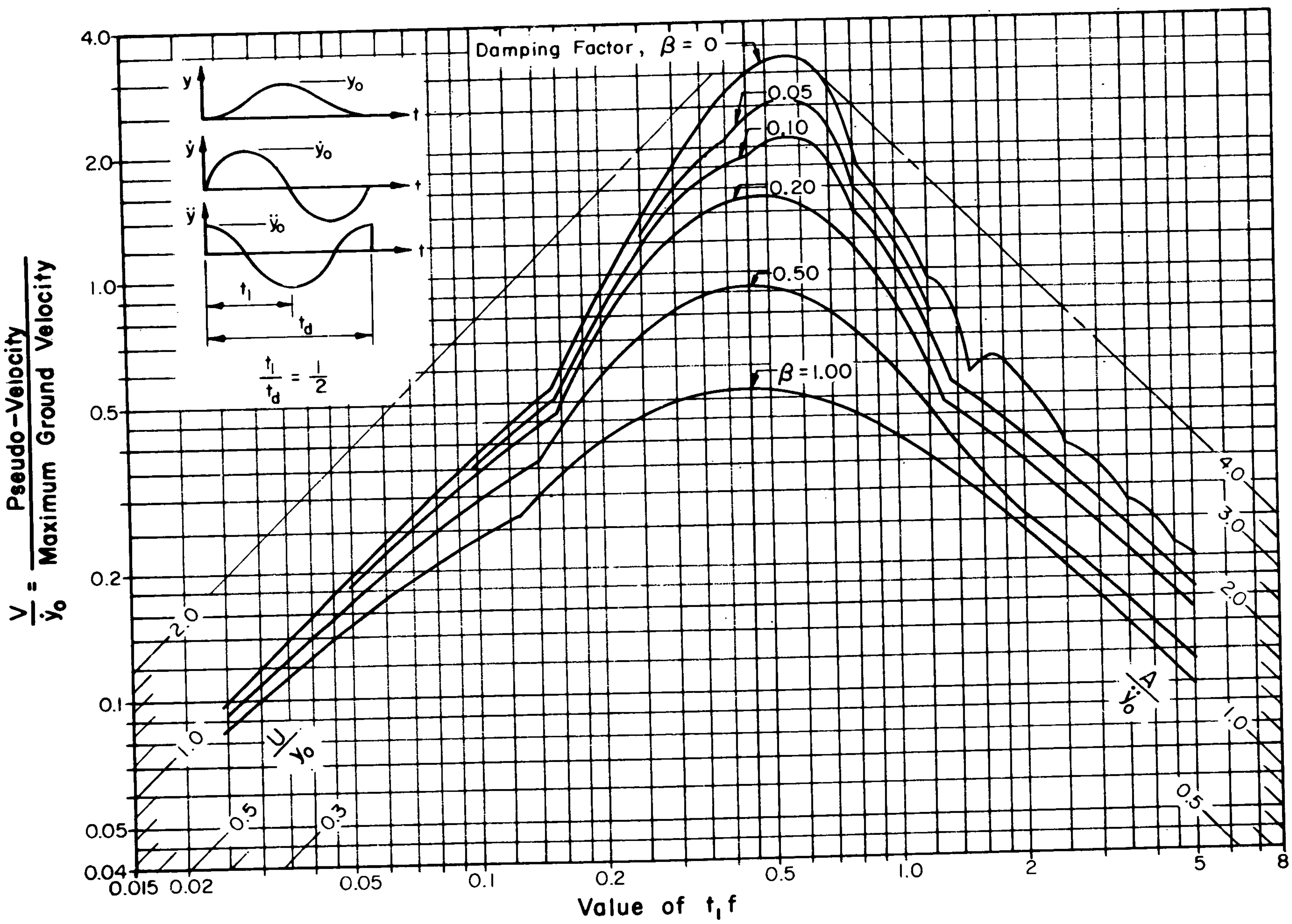


FIG. 2.38 DEFORMATION SPECTRA FOR DAMPED ELASTIC SYSTEMS SUBJECTED TO A VERSED-SINE DISPLACEMENT PULSE

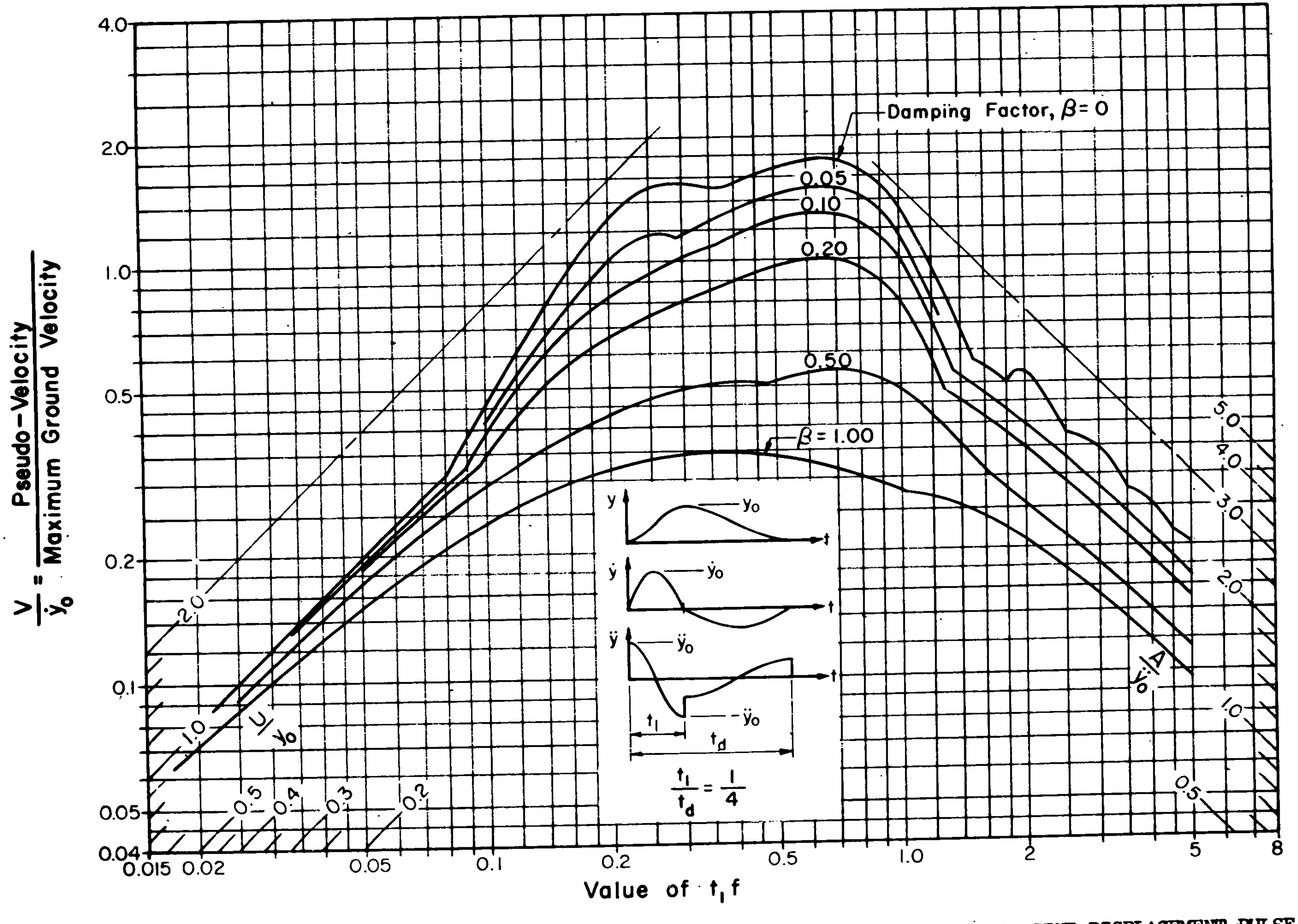


FIG. 2.39a DEFORMATION SPECTRA FOR DAMPED ELASTIC SYSTEMS SUBJECTED TO A SKEWED VERSED-SINE DISPLACEMENT PULSE

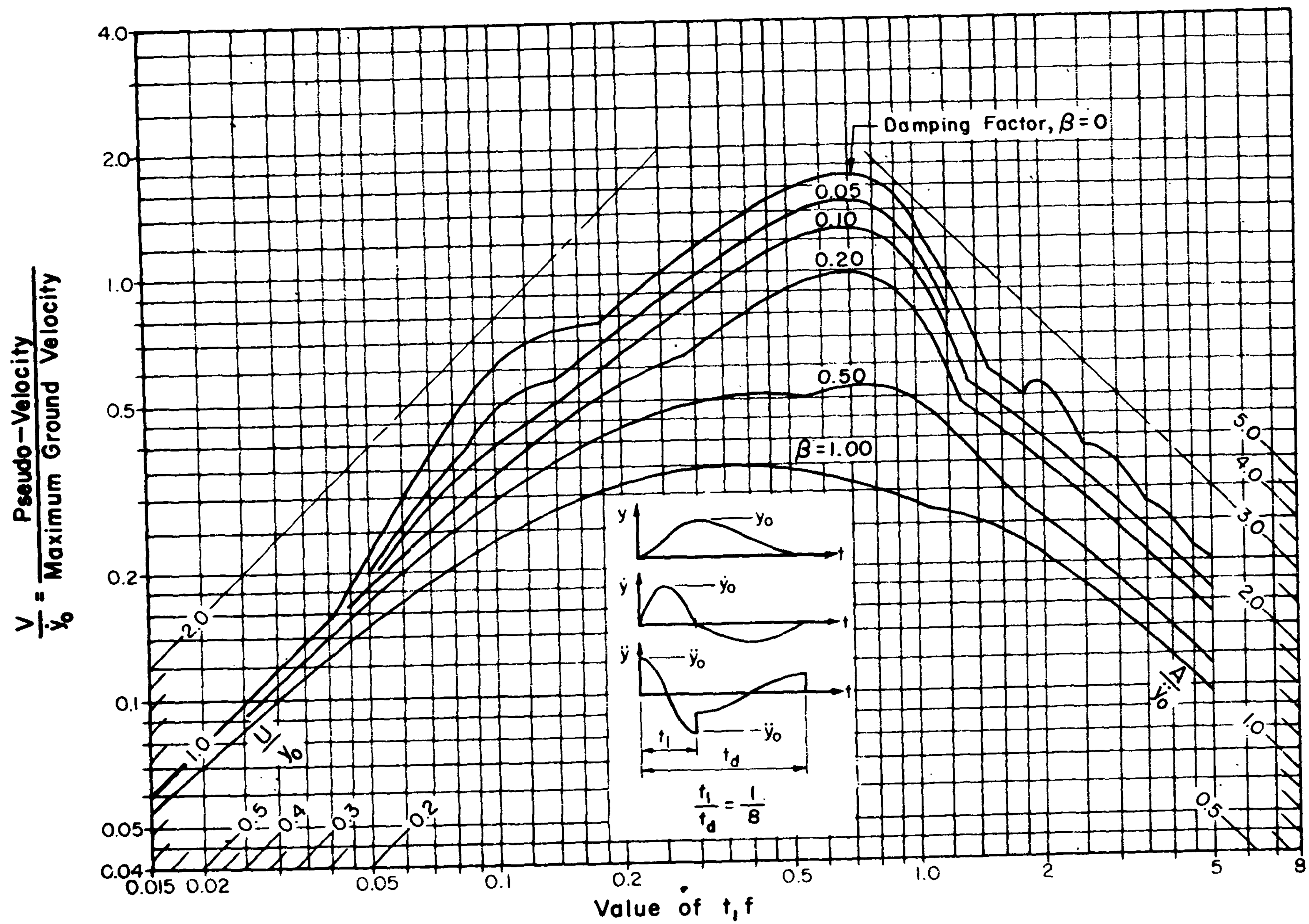


FIG. 2.39b DEFORMATION SPECTRA FOR DAMPED ELASTIC SYSTEMS SUBJECTED TO A SKEWED VERSED-SINE DISPLACEMENT PULSE

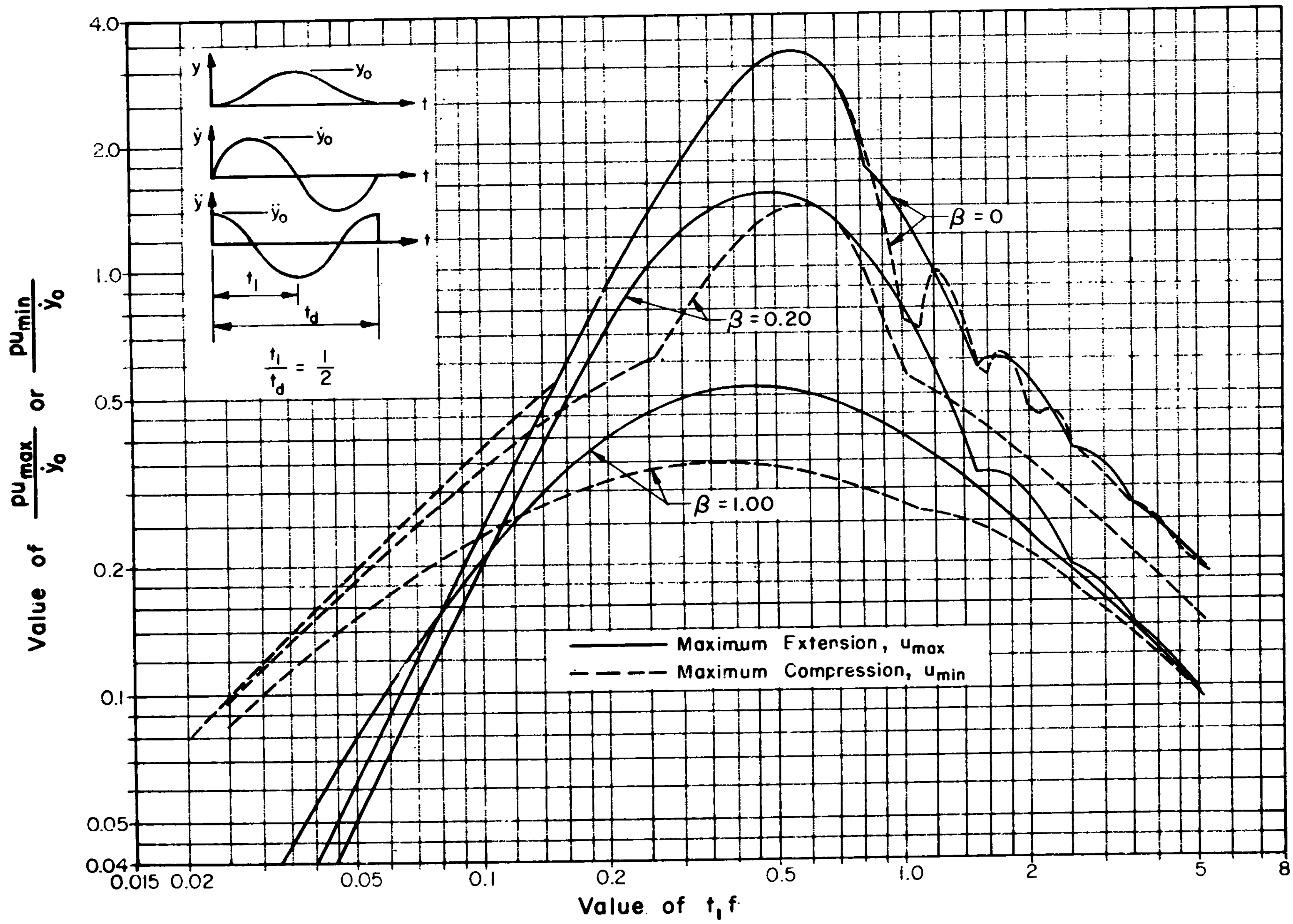


FIG. 2.40 COMPARISON OF SPECTRA FOR ABSOLUTE MAXIMUM AND ABSOLUTE MINIMUM DEFORMATIONS
Damped Elastic Systems Subjected to a Versed-Sine Displacement Pulse

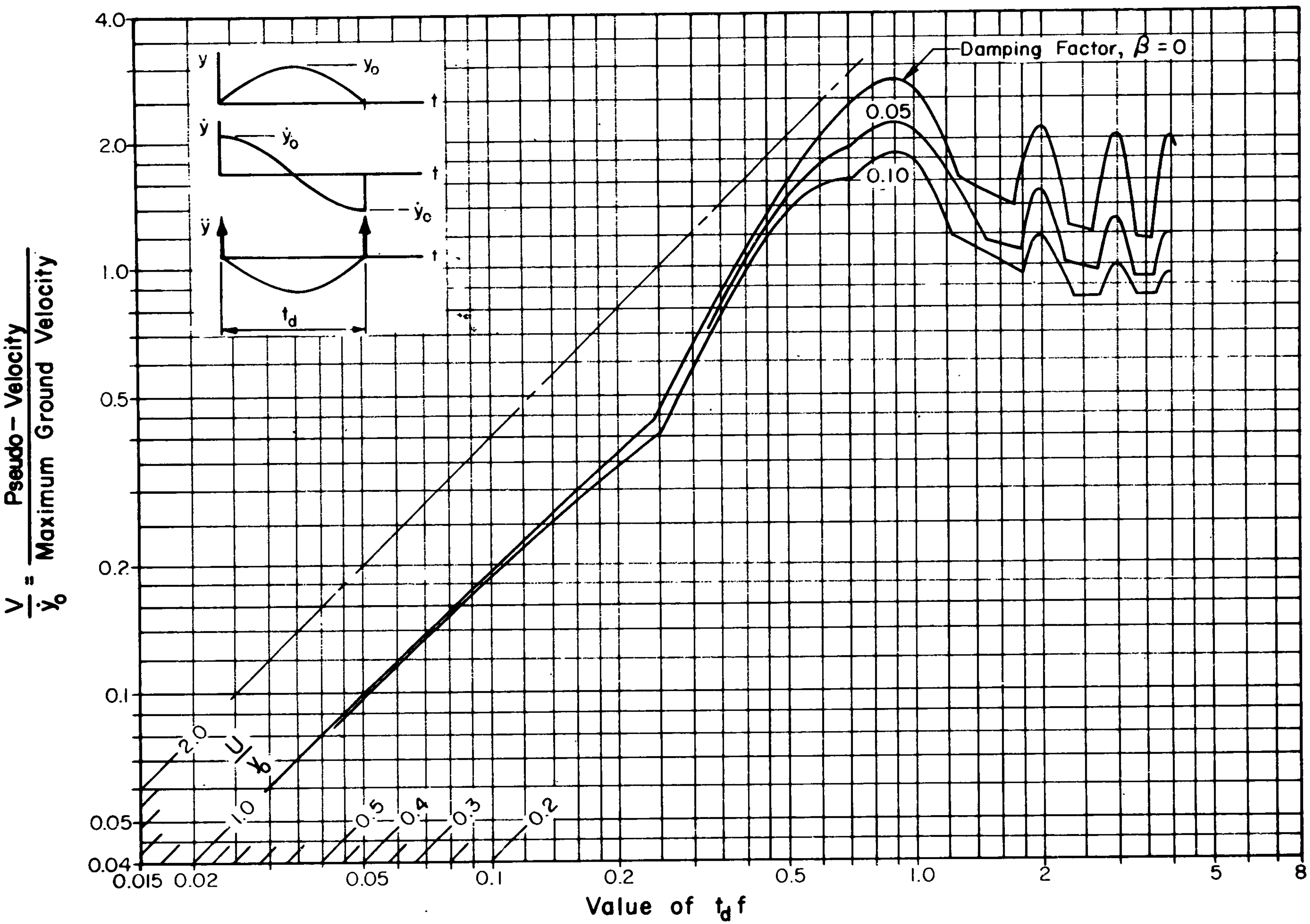


FIG. 2.41 DEFORMATION SPECTRA FOR DAMPED ELASTIC SYSTEMS SUBJECTED TO A HALF-SINE DISPLACEMENT PULSE

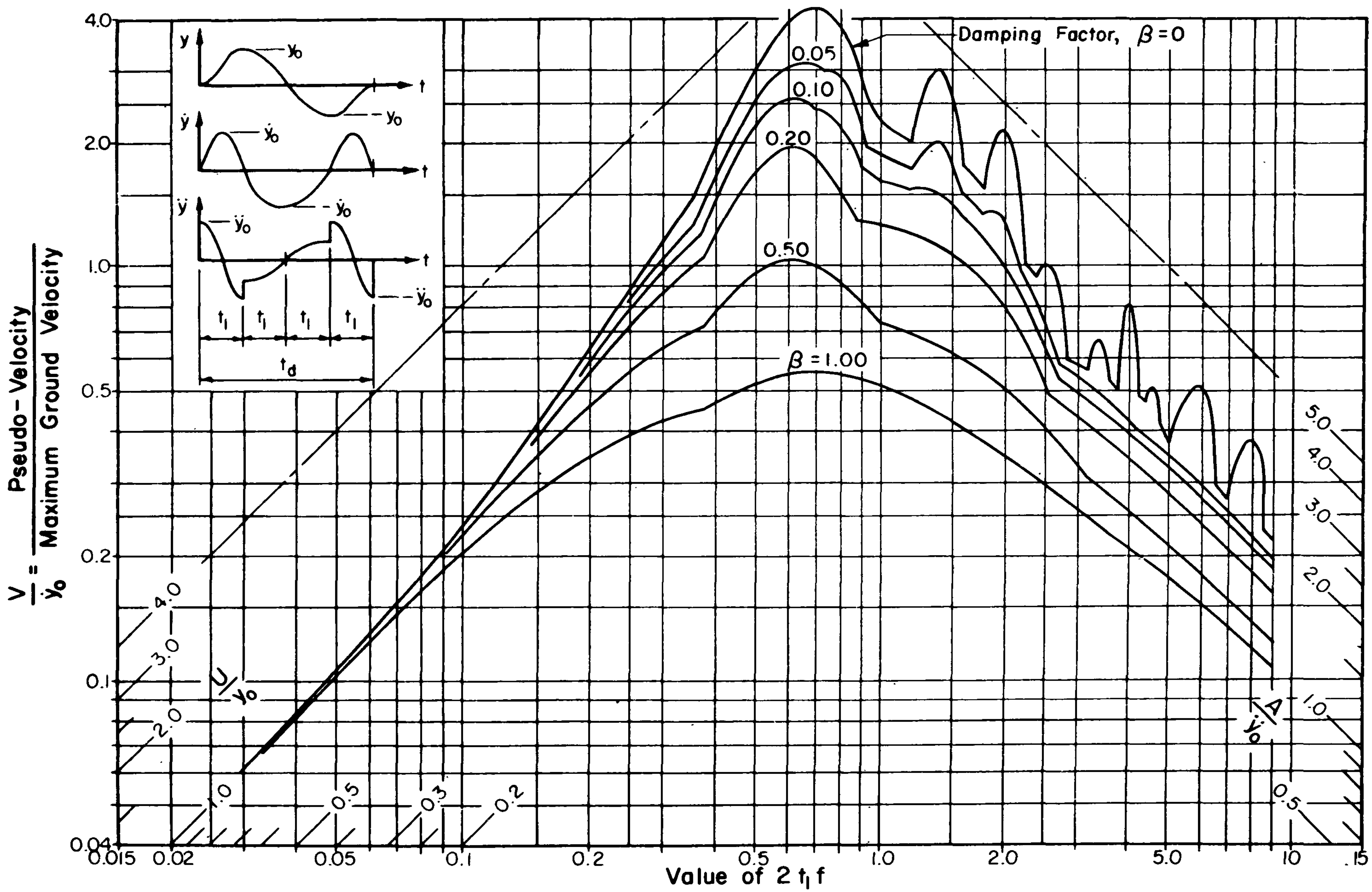


FIG. 2.42 DEFORMATION SPECTRA FOR DAMPED ELASTIC SYSTEMS SUBJECTED TO A FULL-CYCLE DISPLACEMENT PULSE
Velocity Diagram Consists of Three Half-Sine Waves as Shown

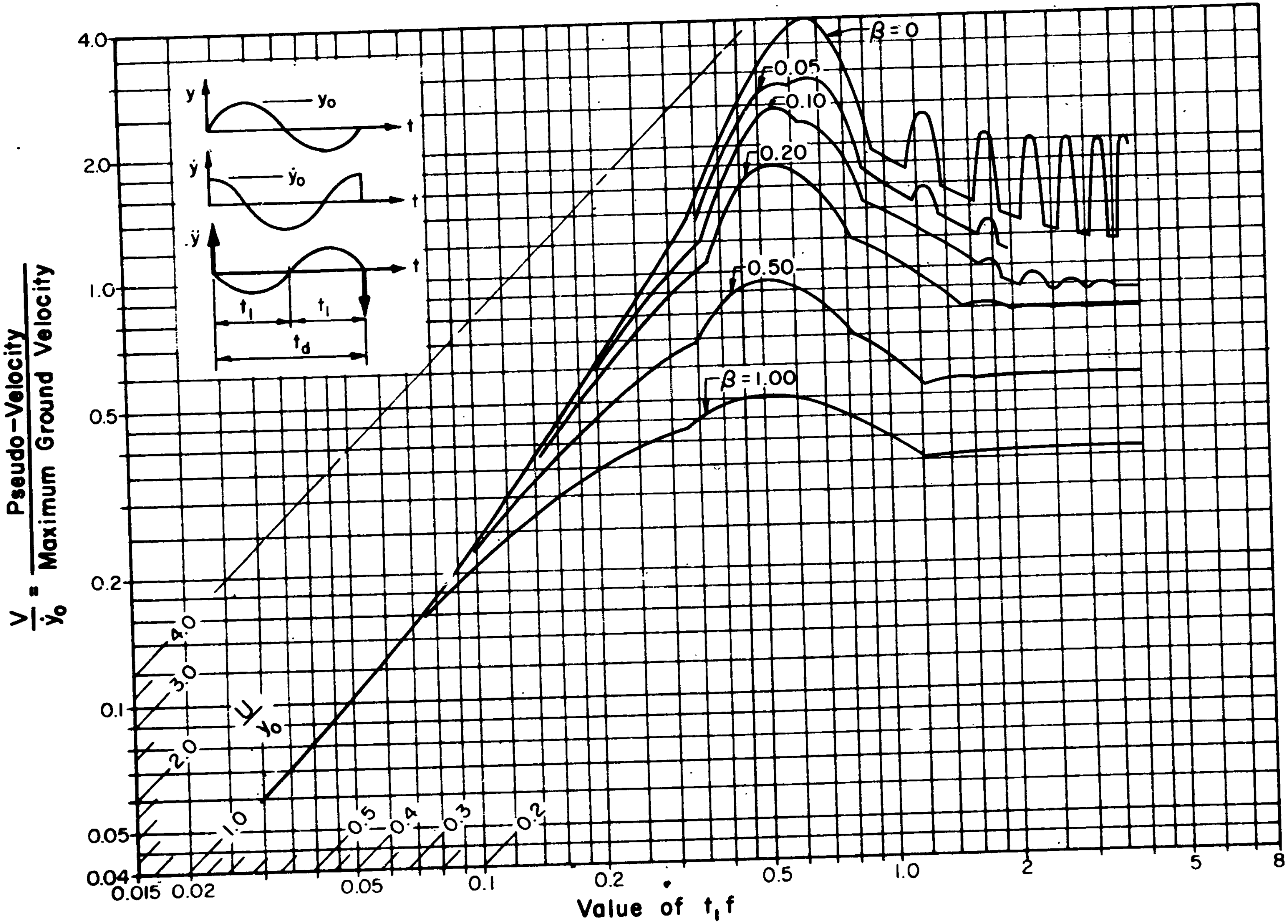


FIG. 2.43a DEFORMATION SPECTRA FOR DAMPED ELASTIC SYSTEMS SUBJECTED TO A FULL-CYCLE SINUSOIDAL DISPLACEMENT PULSE

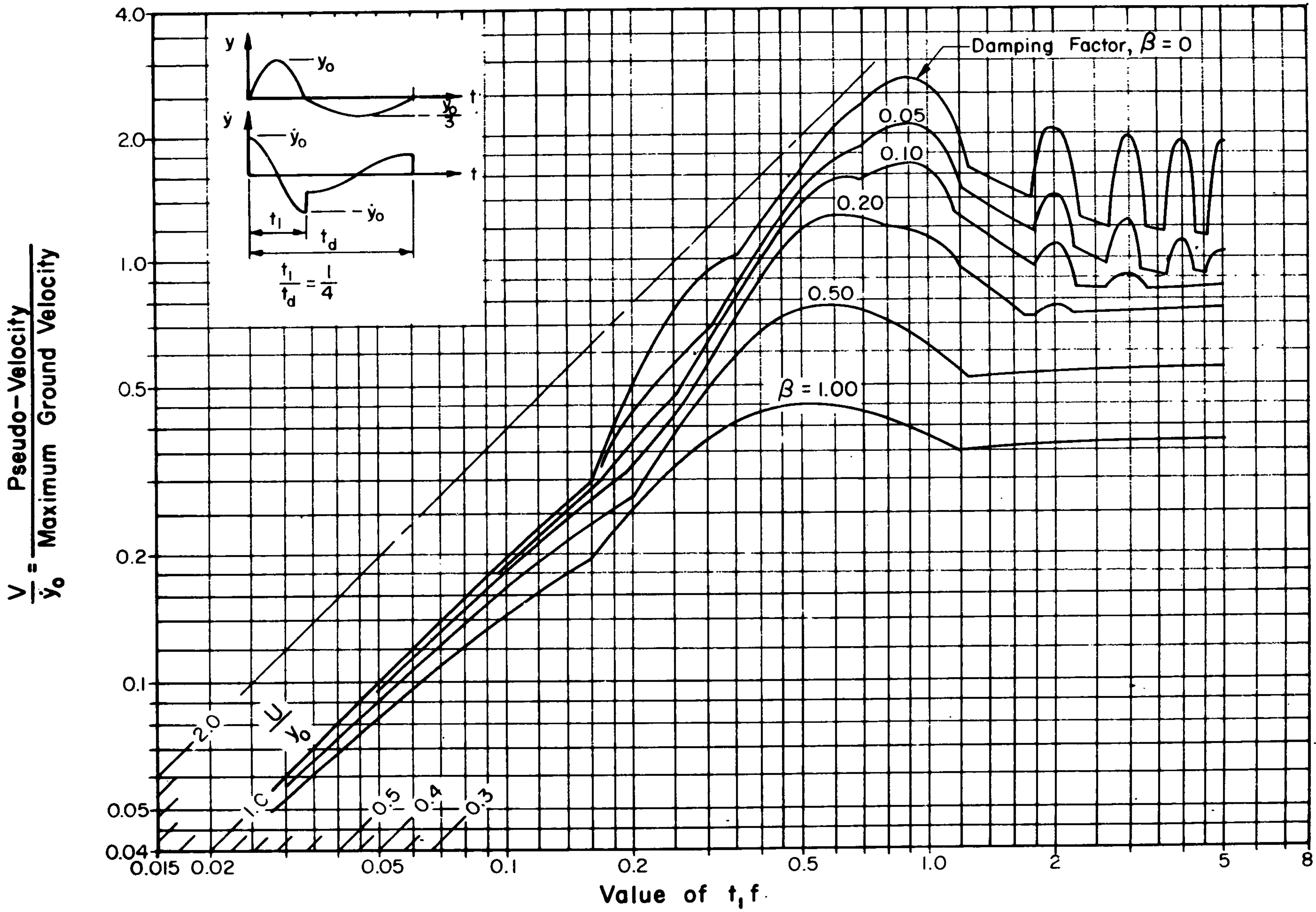


FIG. 2.43b DEFORMATION SPECTRA FOR DAMPED ELASTIC SYSTEMS SUBJECTED TO A FULL-CYCLE DISPLACEMENT PULSE
Displacement Pulse Consists of Two Half-Sine Waves

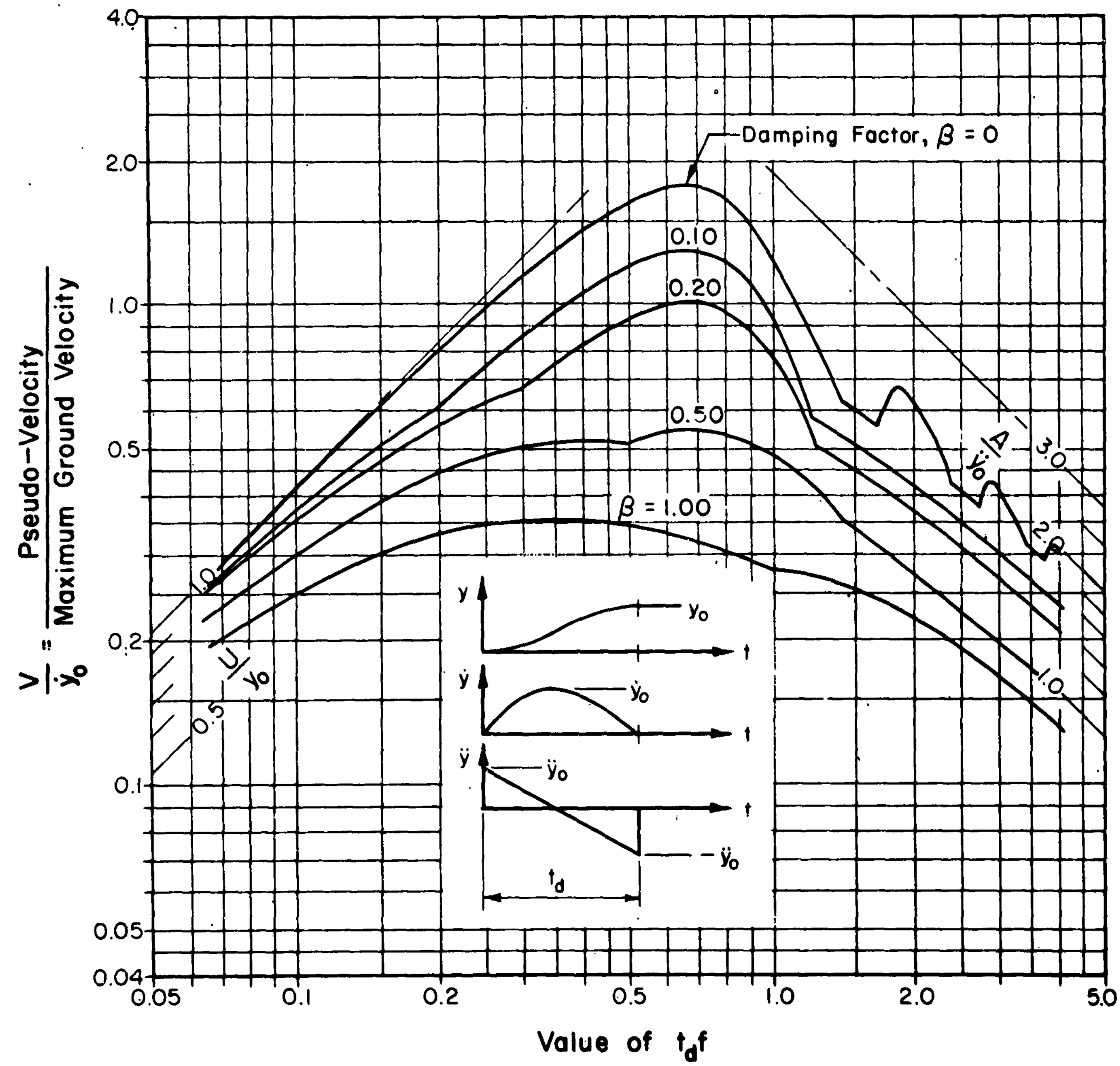


FIG. 2.44a DEFORMATION SPECTRA FOR DAMPED ELASTIC SYSTEMS SUBJECTED TO A HALF-CYCLE PARABOLIC VELOCITY PULSE

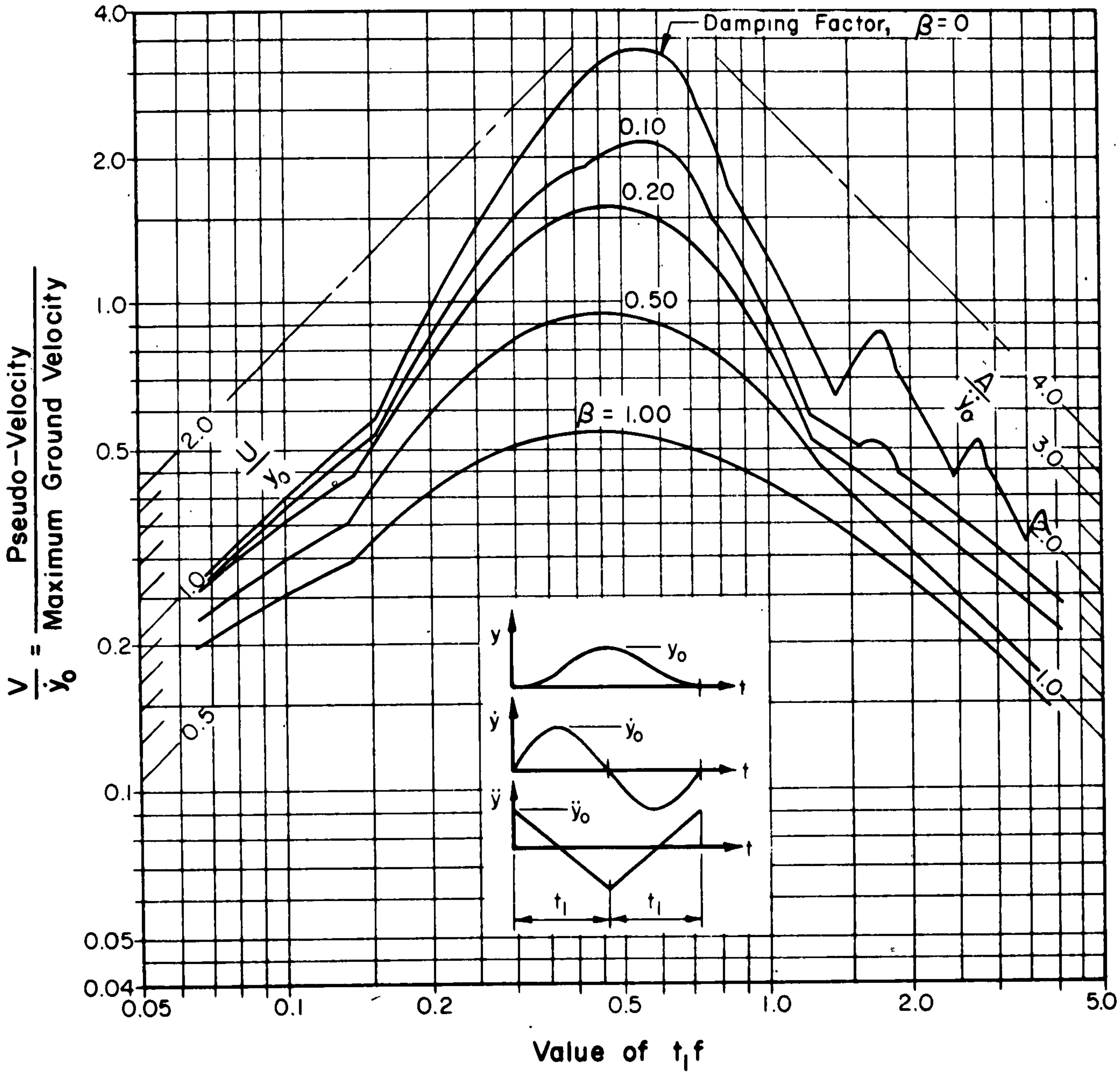


FIG. 2.44b DEFORMATION SPECTRA FOR DAMPED ELASTIC SYSTEMS SUBJECTED TO A FULL-CYCLE PARABOLIC VELOCITY PULSE

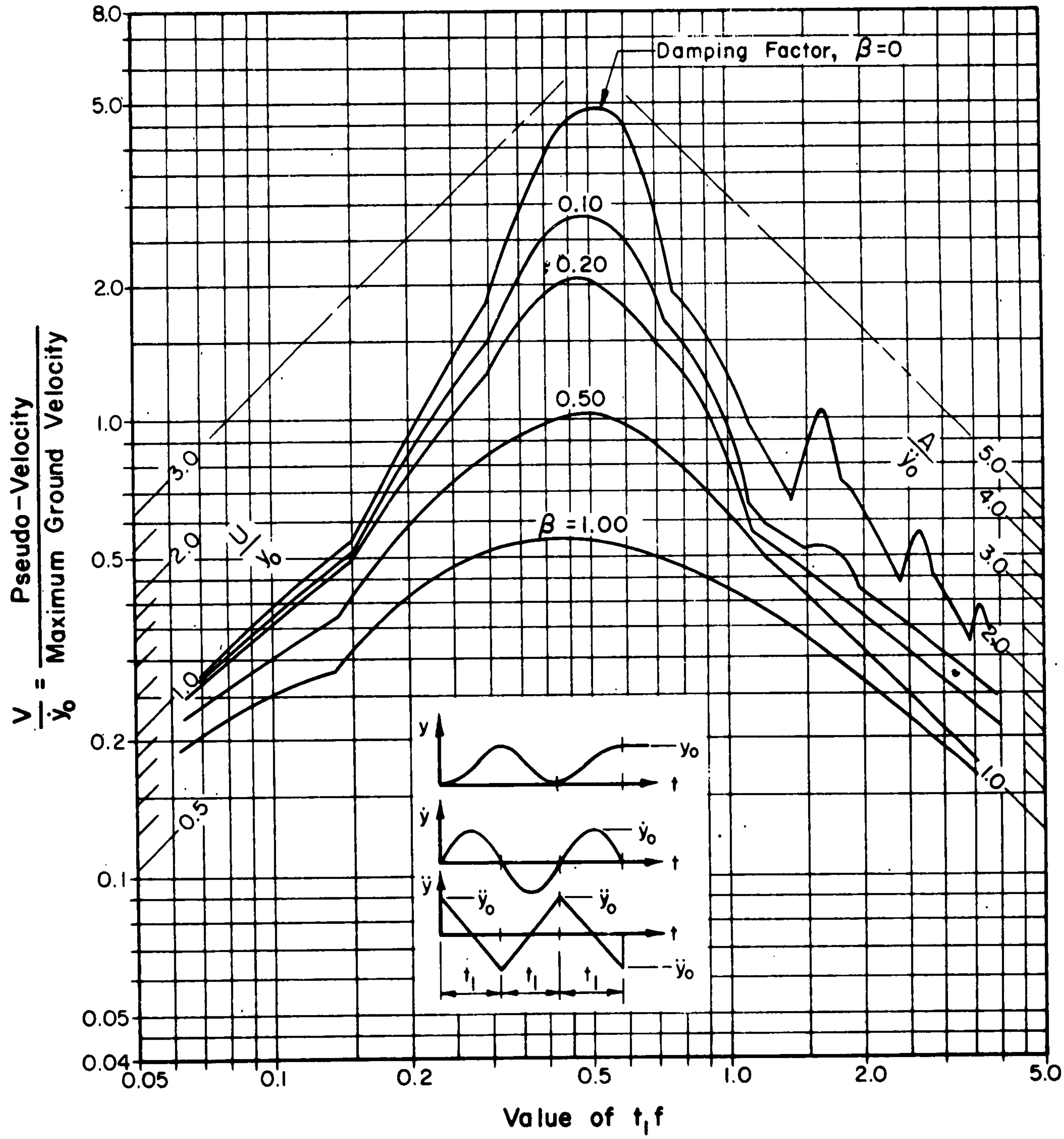


FIG. 2.44c DEFORMATION SPECTRA FOR DAMPED ELASTIC SYSTEMS SUBJECTED TO A SEQUENCE OF THREE PARABOLIC VELOCITY HALF-CYCLES

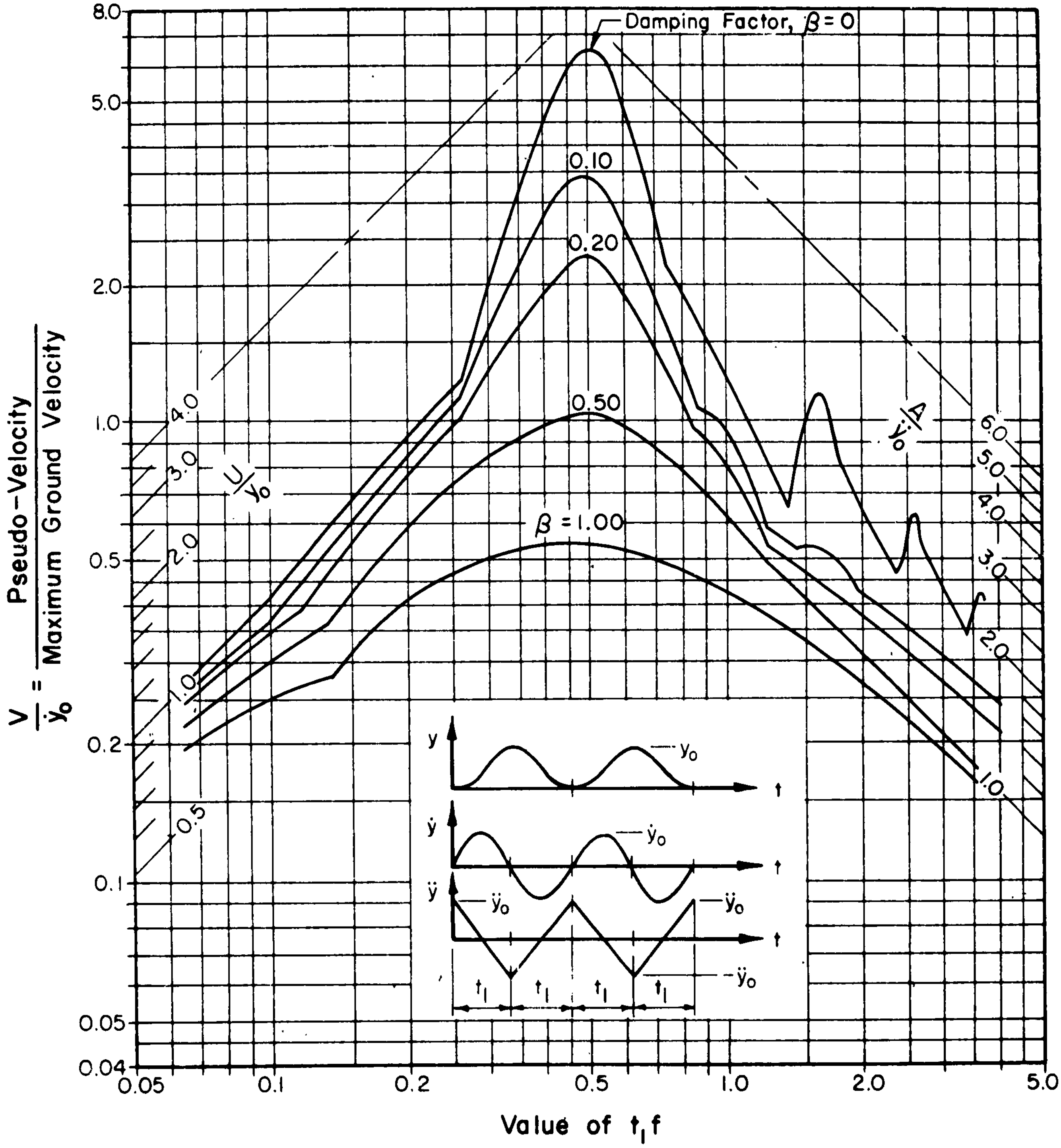


FIG. 2.44d DEFORMATION SPECTRA FOR DAMPED ELASTIC SYSTEMS SUBJECTED TO A SEQUENCE OF FOUR PARABOLIC VELOCITY HALF-CYCLES

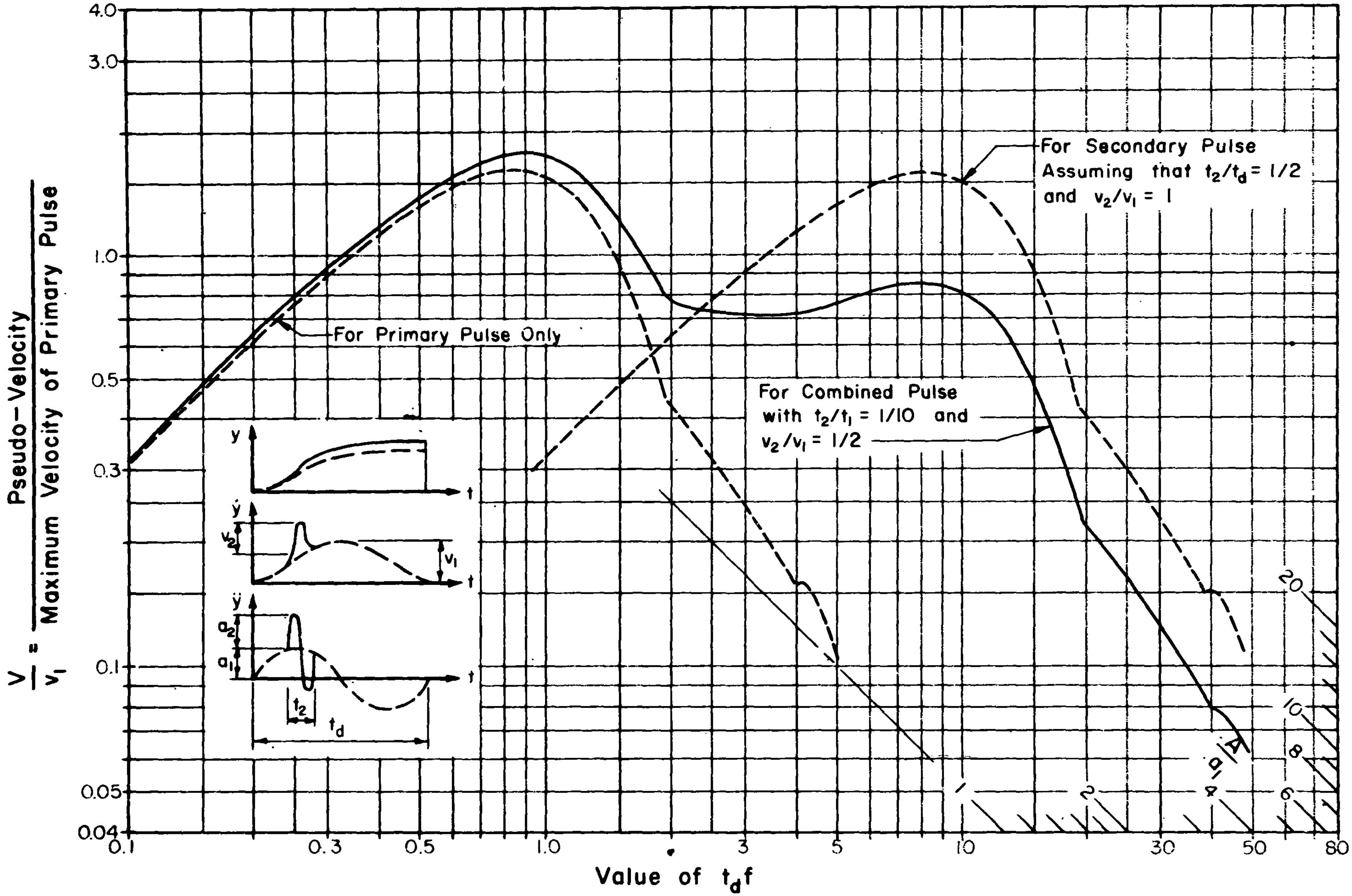


FIG. 2.45a APPROXIMATE DEFORMATION SPECTRA FOR UNDAMPED ELASTIC SYSTEMS SUBJECTED TO A COMBINATION OF TWO FULL-CYCLE, SINUSOIDAL ACCELERATION PULSES

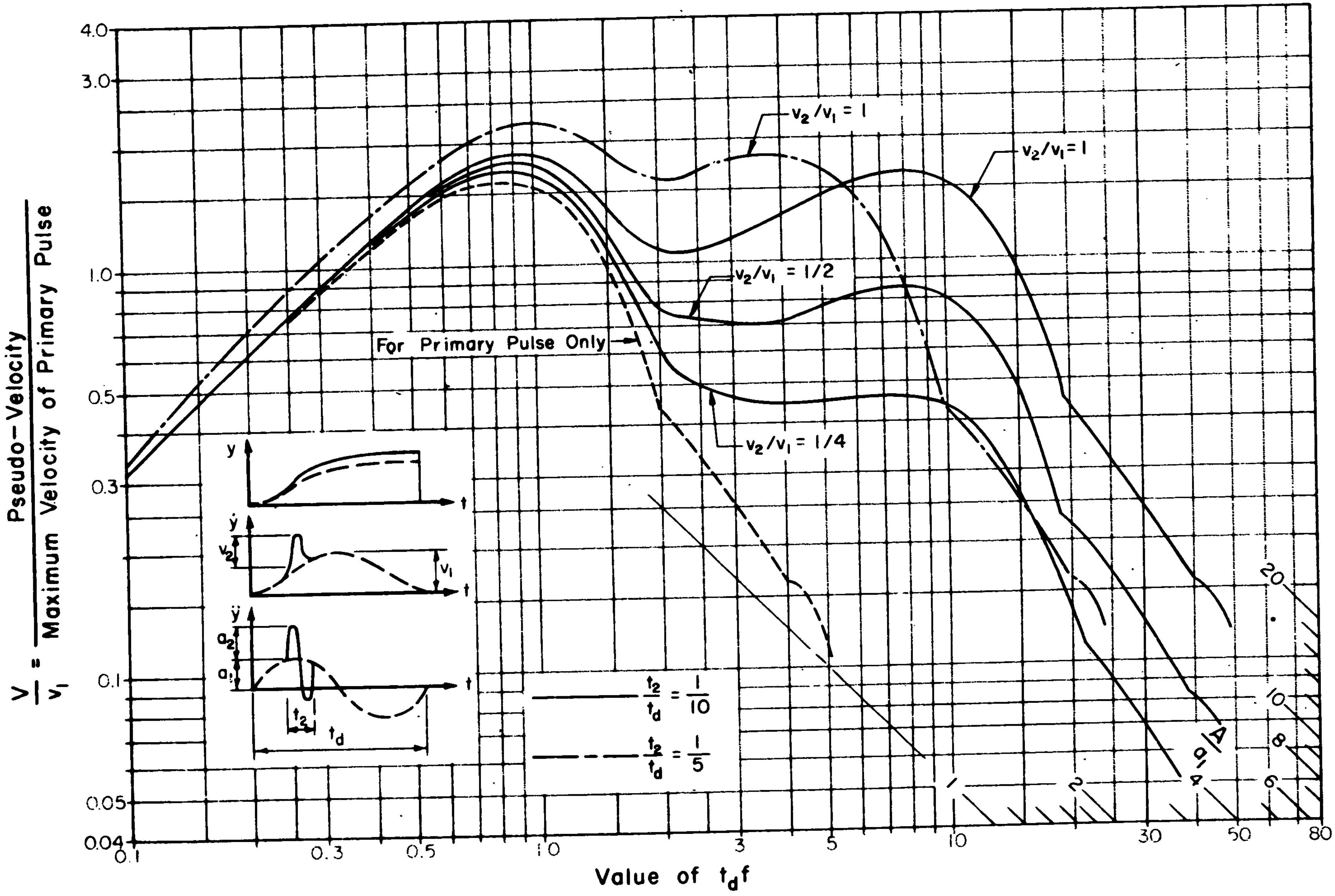


FIG. 2.45b APPROXIMATE DEFORMATION SPECTRA FOR UNDAMPED ELASTIC SYSTEMS SUBJECTED TO A COMBINATION OF TWO FULL-CYCLE, SINUSOIDAL ACCELERATION PULSES

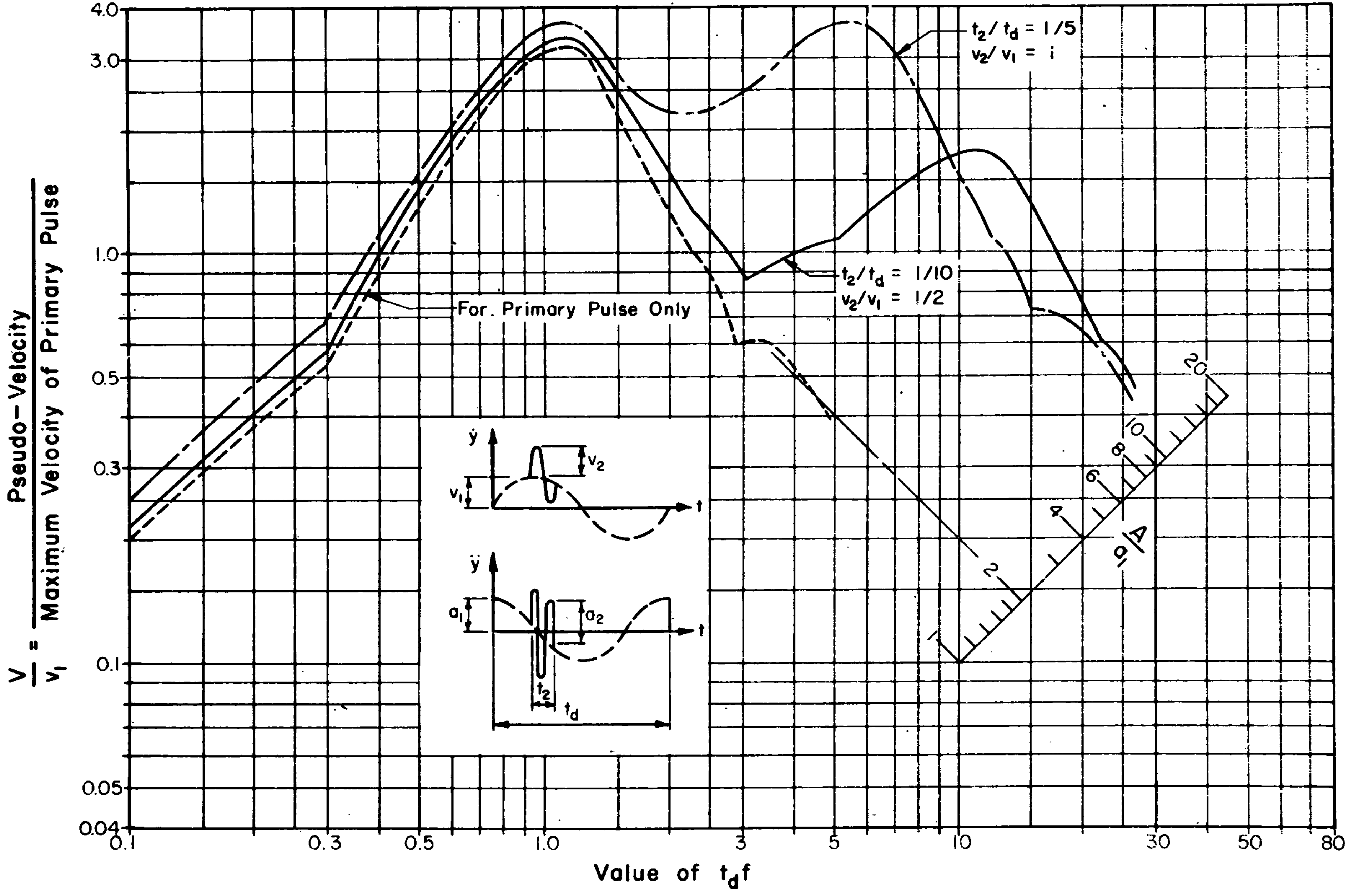


FIG. 2.46 APPROXIMATE DEFORMATION SPECTRA FOR UNDAMPED ELASTIC SYSTEMS SUBJECTED TO A COMBINATION OF TWO FULL-CYCLE, SINUSOIDAL VELOCITY PULSES

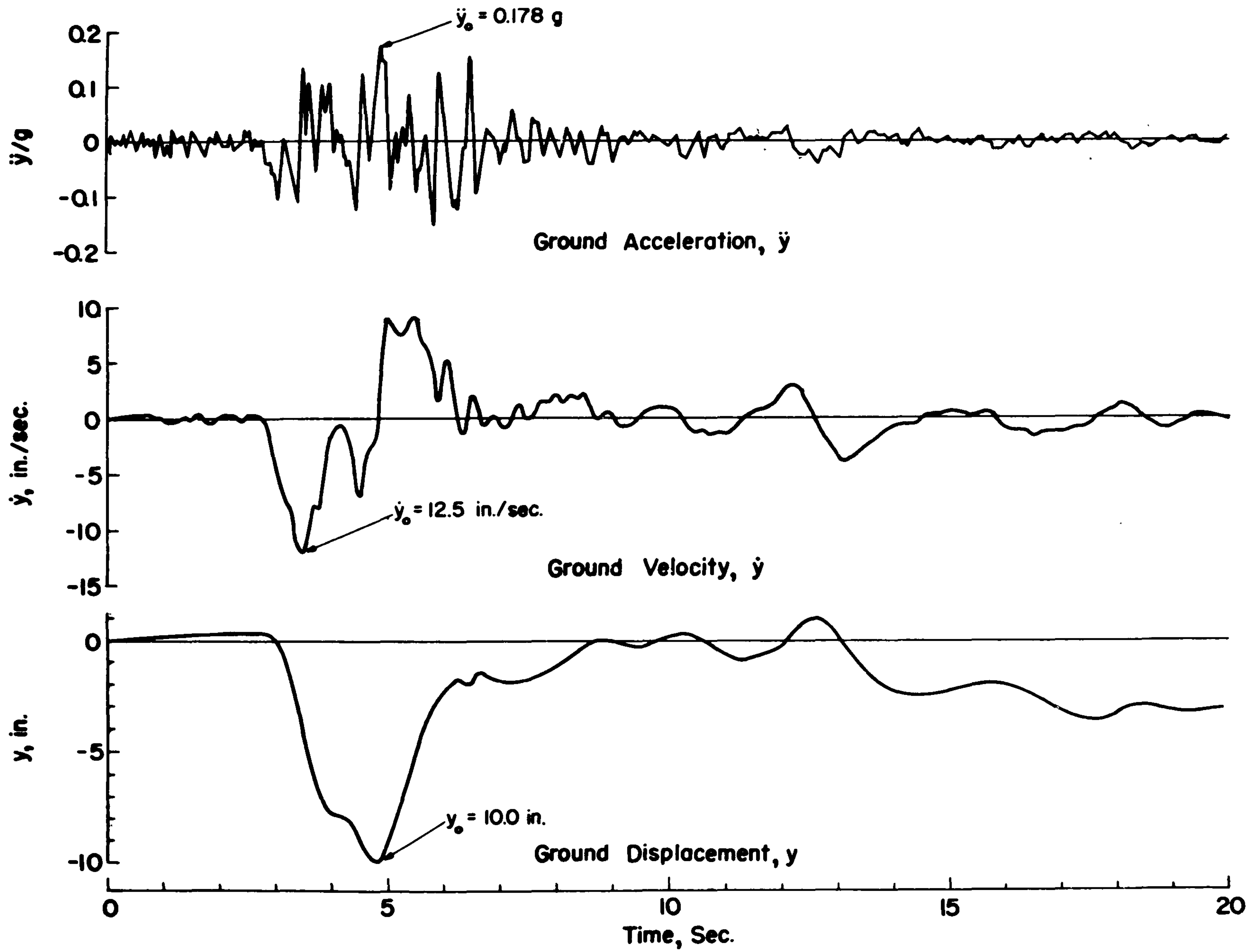


FIG. 2.47 EUREKA, CALIFORNIA EARTHQUAKE OF DEC. 21, 1954, S11°E COMPONENT

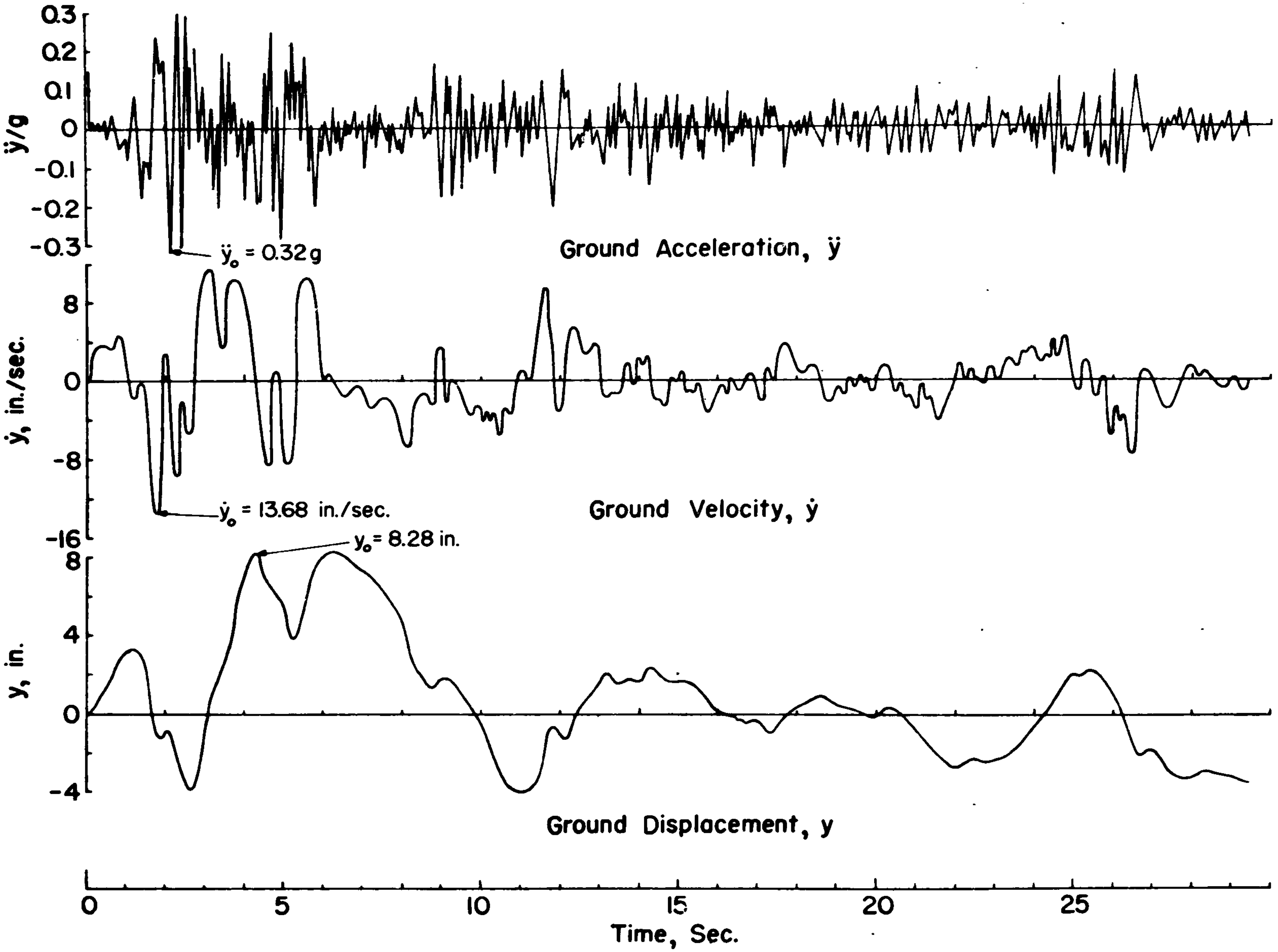


FIG. 2.48 EL CENTRO, CALIFORNIA EARTHQUAKE OF MAY 18, 1940, N-S COMPONENT

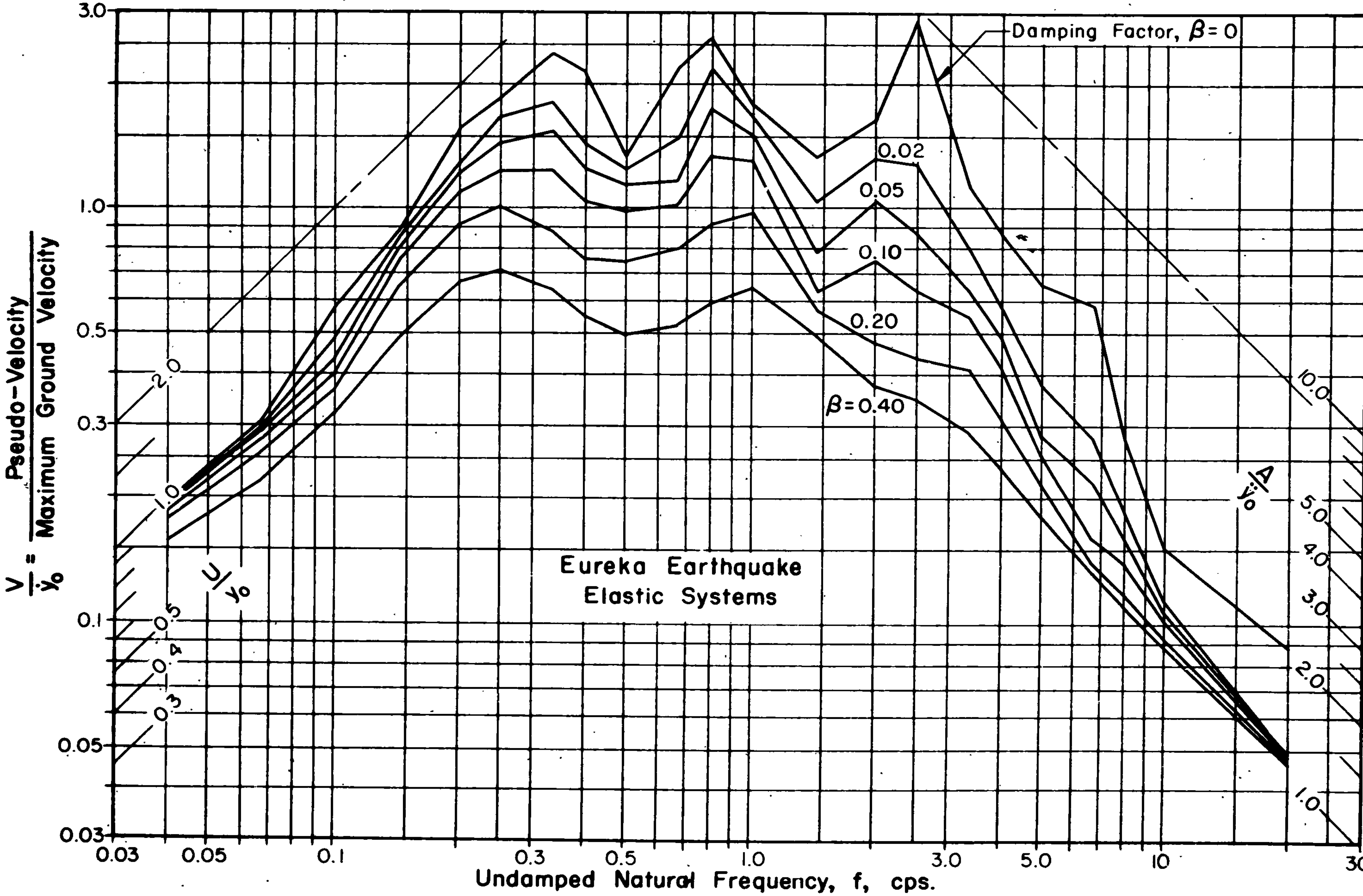


FIG. 2.49 DEFORMATION SPECTRA FOR ELASTIC SYSTEMS SUBJECTED TO THE EUREKA QUAKE

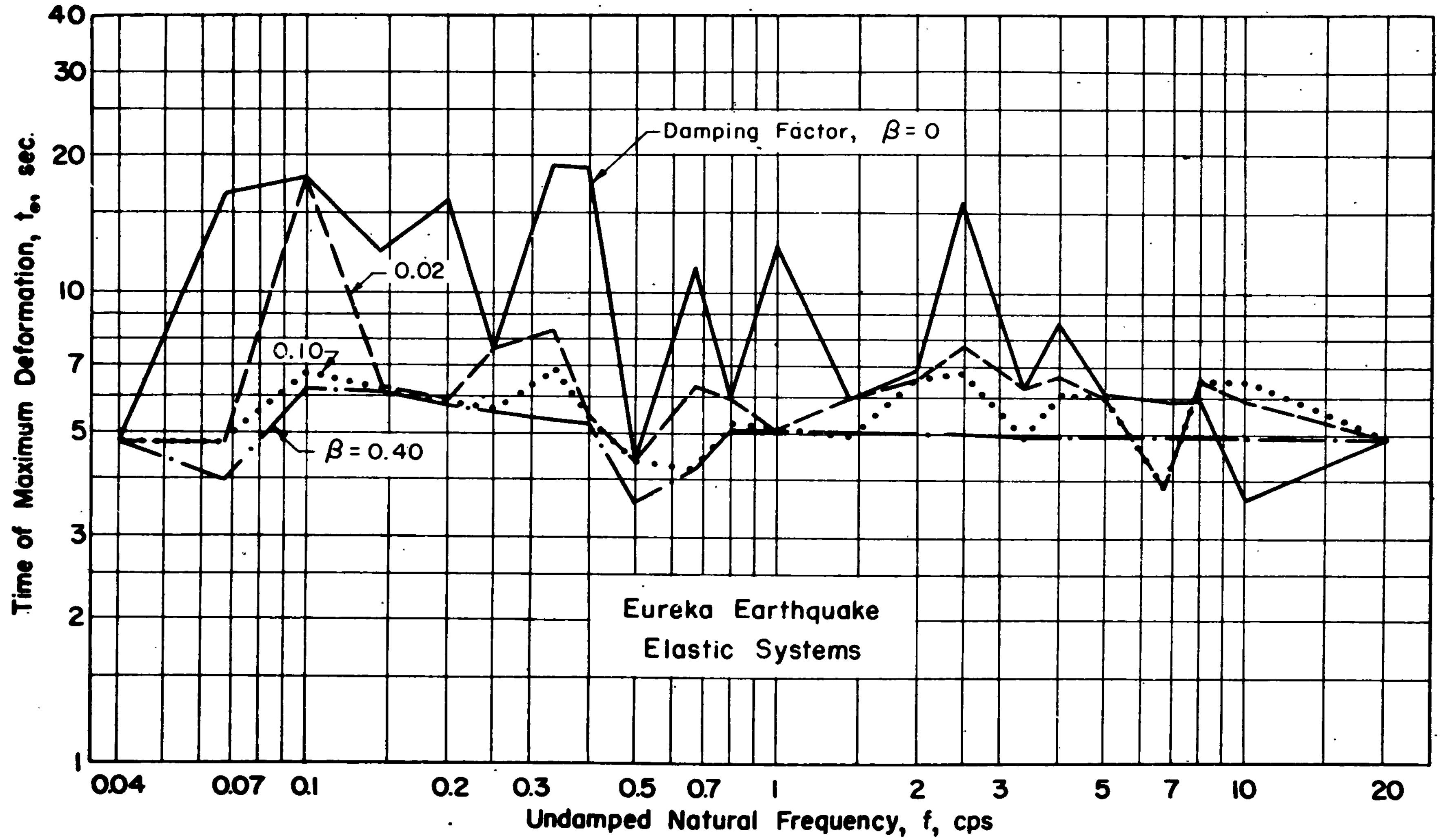


FIG. 2.50 SPECTRA FOR TIME OF MAXIMUM DEFORMATION FOR ELASTIC SYSTEMS SUBJECTED TO THE EUREKA QUAKE

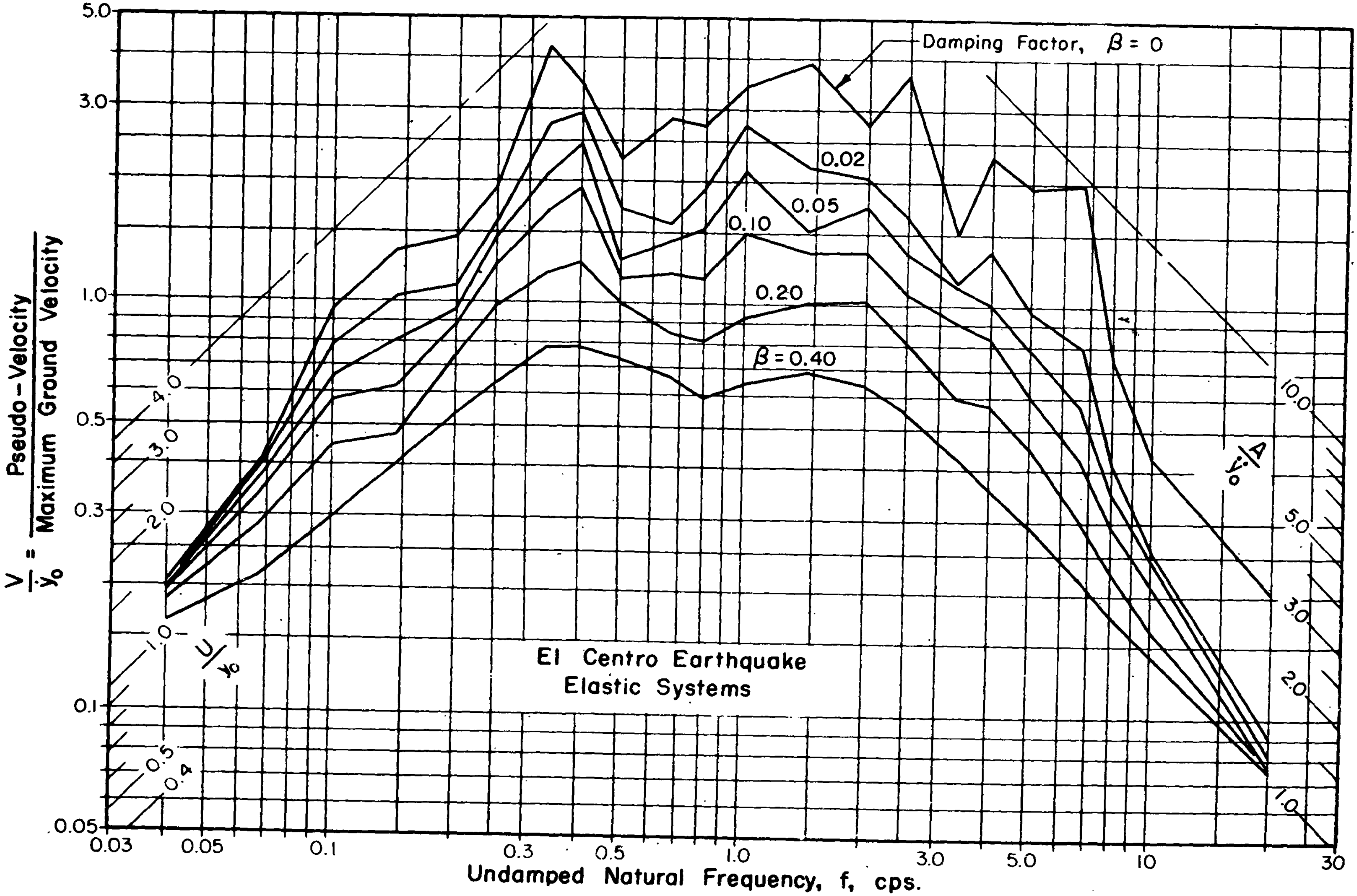


FIG. 2.51 DEFORMATION SPECTRA FOR ELASTIC SYSTEMS SUBJECTED TO THE EL CENTRO QUAKE

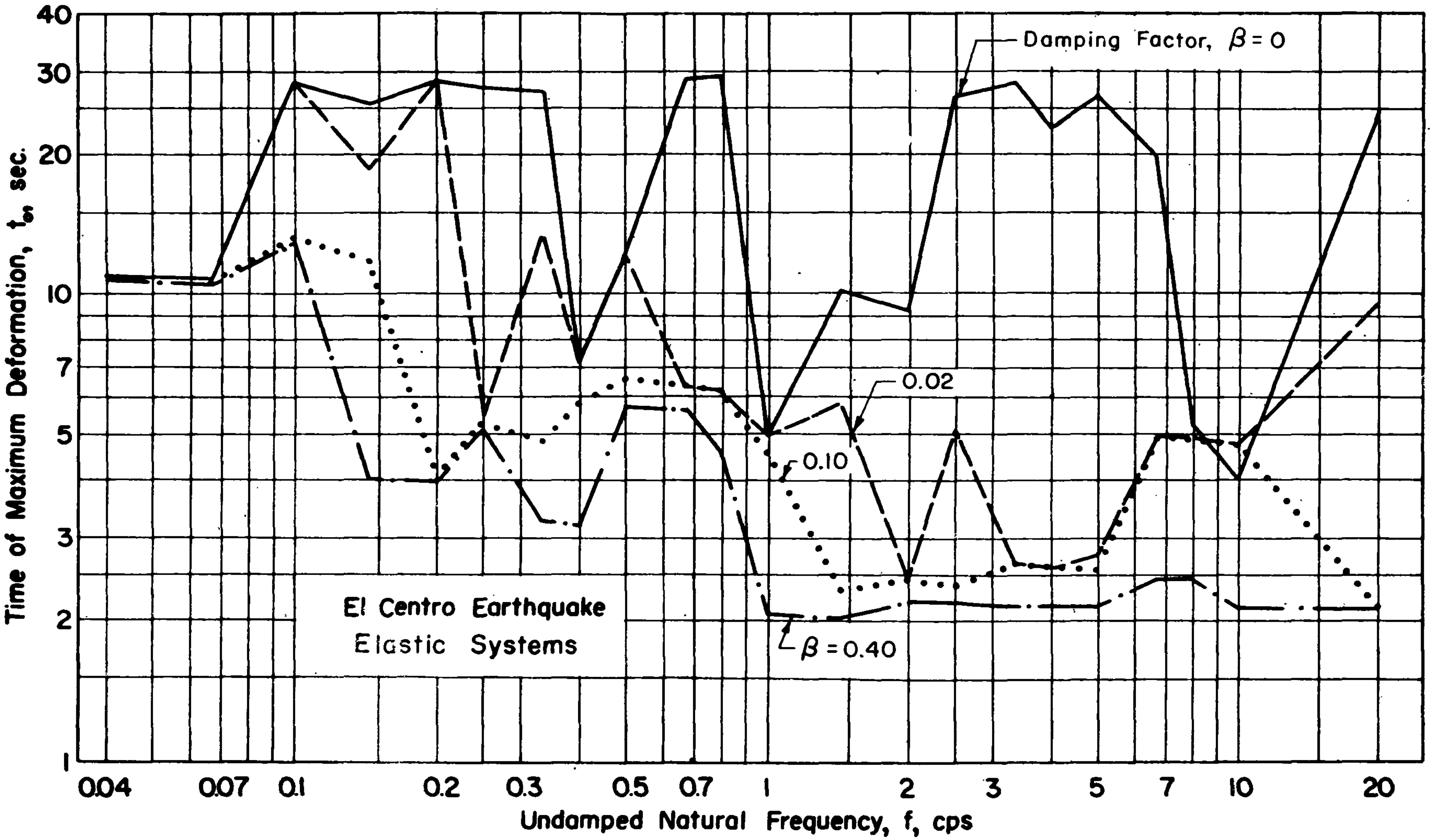


FIG. 2.52 SPECTRA FOR TIME OF MAXIMUM DEFORMATION FOR ELASTIC SYSTEMS SUBJECTED TO THE EL CENTRO QUAKE

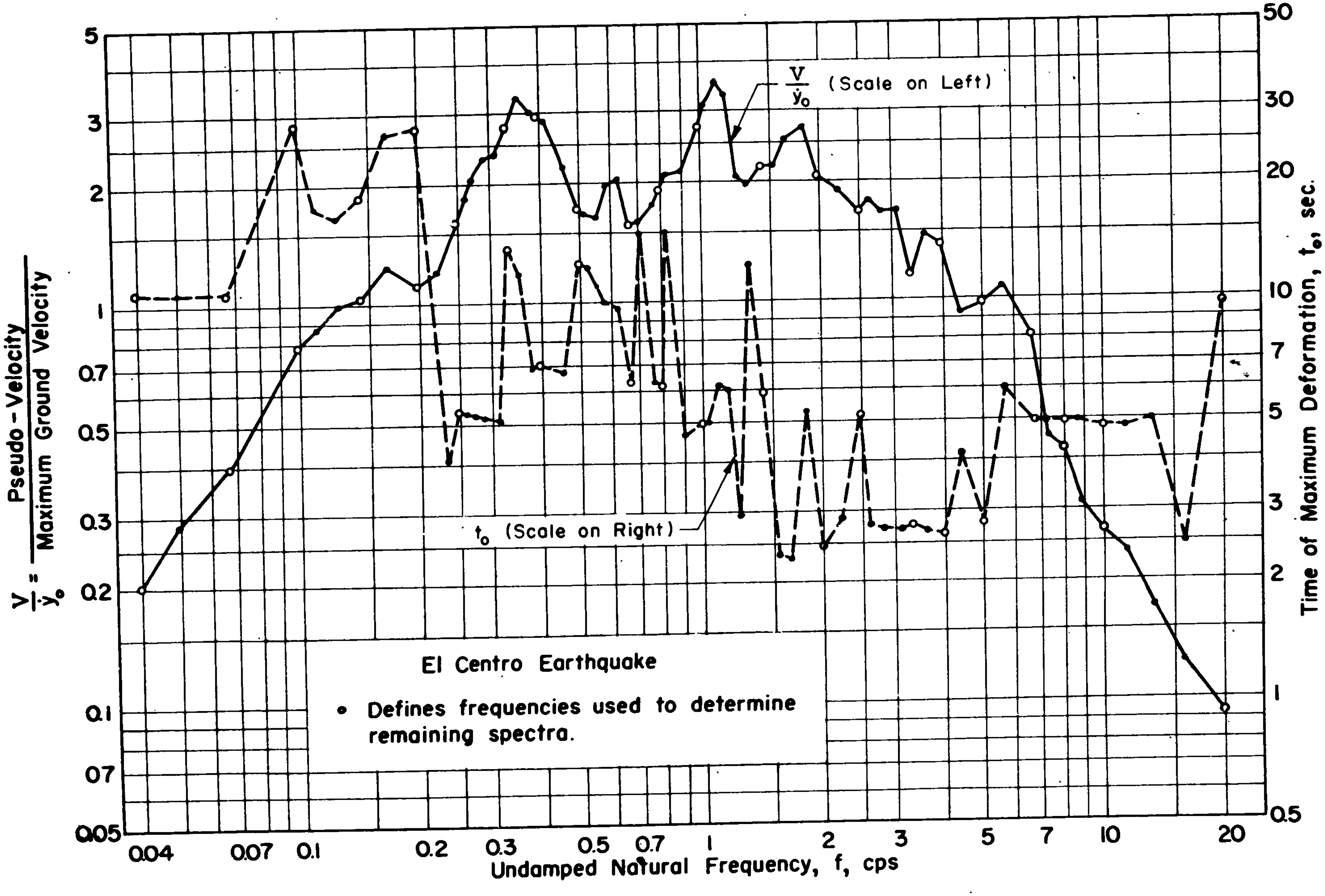


FIG. 2.53 SPECTRA FOR MAXIMUM DEFORMATION AND ASSOCIATED TIME OF SYSTEMS SUBJECTED TO THE EL CENTRO QUAKE
Elastic Systems with 2 Percent Critical Damping

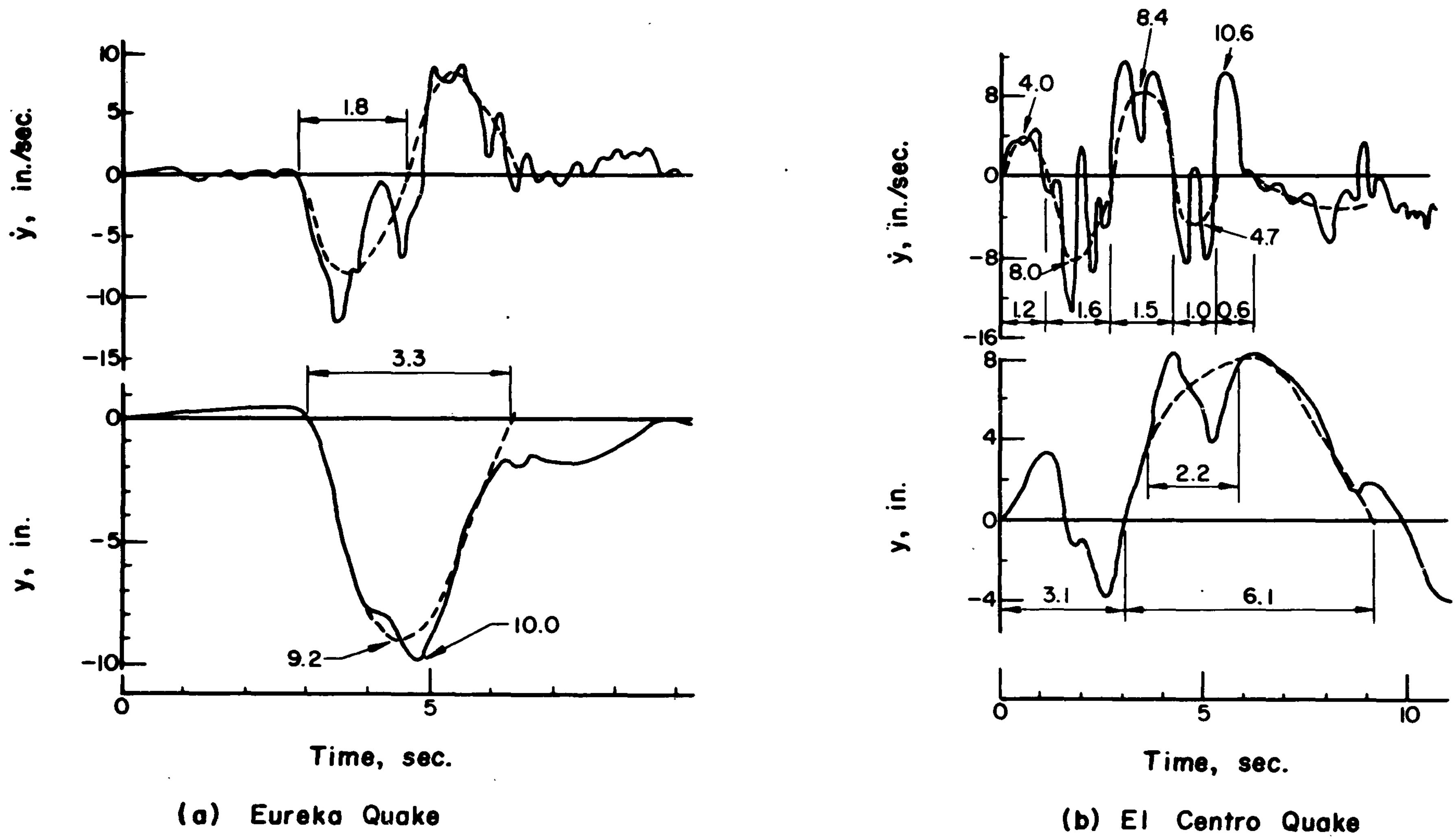


FIG. 2.54 COMPONENTS OF GROUND MOTION FOR SIGNIFICANT PORTIONS OF THE EUREKA AND EL CENTRO EARTHQUAKE RECORDS

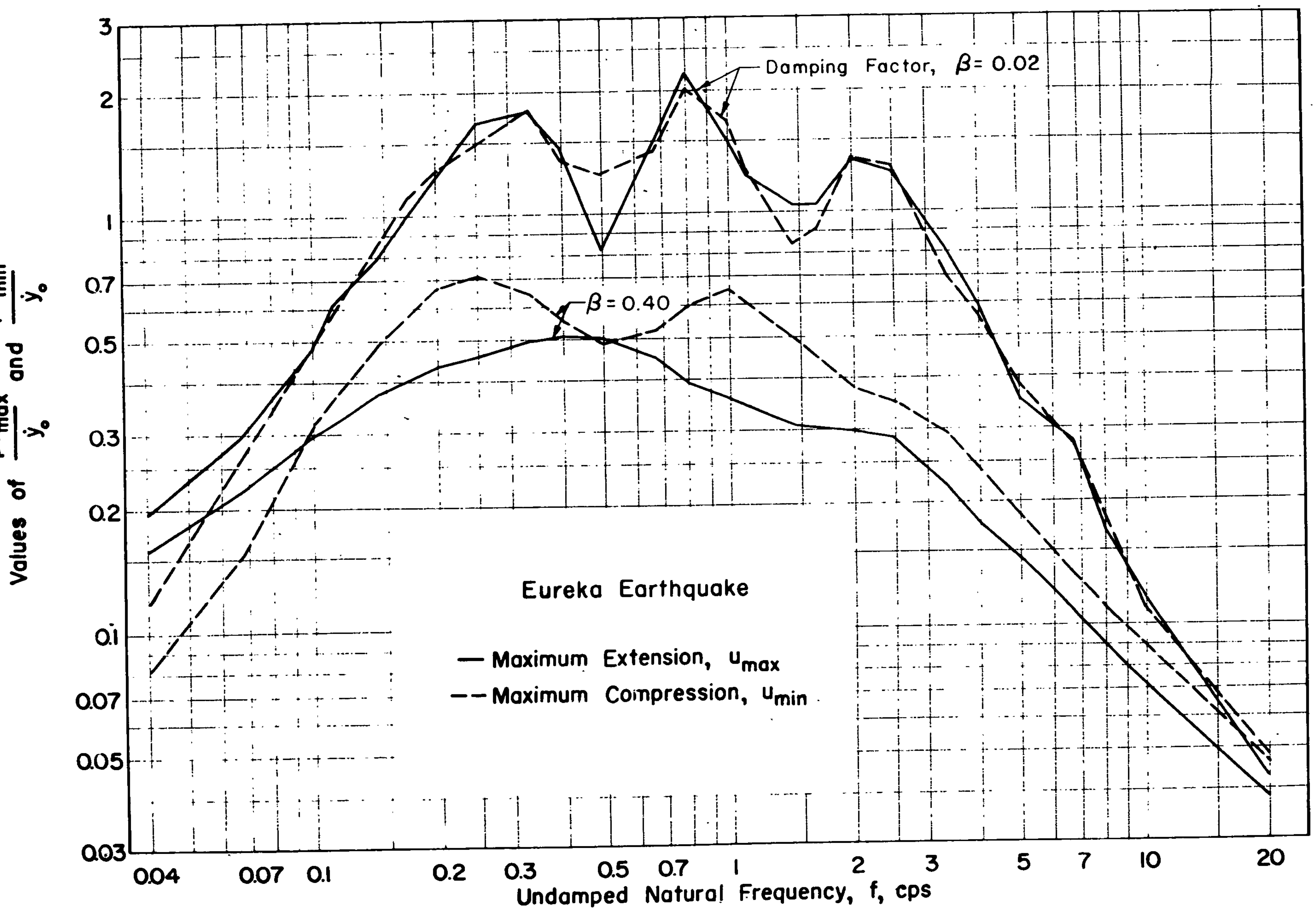


FIG. 2.55 COMPARISON OF MAXIMUM POSITIVE AND MAXIMUM NEGATIVE VALUES OF PSEUDO-VELOCITY Elastic Systems Subjected to the Eureka Quake

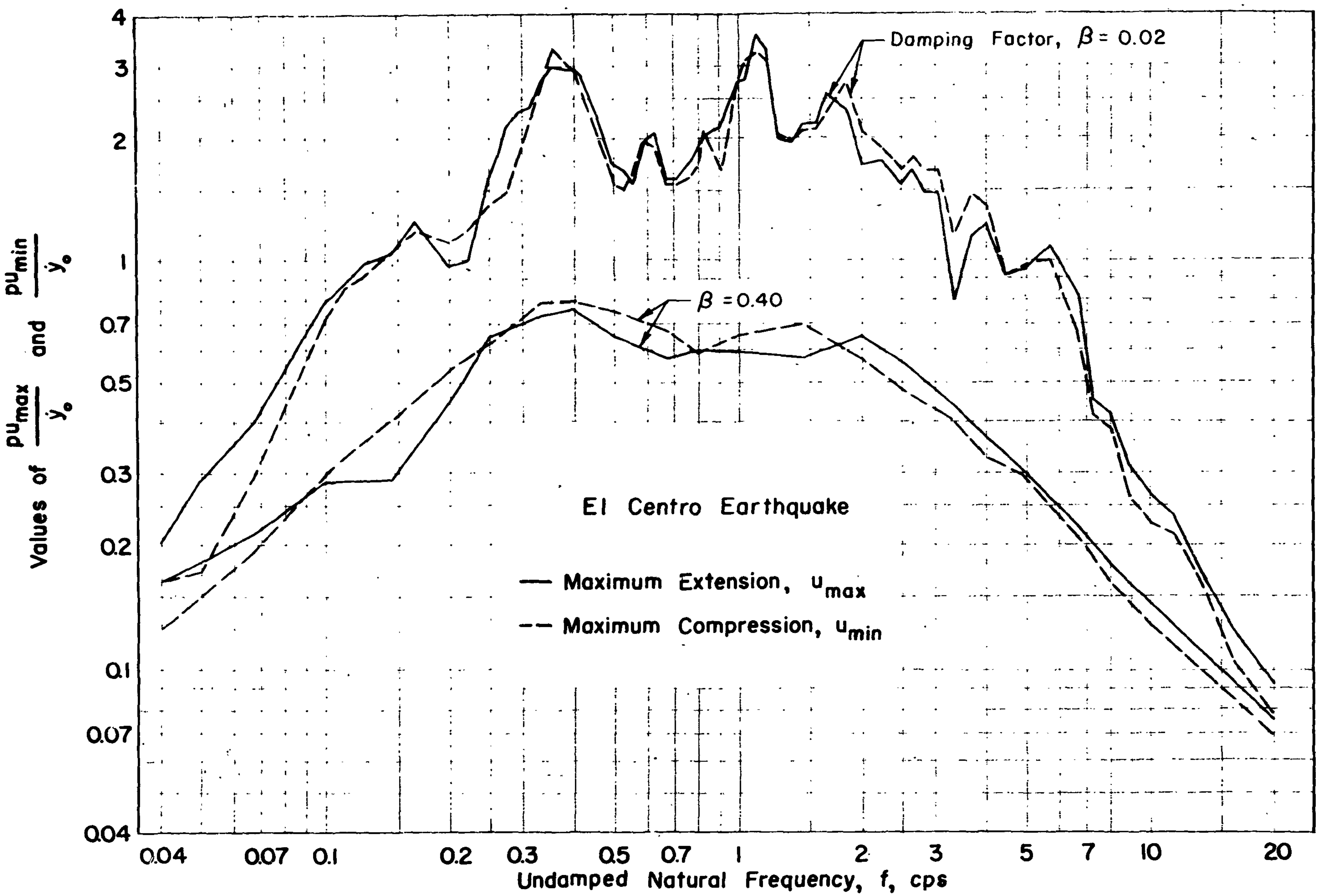


FIG. 2.56 COMPARISON OF MAXIMUM POSITIVE AND MAXIMUM NEGATIVE VALUES OF PSEUDO-VELOCITY Elastic Systems Subjected to the El Centro Quake

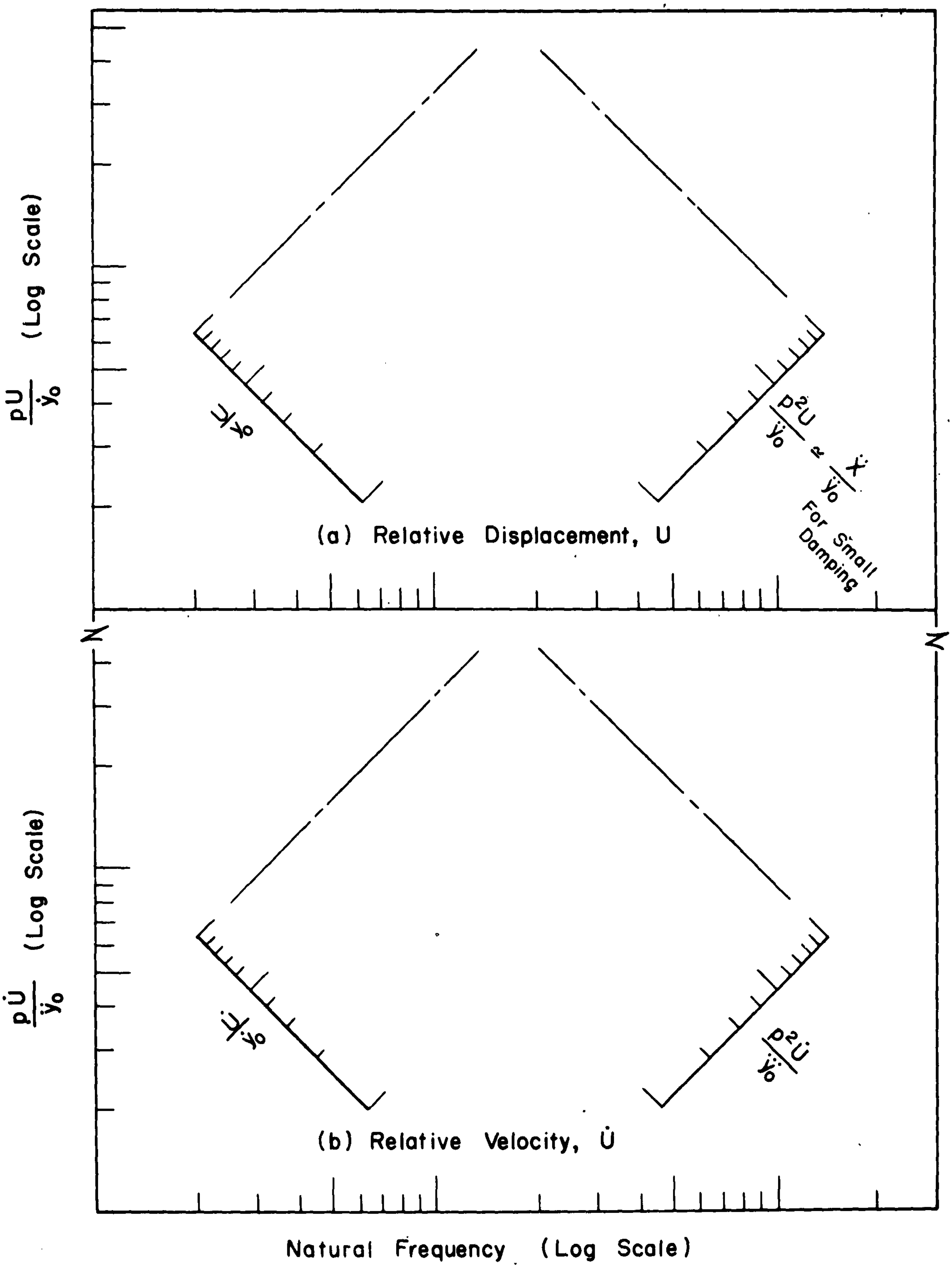


FIG. 2.57 RECOMMENDED FORM FOR PLOTTING SPECTRA FOR VARIOUS RESPONSE QUANTITIES (Continued on Next Page)

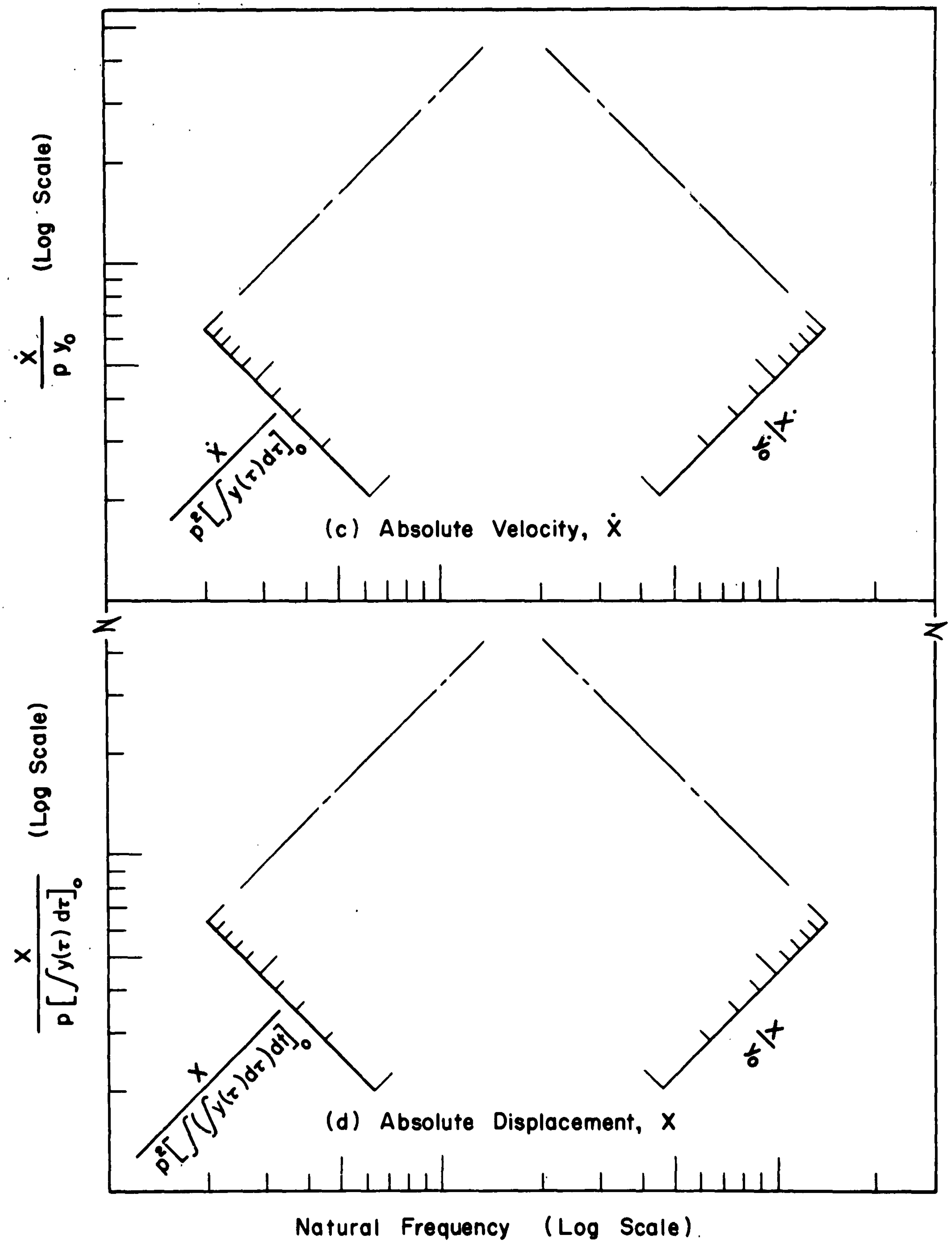


FIG. 2.57 (Continued)

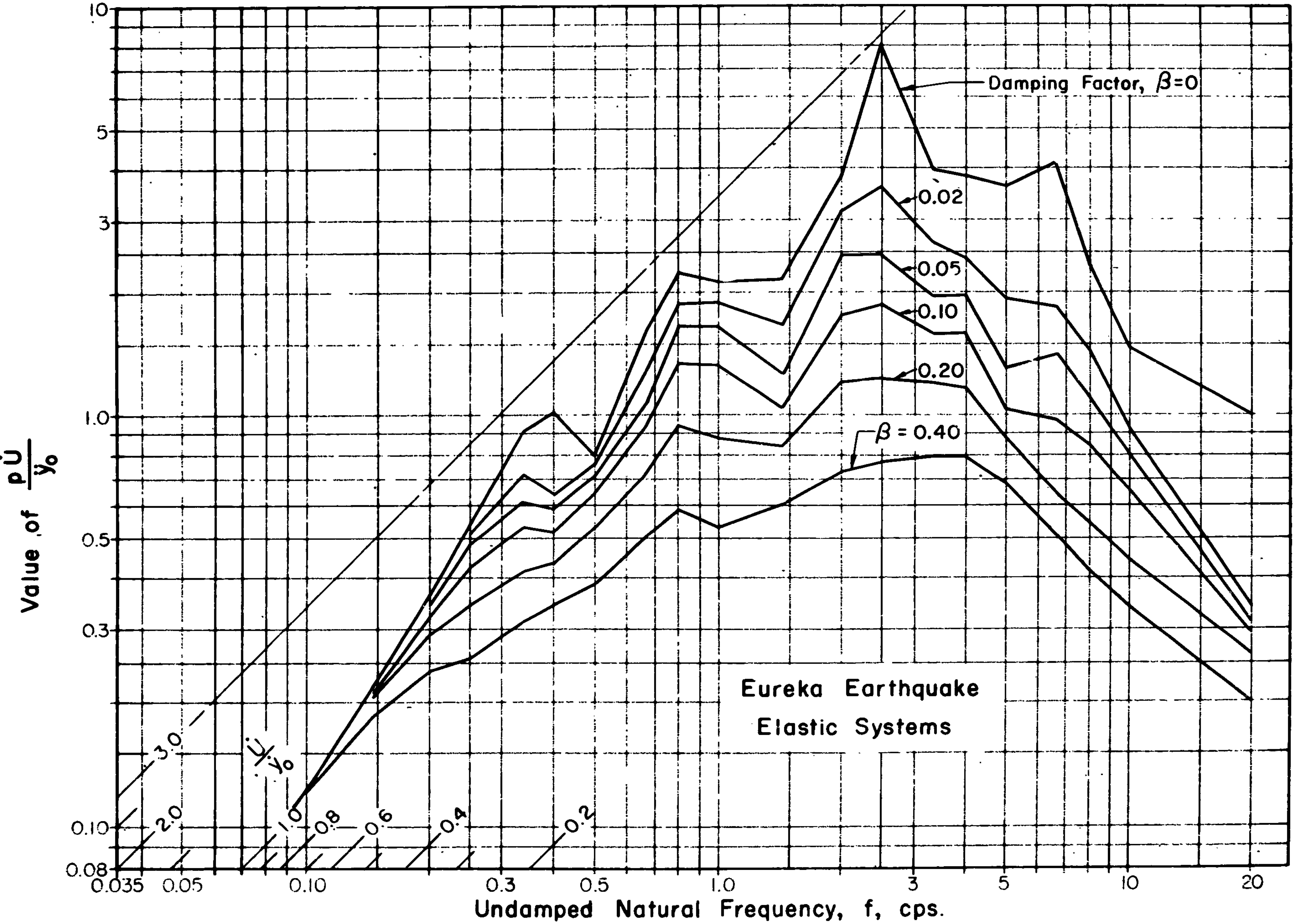


FIG. 2.58a RELATIVE VELOCITY SPECTRA FOR DAMPED ELASTIC SYSTEMS SUBJECTED TO THE EUREKA EARTHQUAKE

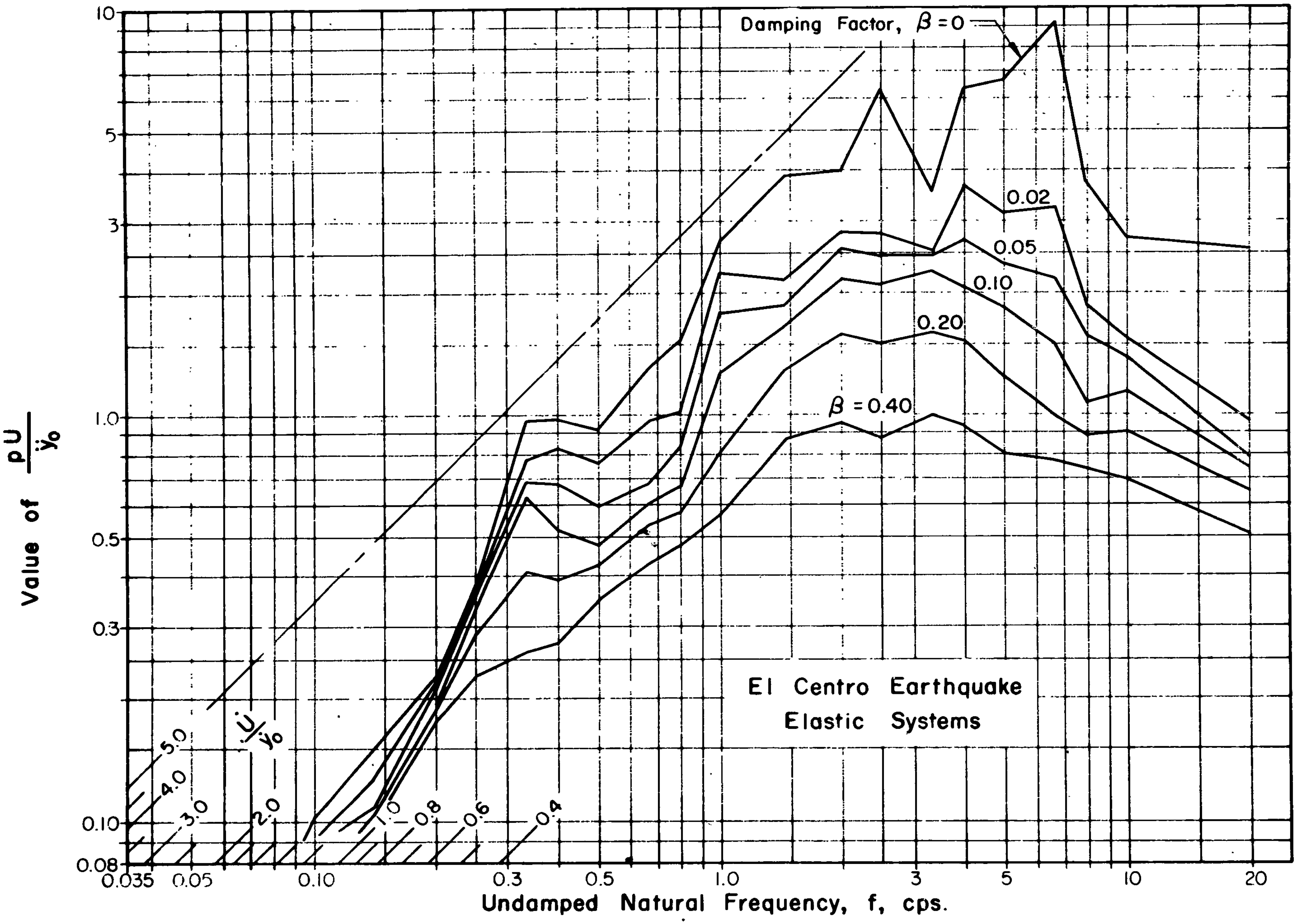


FIG. 2.58b RELATIVE VELOCITY SPECTRA FOR DAMPED ELASTIC SYSTEMS SUBJECTED TO THE EL CENTRO EARTHQUAKE

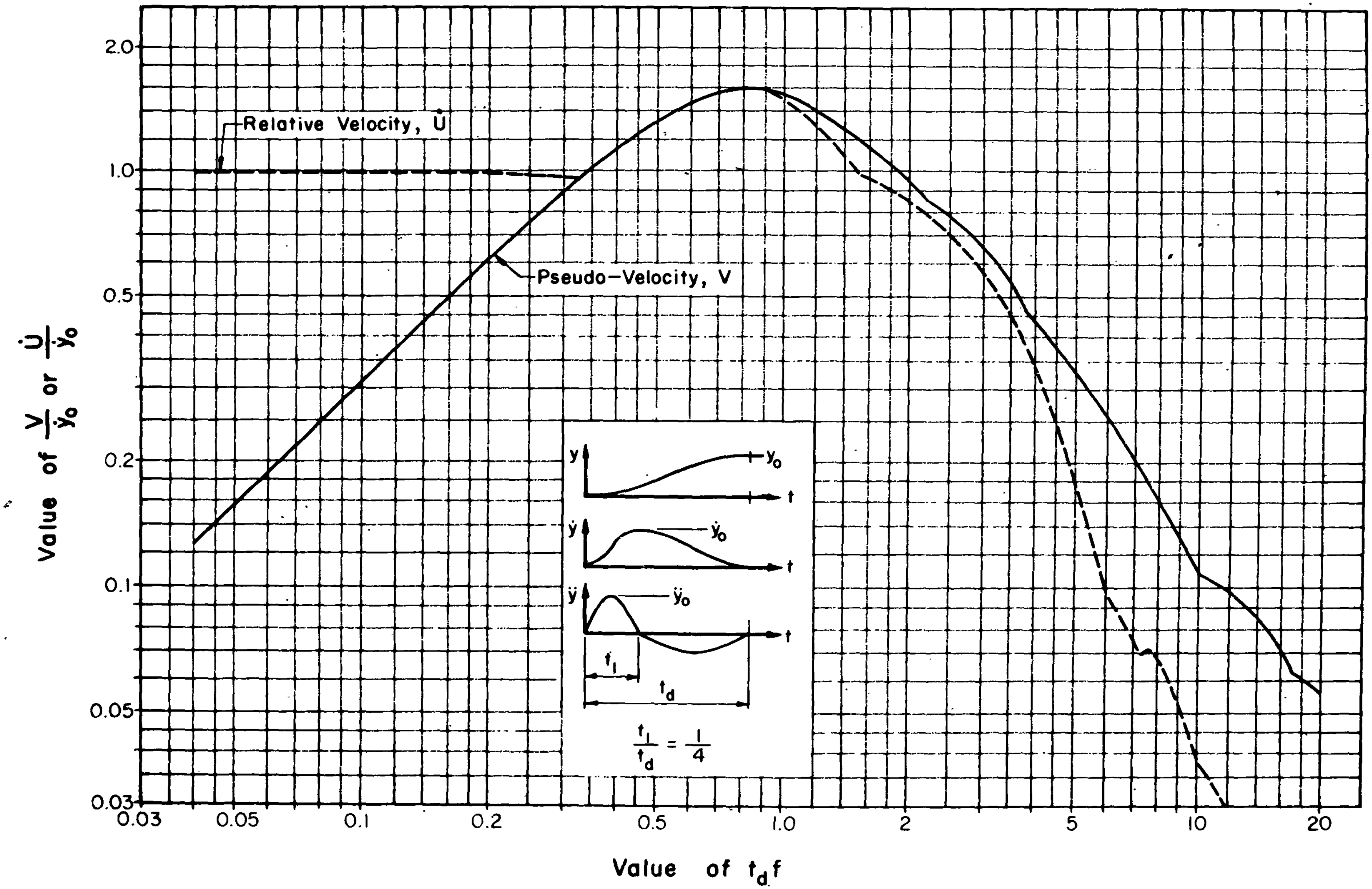


FIG. 2.59 COMPARISON OF SPECTRA FOR RELATIVE VELOCITY AND PSEUDO-VELOCITY
Undamped Elastic Systems Subjected to a Skewed-Versed-Sine Velocity Pulse

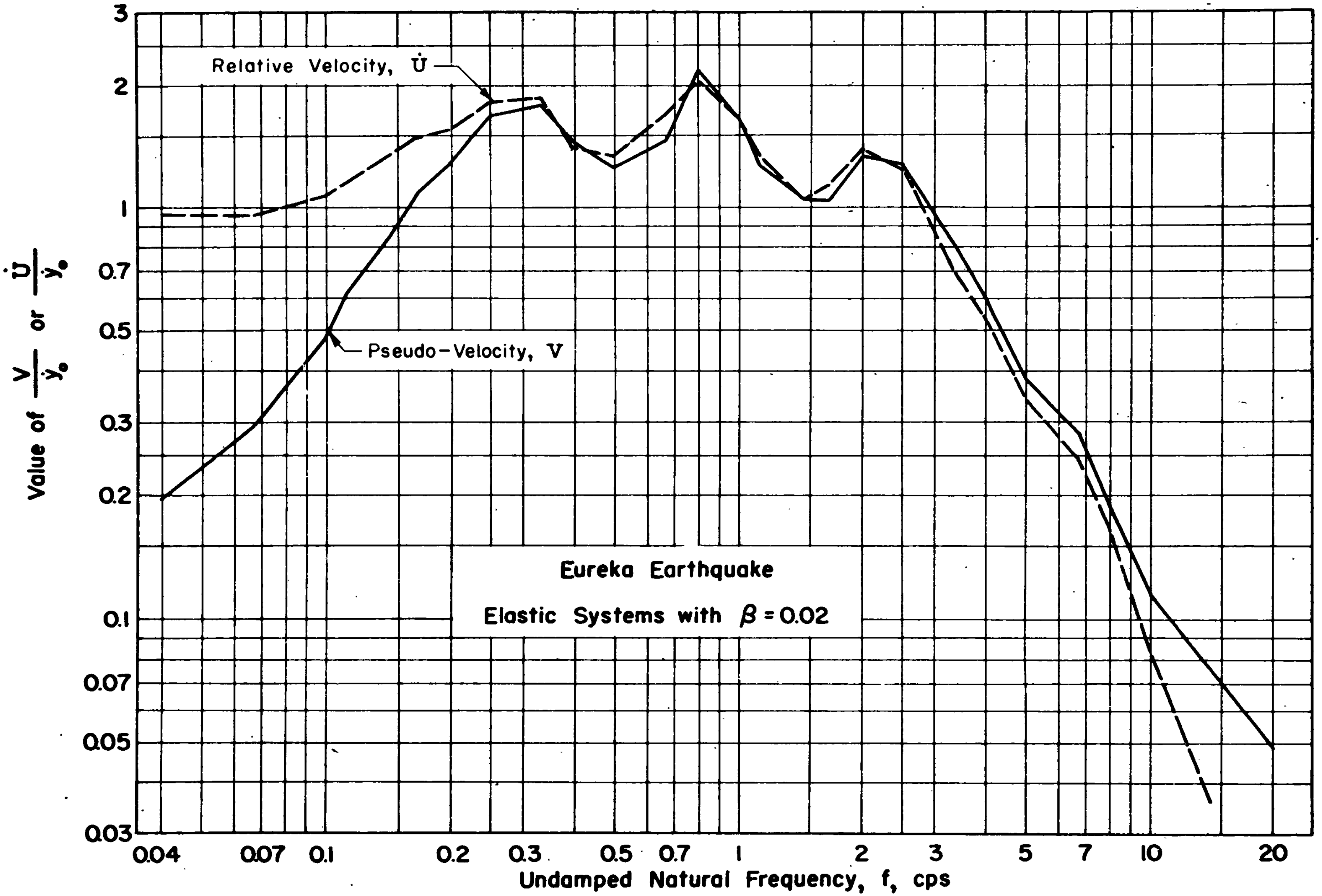


FIG. 2.60a COMPARISON OF SPECTRA FOR RELATIVE VELOCITY AND PSEUDO-VELOCITY
Elastic Systems with 2 Percent Critical Damping Subjected to the Eureka Quake

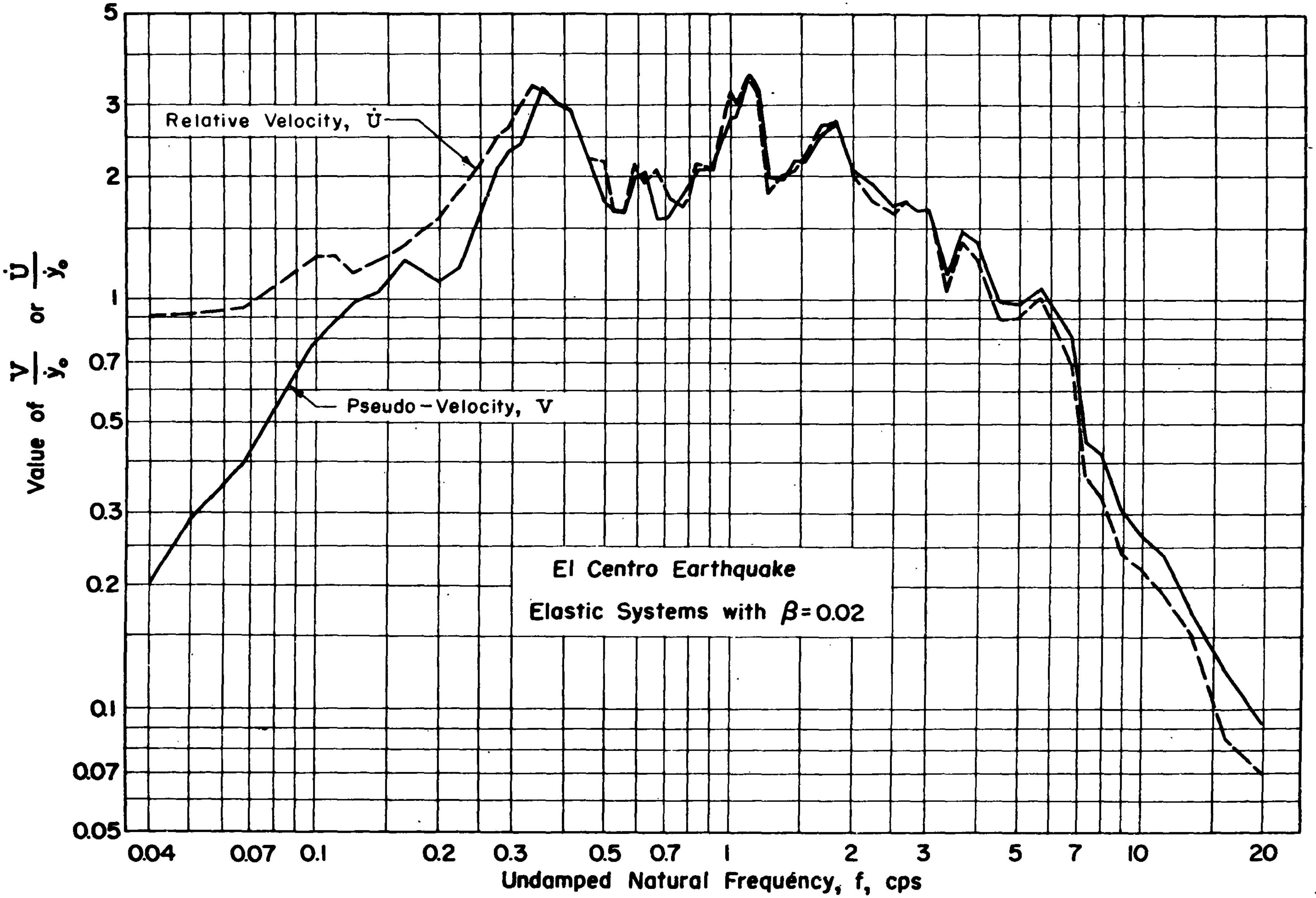


FIG. 2.60b COMPARISON OF SPECTRA FOR RELATIVE VELOCITY AND PSEUDO-VELOCITY
Elastic Systems with 2 Percent Critical Damping Subjected to the El Centro Quake

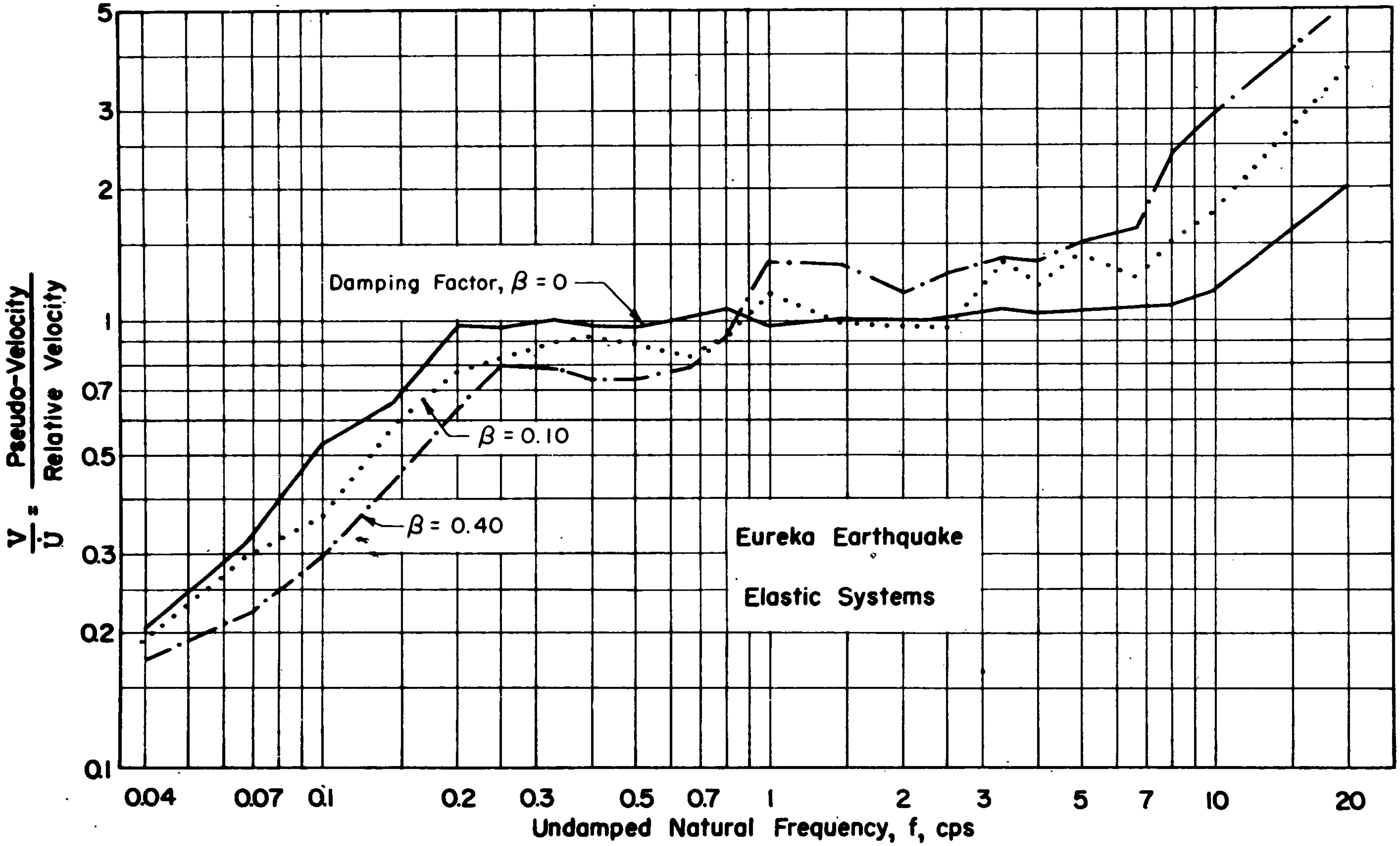


FIG. 2.61a EFFECT OF DAMPING ON RATIO OF PSEUDO-VELOCITY AND RELATIVE VELOCITY
Elastic Systems Subjected to the Eureka Quake

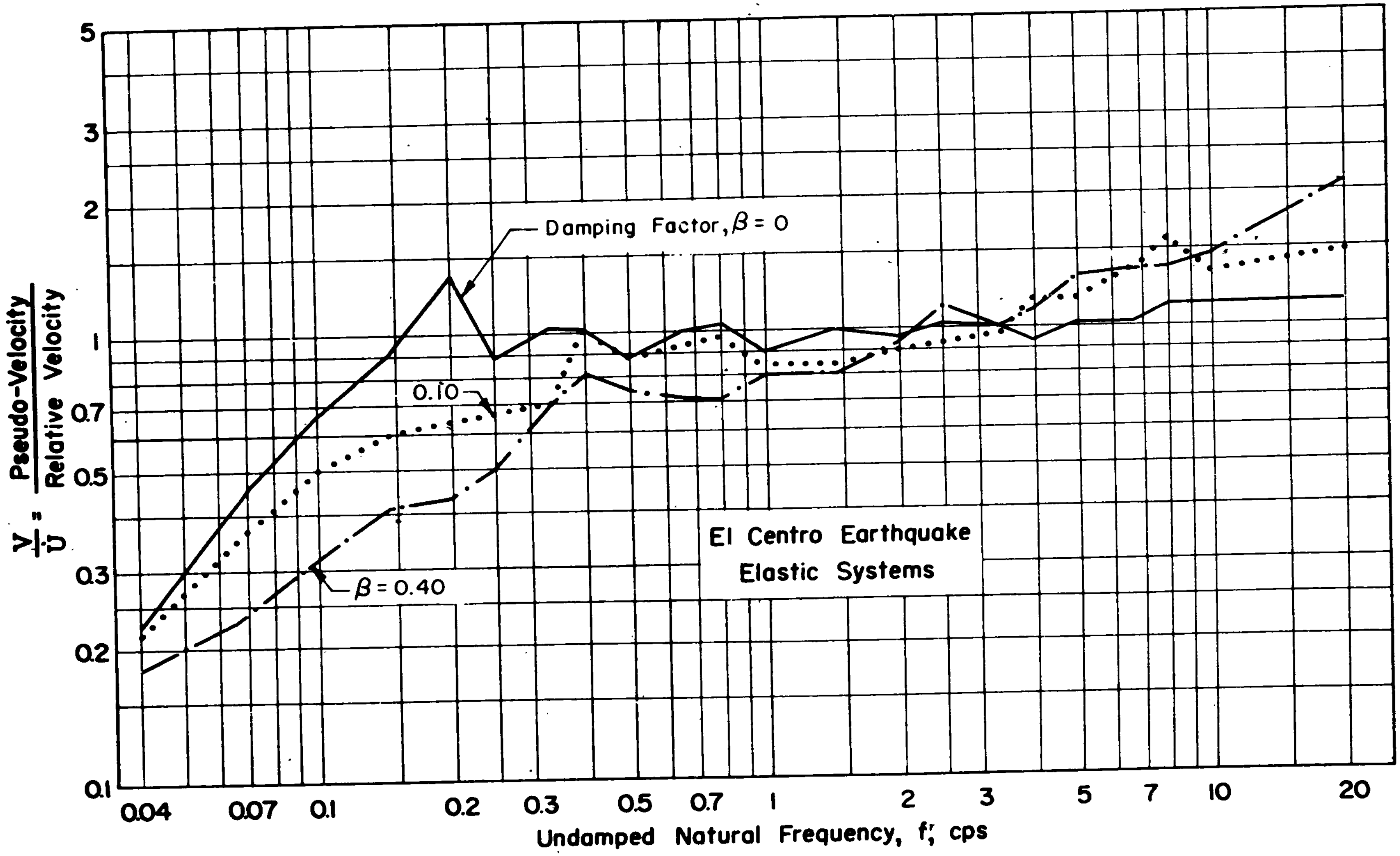


FIG. 2.61b EFFECT OF DAMPING ON RATIO OF PSEUDO-VELOCITY AND RELATIVE VELOCITY
Elastic Systems Subjected to the El Centro Quake

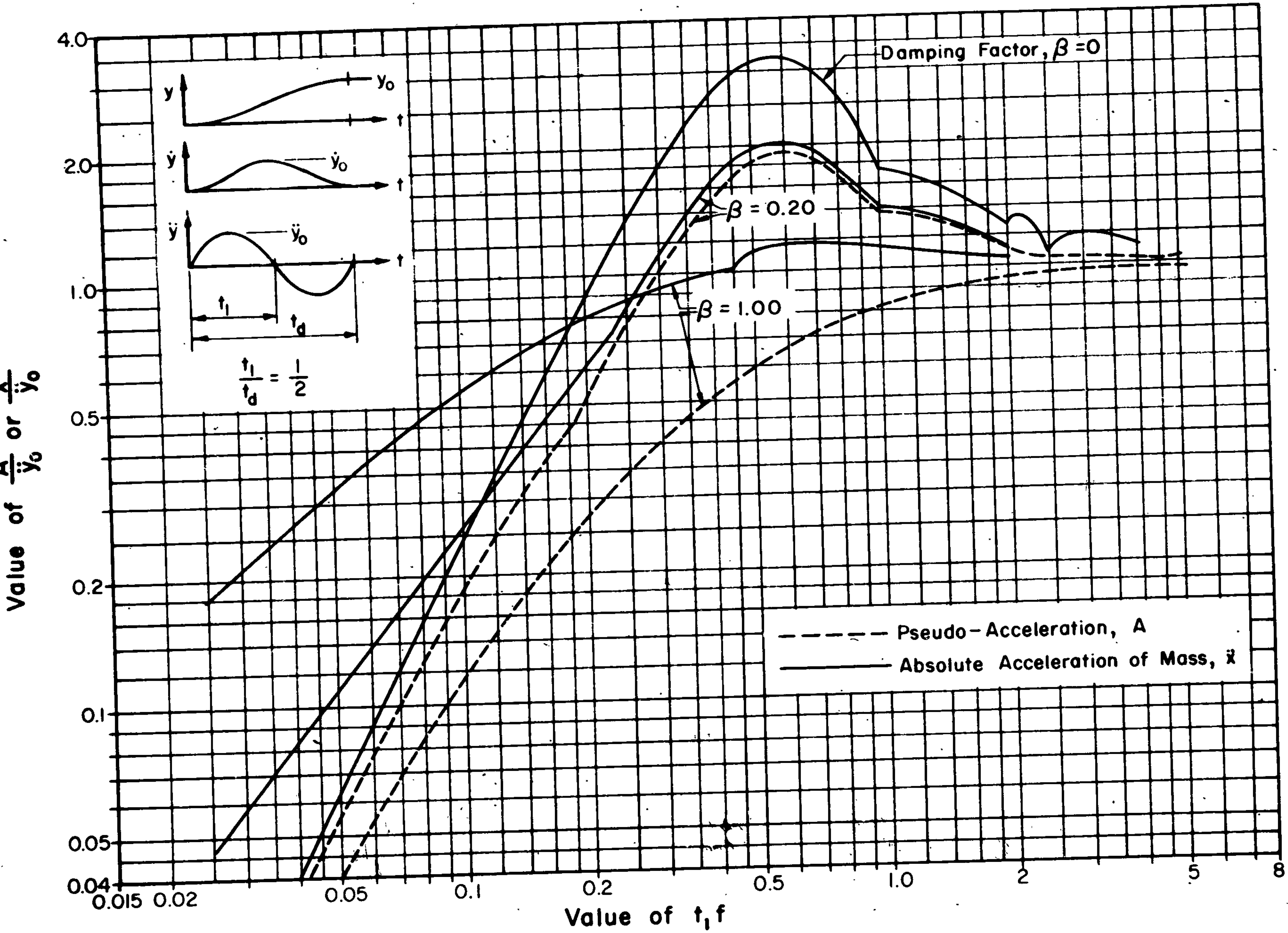


FIG. 2.62a COMPARISON OF SPECTRA FOR ABSOLUTE ACCELERATION OF MASS AND PSEUDO-ACCELERATION
Elastic Systems Subjected to a Versed-Sine Velocity Pulse

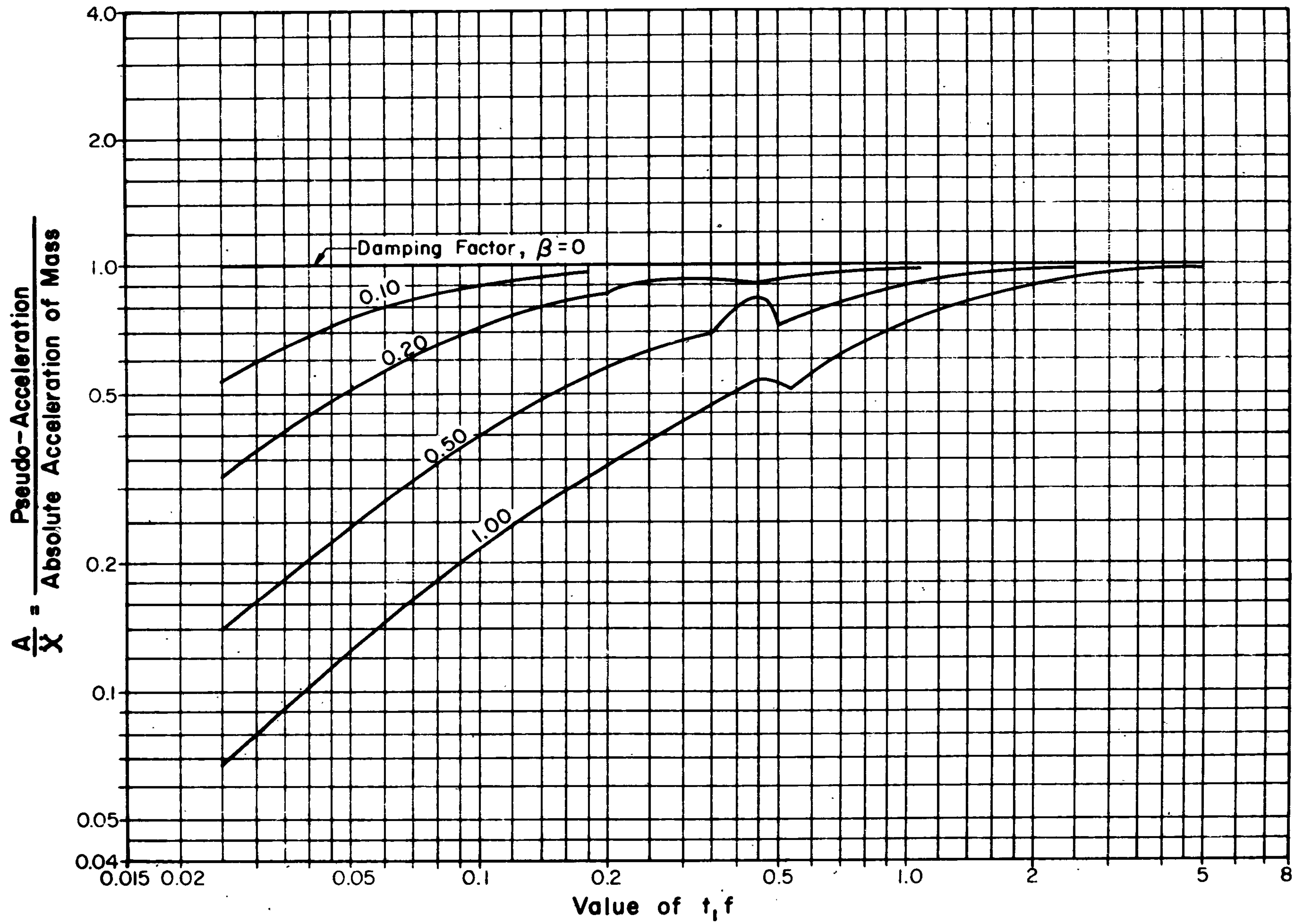


FIG. 2.62b COMPARISON OF SPECTRA FOR ABSOLUTE ACCELERATION OF MASS AND PSEUDO-ACCELERATION
Elastic Systems Subjected to a Versed-Sine Velocity Pulse

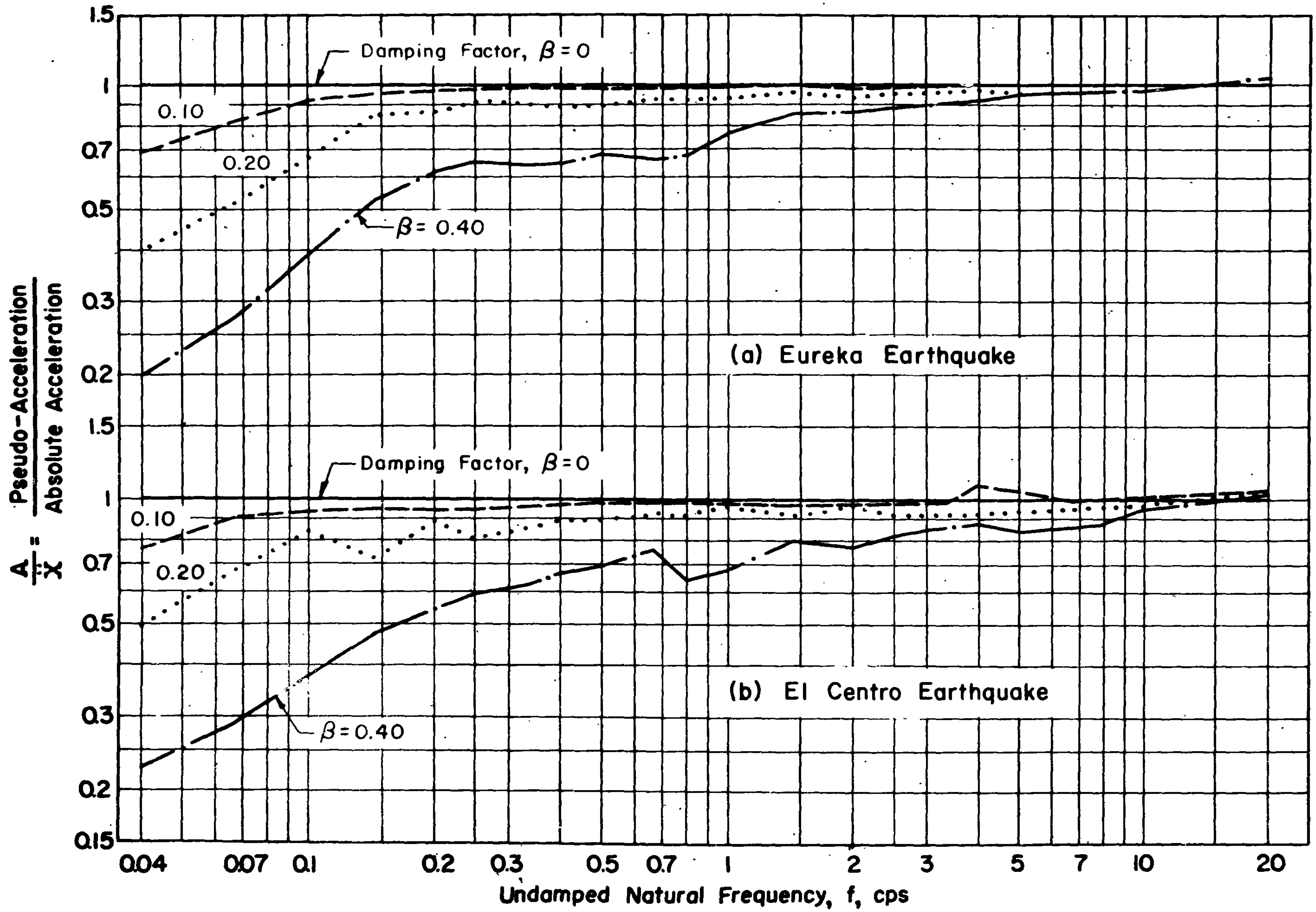


FIG. 2.63 EFFECT OF DAMPING ON RATIO OF PSEUDO-ACCELERATION AND ABSOLUTE ACCELERATION OF MASS Elastic Systems Subjected to the Eureka and El Centro Quakes

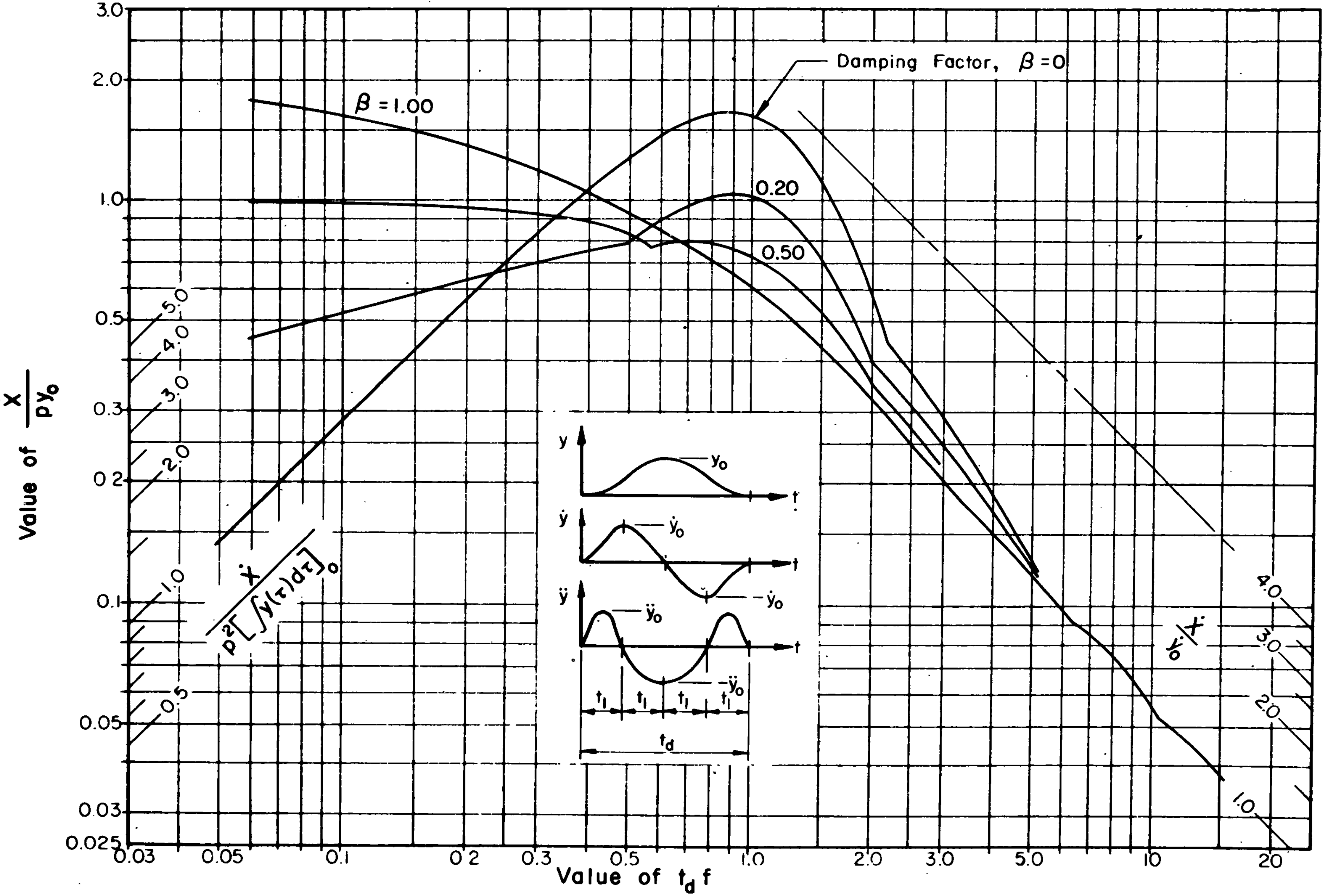


FIG. 2.64 ABSOLUTE VELOCITY SPECTRA FOR ELASTIC SYSTEMS SUBJECTED TO A HALF-CYCLE DISPLACEMENT PULSE
Acceleration Diagram Consists of a Sequence of Three Half-Sine Waves as Shown

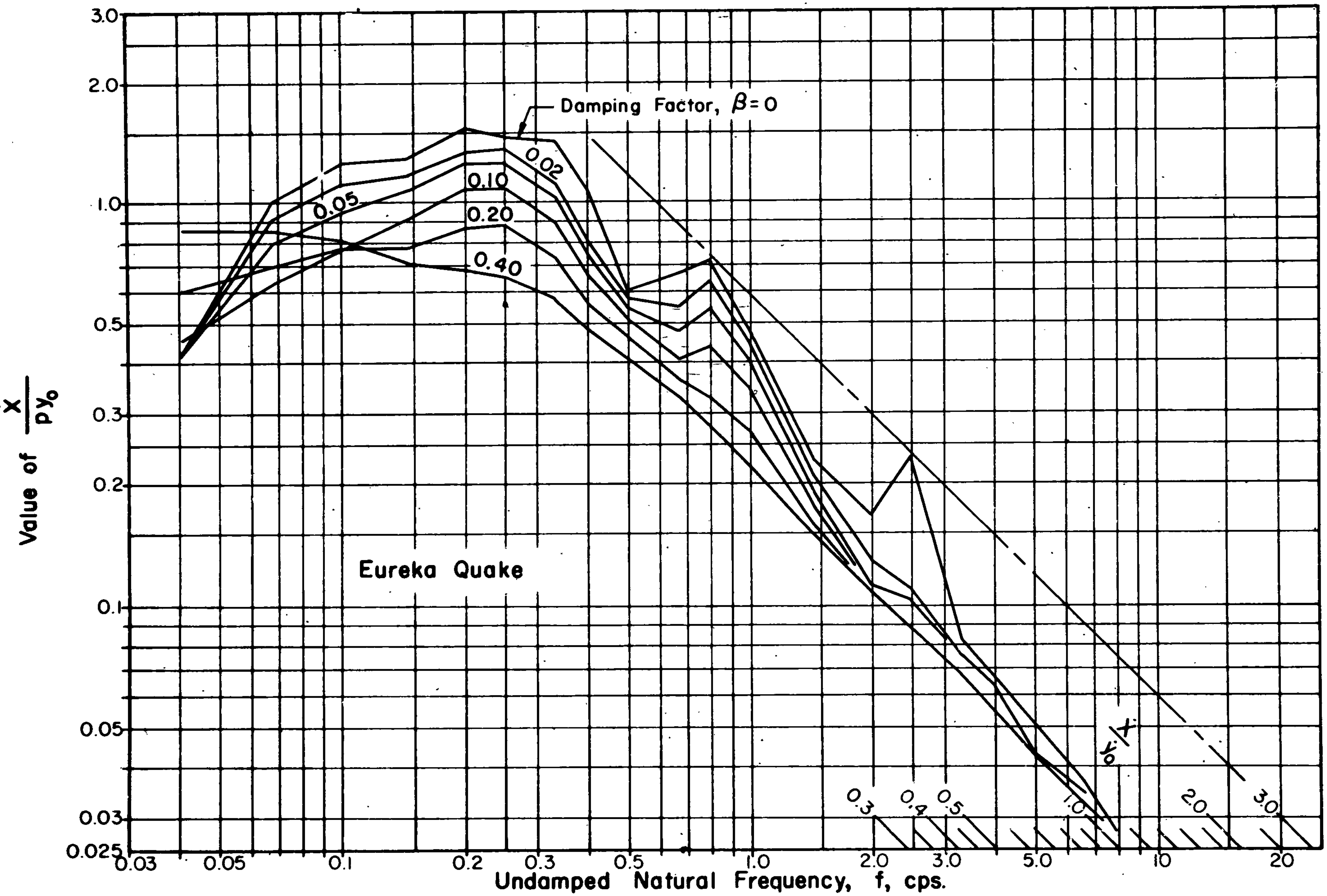


FIG. 2.65a ABSOLUTE VELOCITY SPECTRA FOR ELASTIC SYSTEMS SUBJECTED TO THE EUREKA EARTHQUAKE

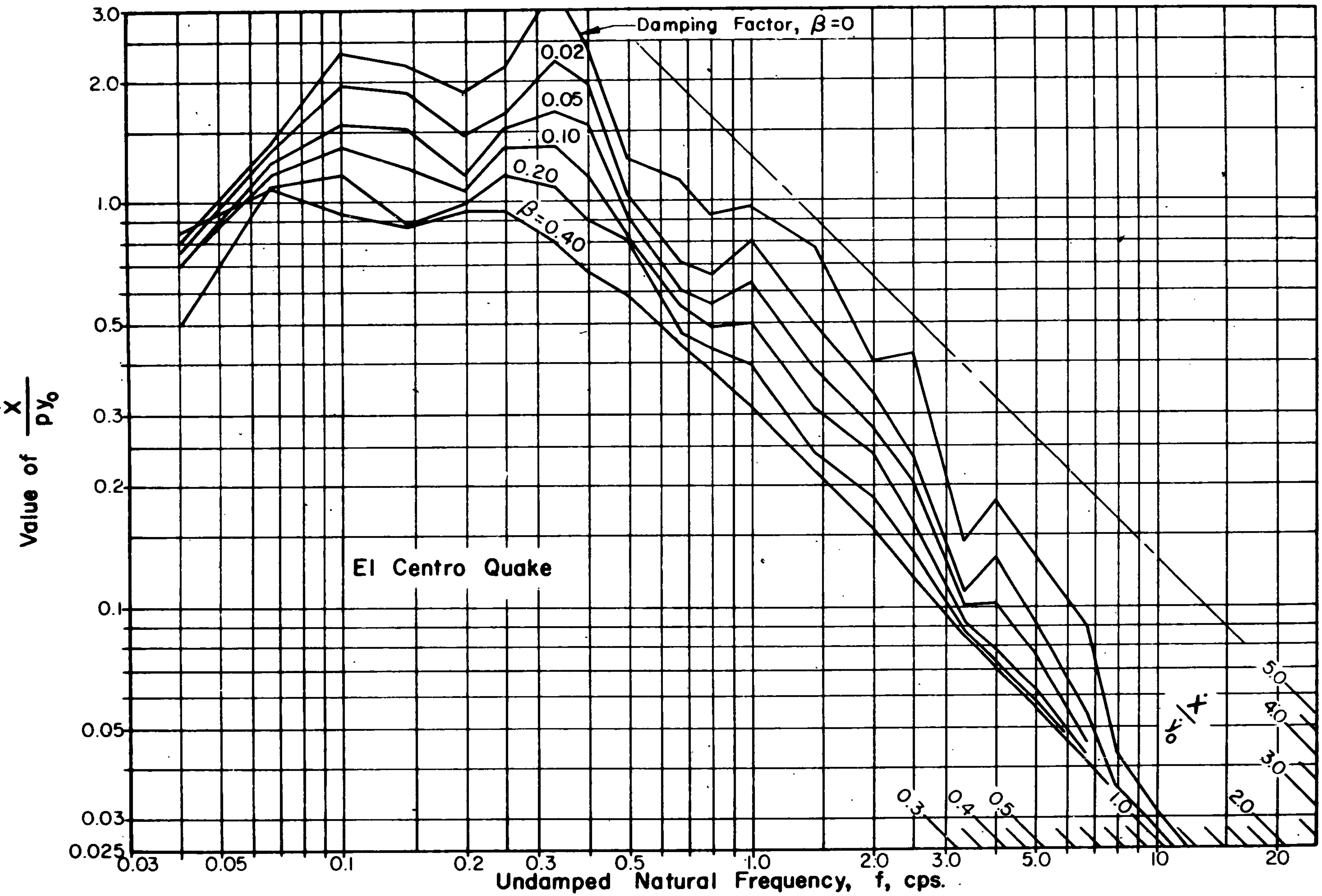


FIG. 2.65b ABSOLUTE VELOCITY SPECTRA FOR ELASTIC SYSTEMS SUBJECTED TO THE EL CENTRO EARTHQUAKE

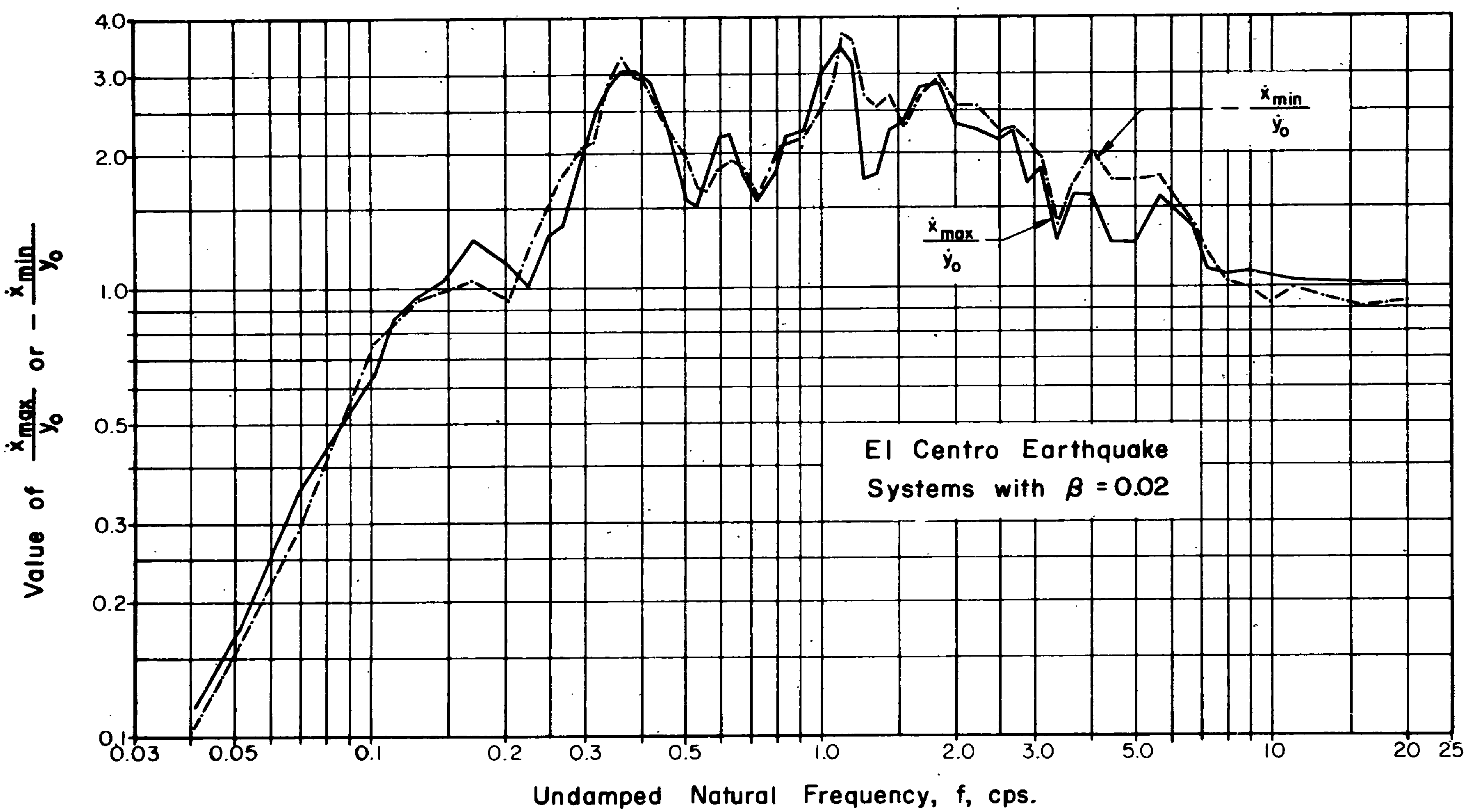


FIG. 2.66 COMPARISON OF SPECTRA FOR THE MAXIMUM AND MINIMUM VELOCITIES OF THE MASS
Damped Elastic Systems Subjected to the El Centro Earthquake

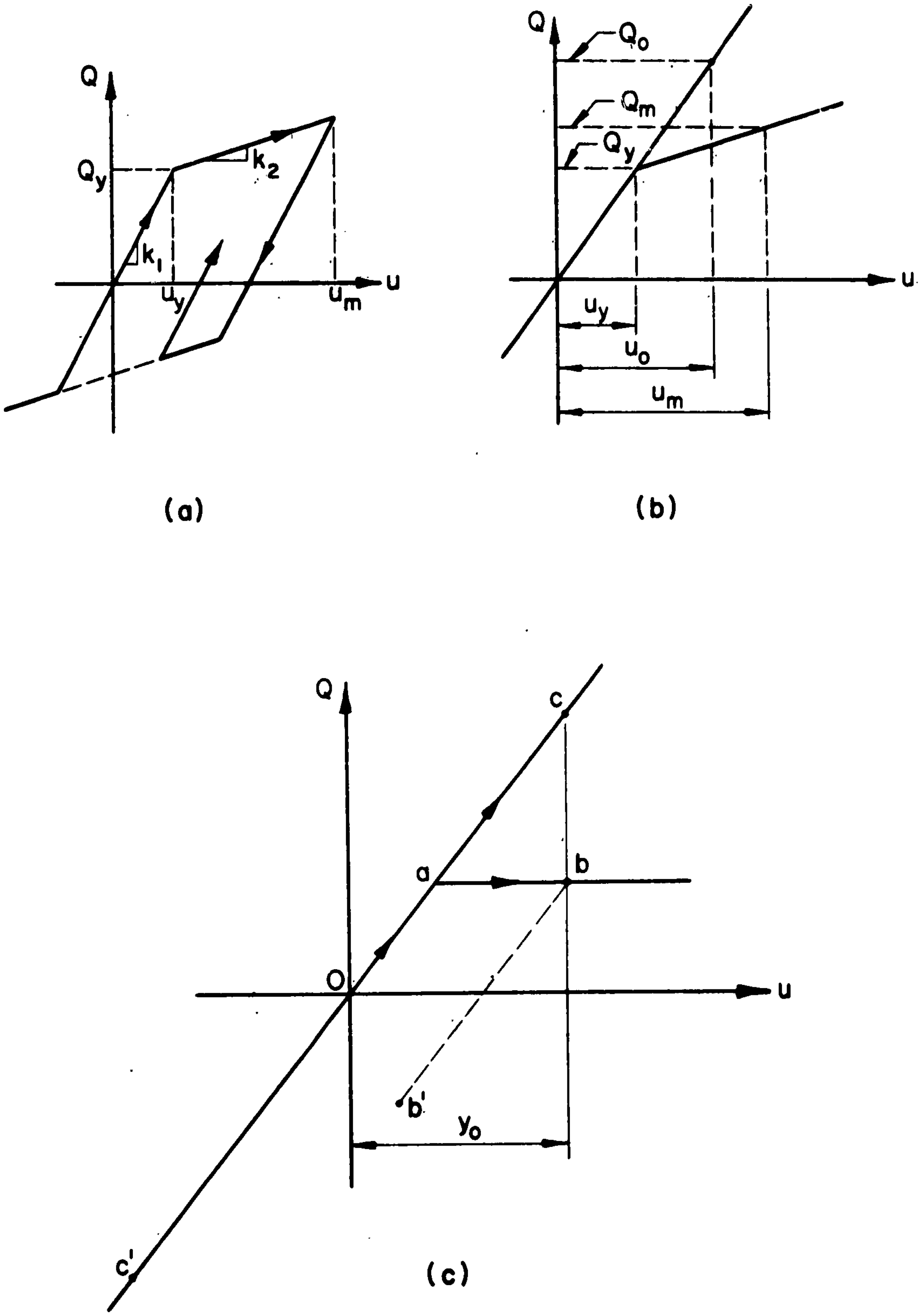


FIG. 3.1 RESISTANCE-DEFORMATION DIAGRAMS FOR SYSTEMS CONSIDERED

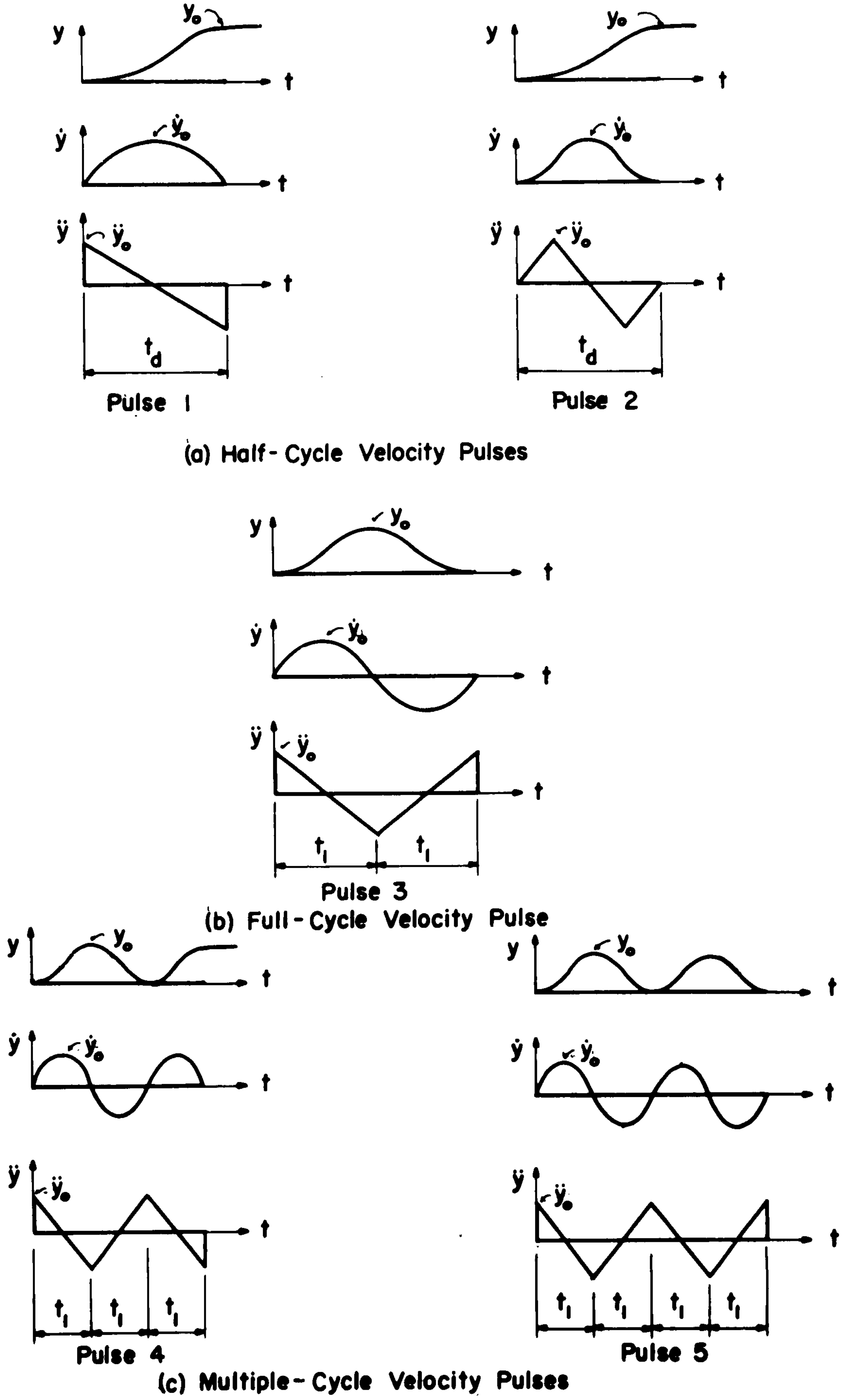


FIG. 3.2 SIMPLE PULSES CONSIDERED

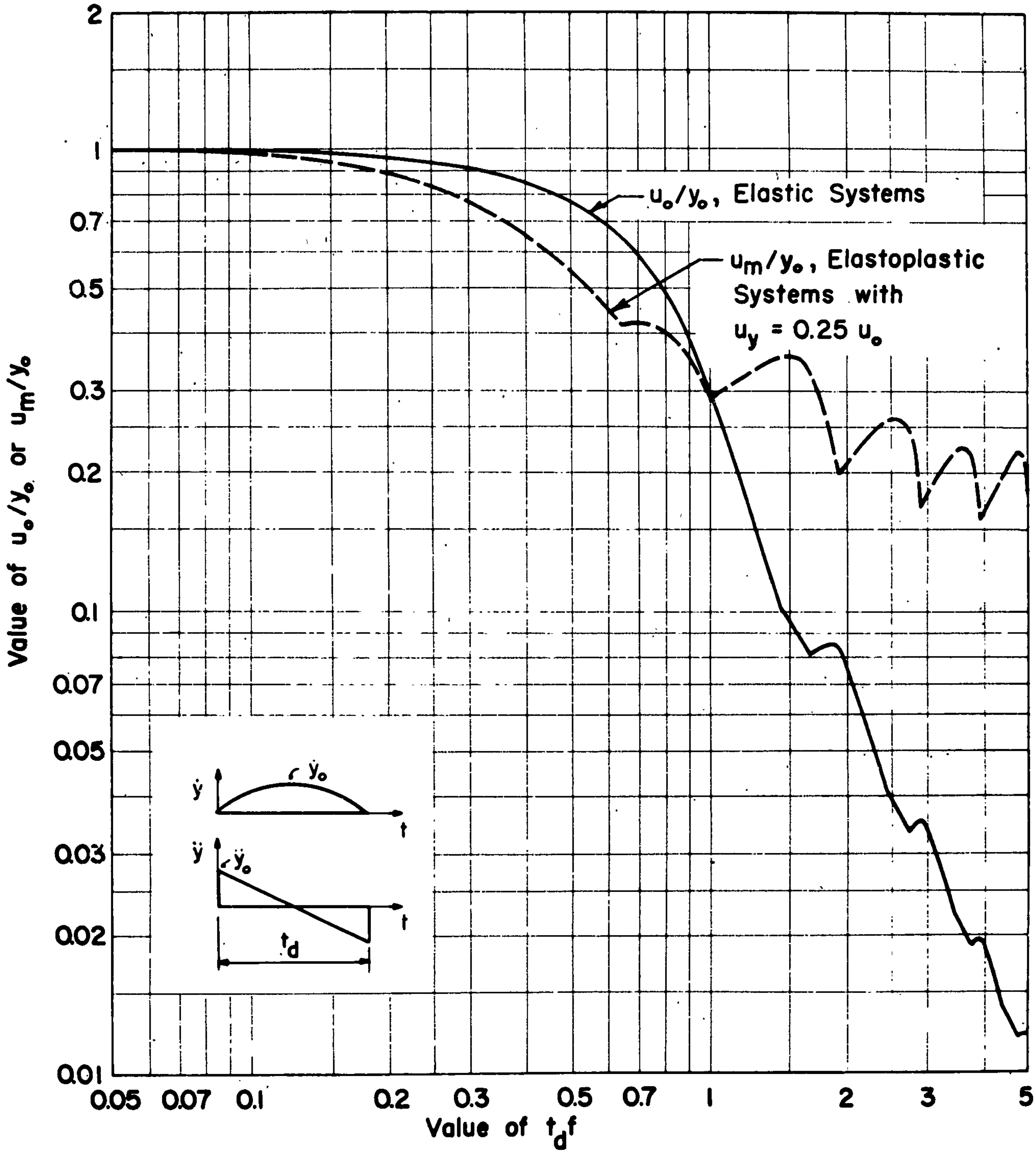


FIG. 3.3 COMPARISON OF MAXIMUM DEFORMATIONS OF ELASTIC SYSTEMS AND ELASTOPLASTIC SYSTEMS WITH u_y = 0.25 u_o--Undamped Systems Subjected to a Half-Cycle Parabolic Velocity Pulse

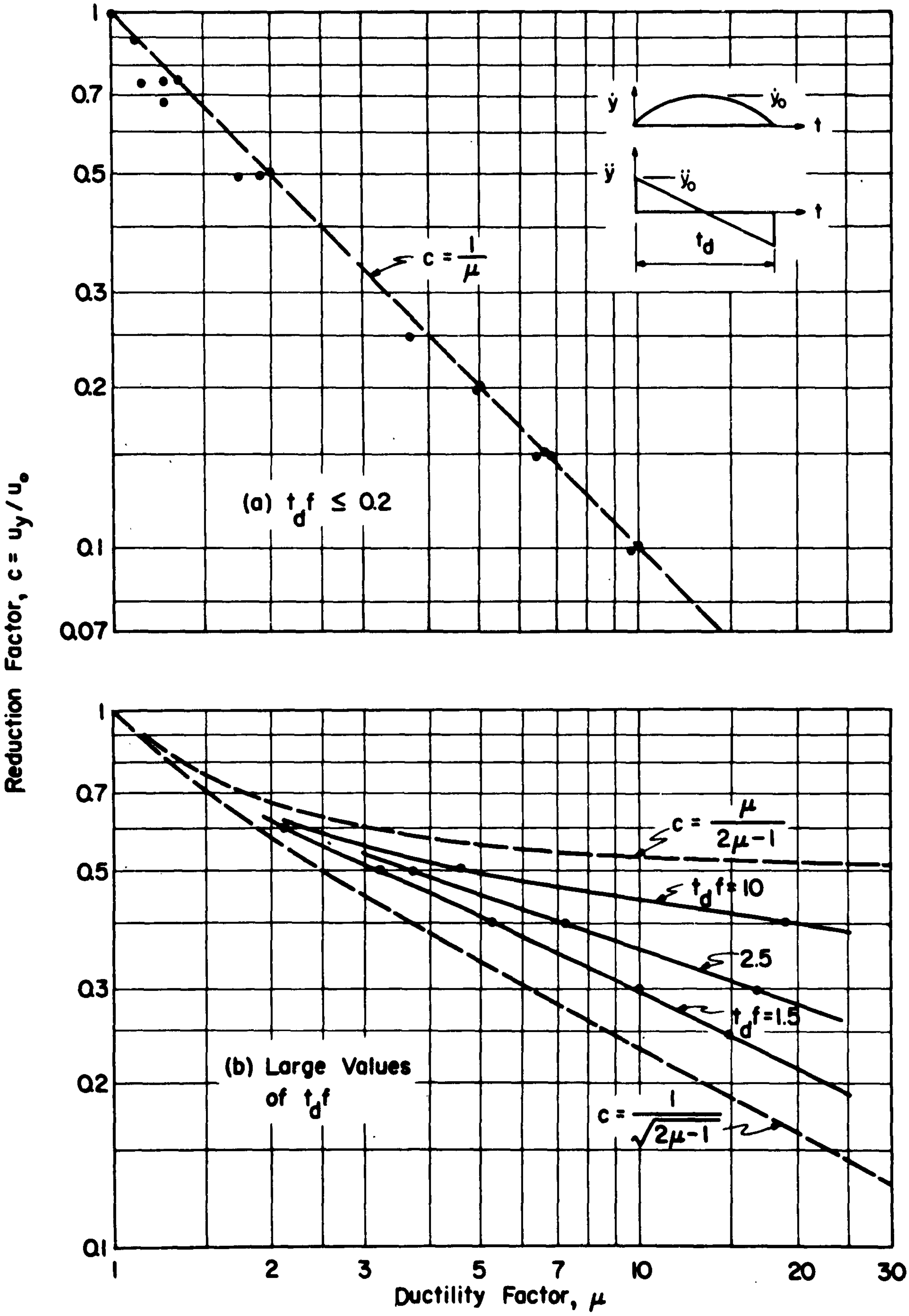


FIG. 3.4 RELATION BETWEEN REDUCTION FACTOR AND DUCTILITY FACTOR Undamped Elastoplastic Systems Subjected to a Half-Cycle Parabolic Velocity Pulse (Continued on Next Page)

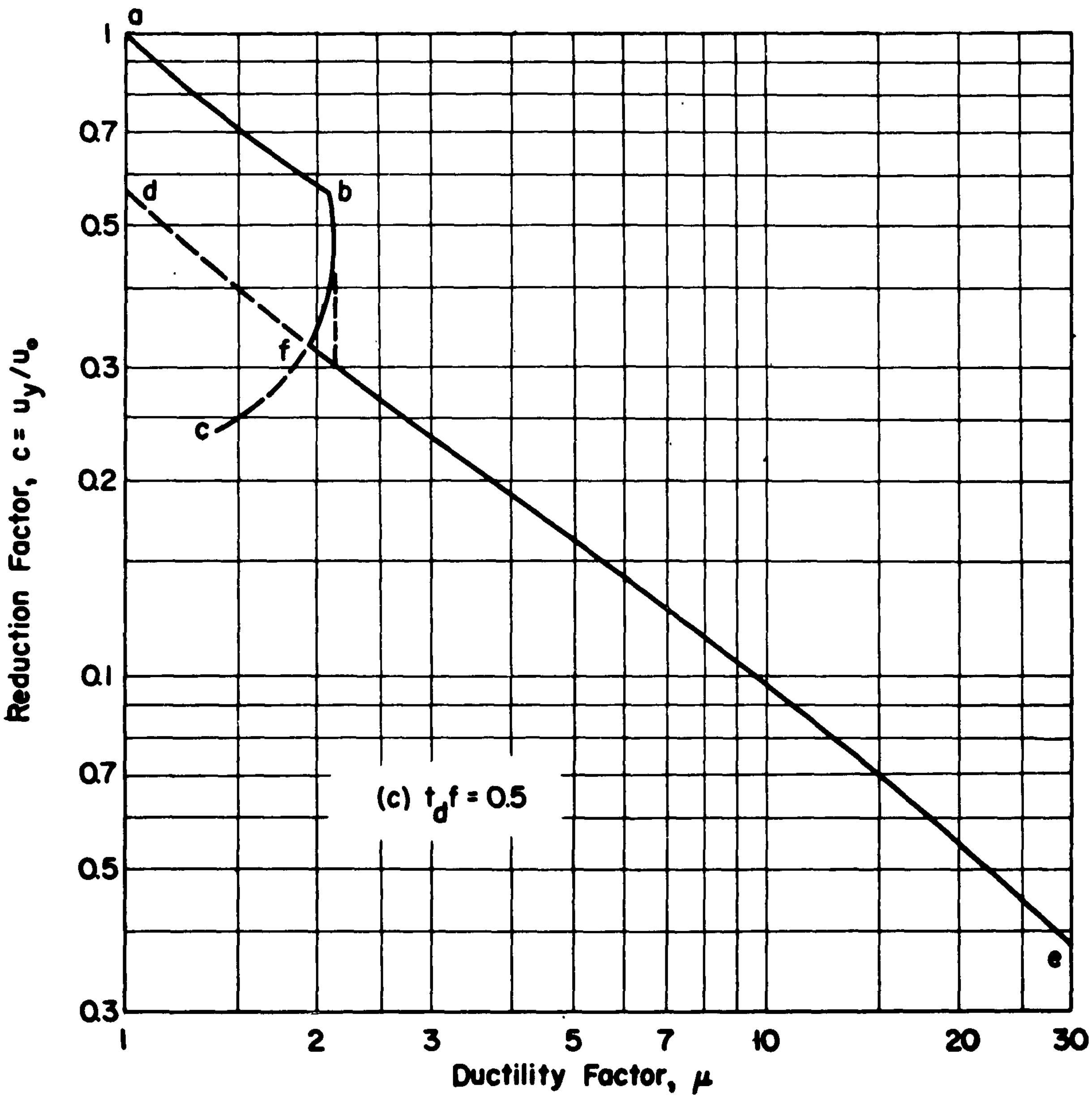


FIG. 3.4 (Continued)

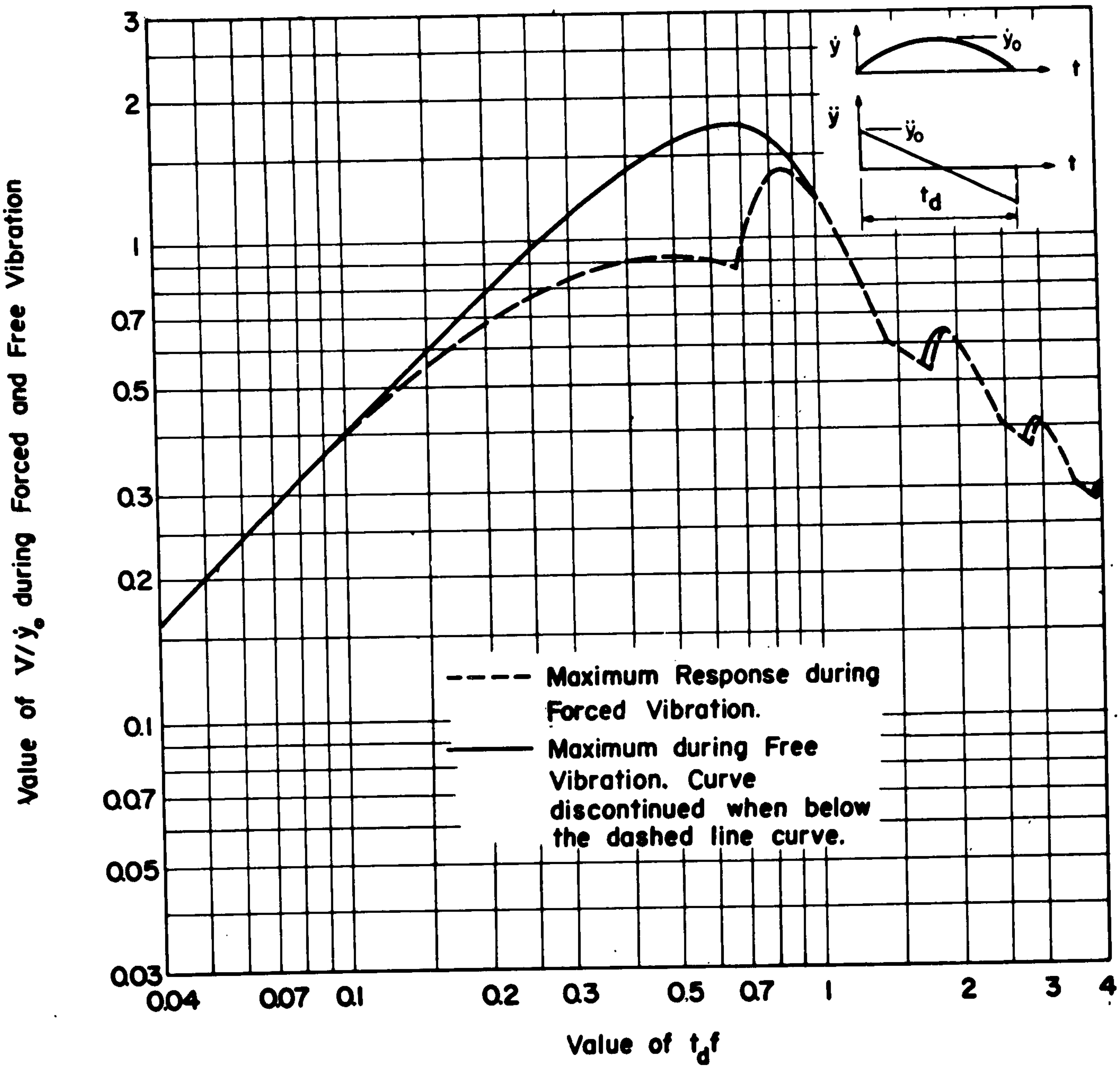


FIG. 3.5 SPECTRA FOR MAXIMUM DEFORMATION DURING FORCED VIBRATION AND FREE VIBRATION Undamped Elastic Systems Subjected to a Half-Cycle Parabolic Velocity Pulse

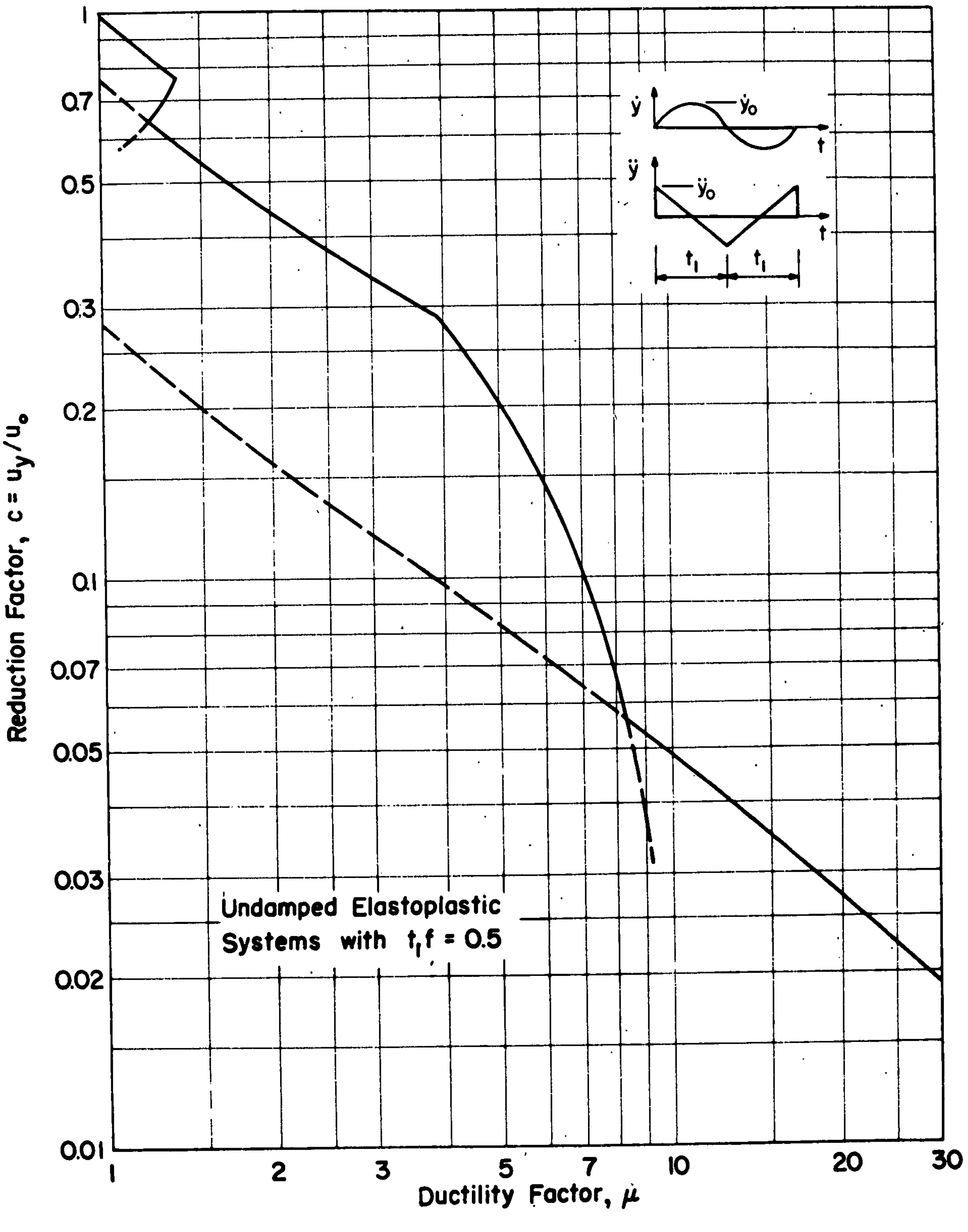


FIG. 3.6a RELATION BETWEEN REDUCTION FACTOR AND DUCTILITY FACTOR
Undamped Elastoplastic Systems Subjected to a Full-Cycle Parabolic Velocity Pulse

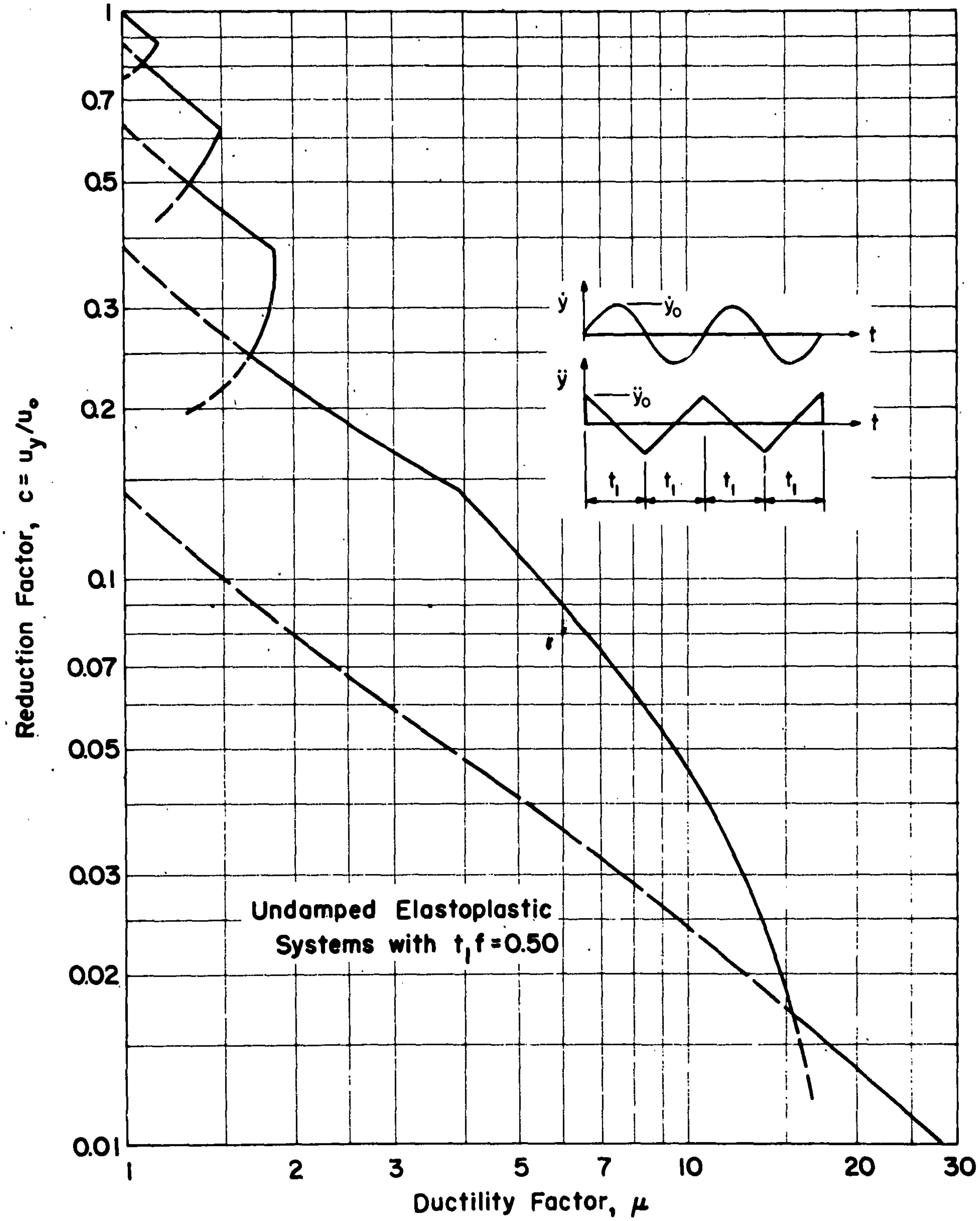


FIG. 3.6b RELATION BETWEEN REDUCTION FACTOR AND DUCTILITY FACTOR
Undamped Elastoplastic Systems Subjected to a Multiple-Cycle Parabolic Velocity Pulse

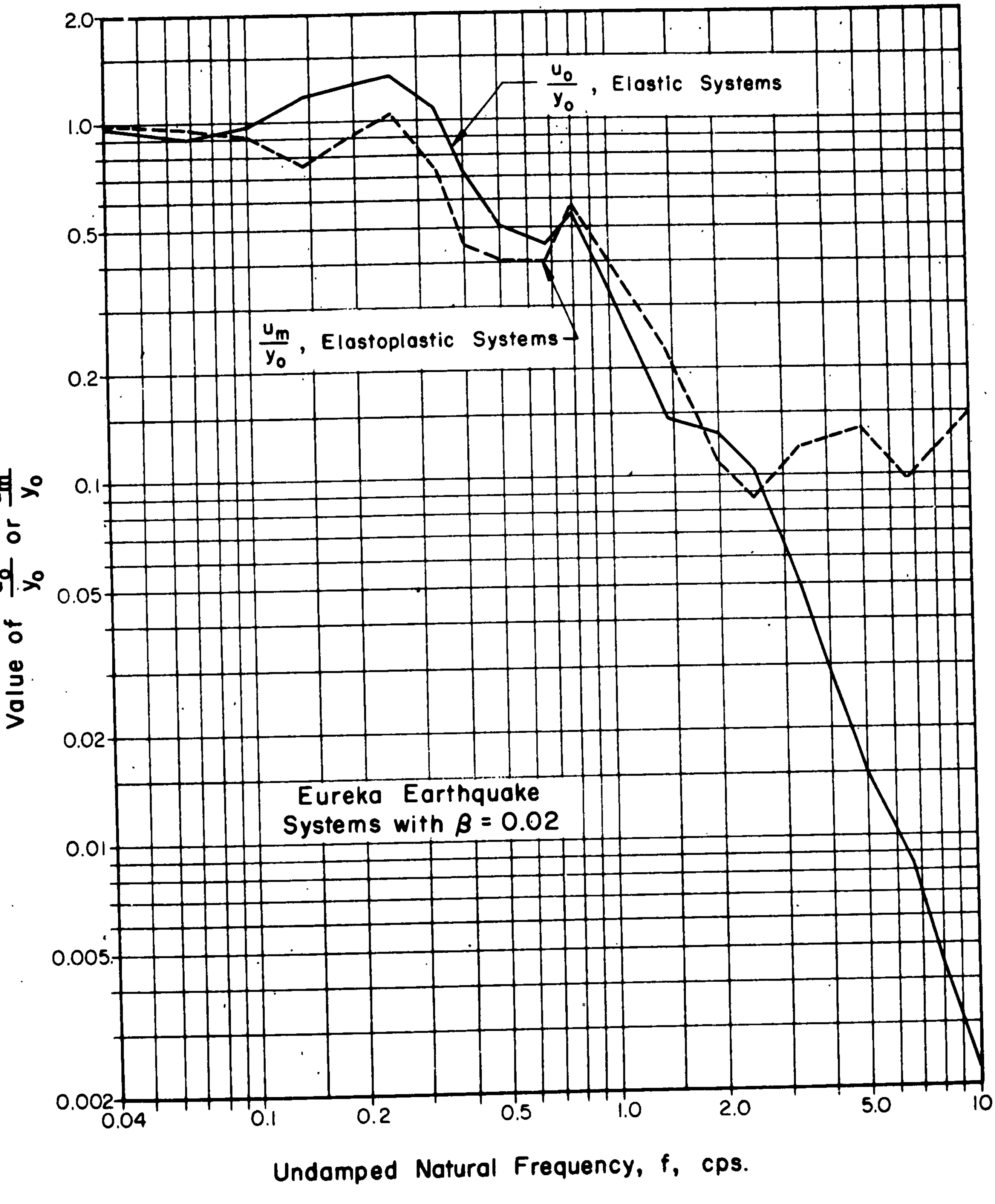


FIG. 3.7a COMPARISON OF MAXIMUM DEFORMATIONS OF ELASTIC SYSTEMS AND ELASTOPLASTIC SYSTEMS WITH $u_y = 0.25\ u_o$--Systems with 2 Percent Critical Damping Subjected to the Eureka Quake

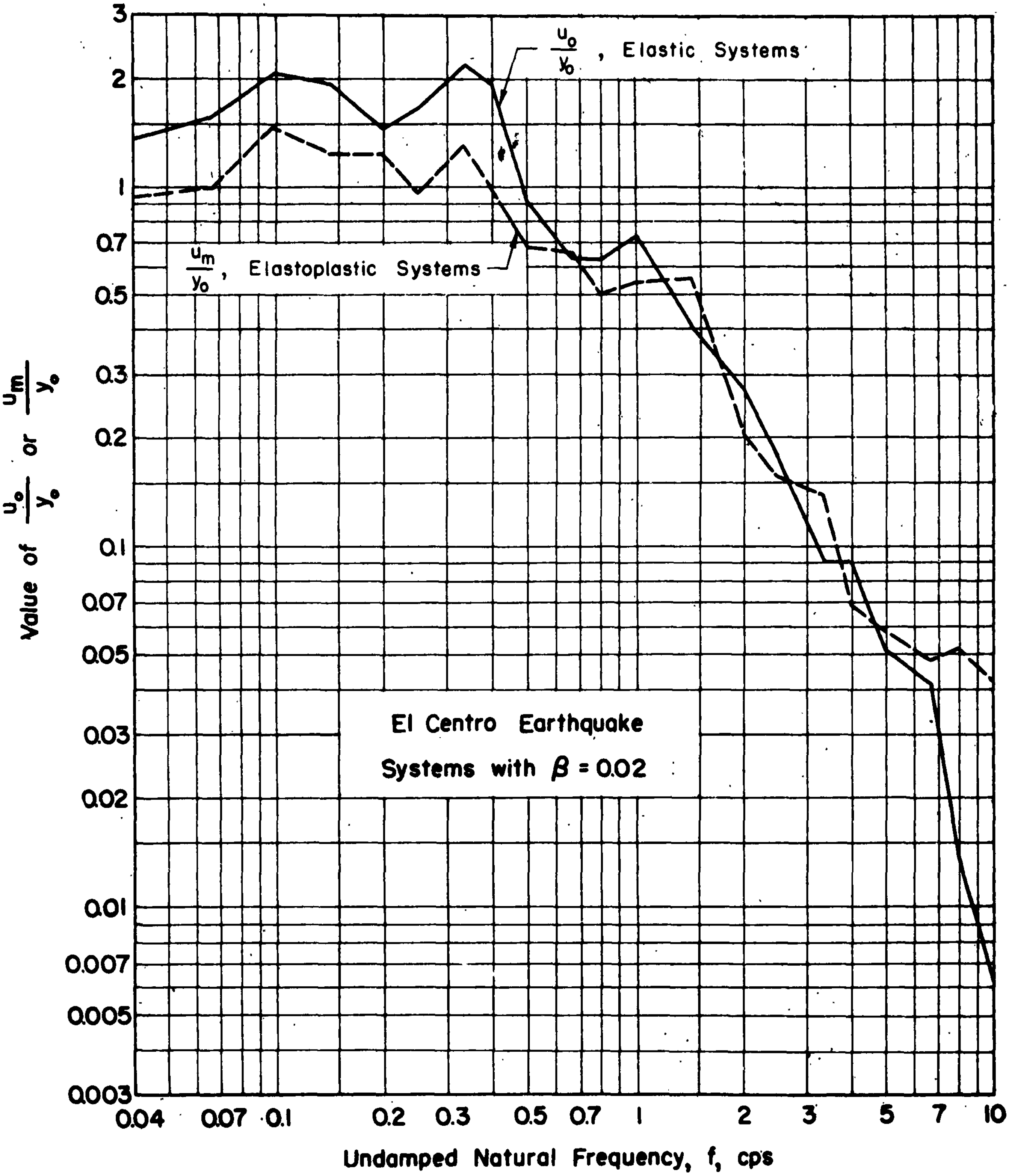


FIG. 3.7b COMPARISON OF MAXIMUM DEFORMATIONS OF ELASTIC SYSTEMS AND ELASTOPLASTIC SYSTEMS WITH $u_y = 0.25\ u_o$--Systems with 2 Percent Critical Damping Subjected to the El Centro Quake

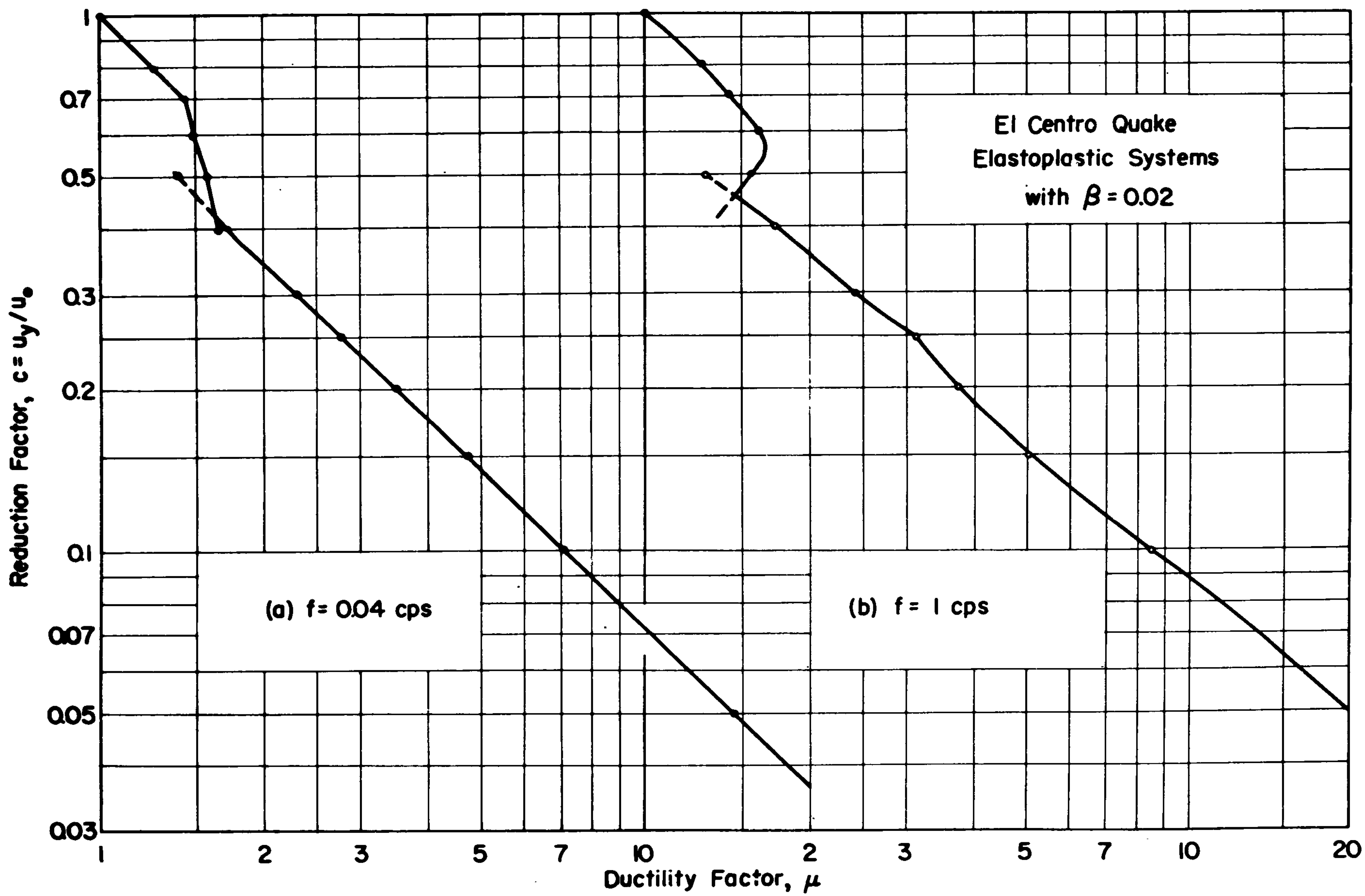


FIG. 3.8 RELATION BETWEEN REDUCTION FACTOR AND DUCTILITY FACTOR--Elastoplastic Systems with 2 Percent Critical Damping Subjected to the El Centro Quake

(Continued on Next Page)

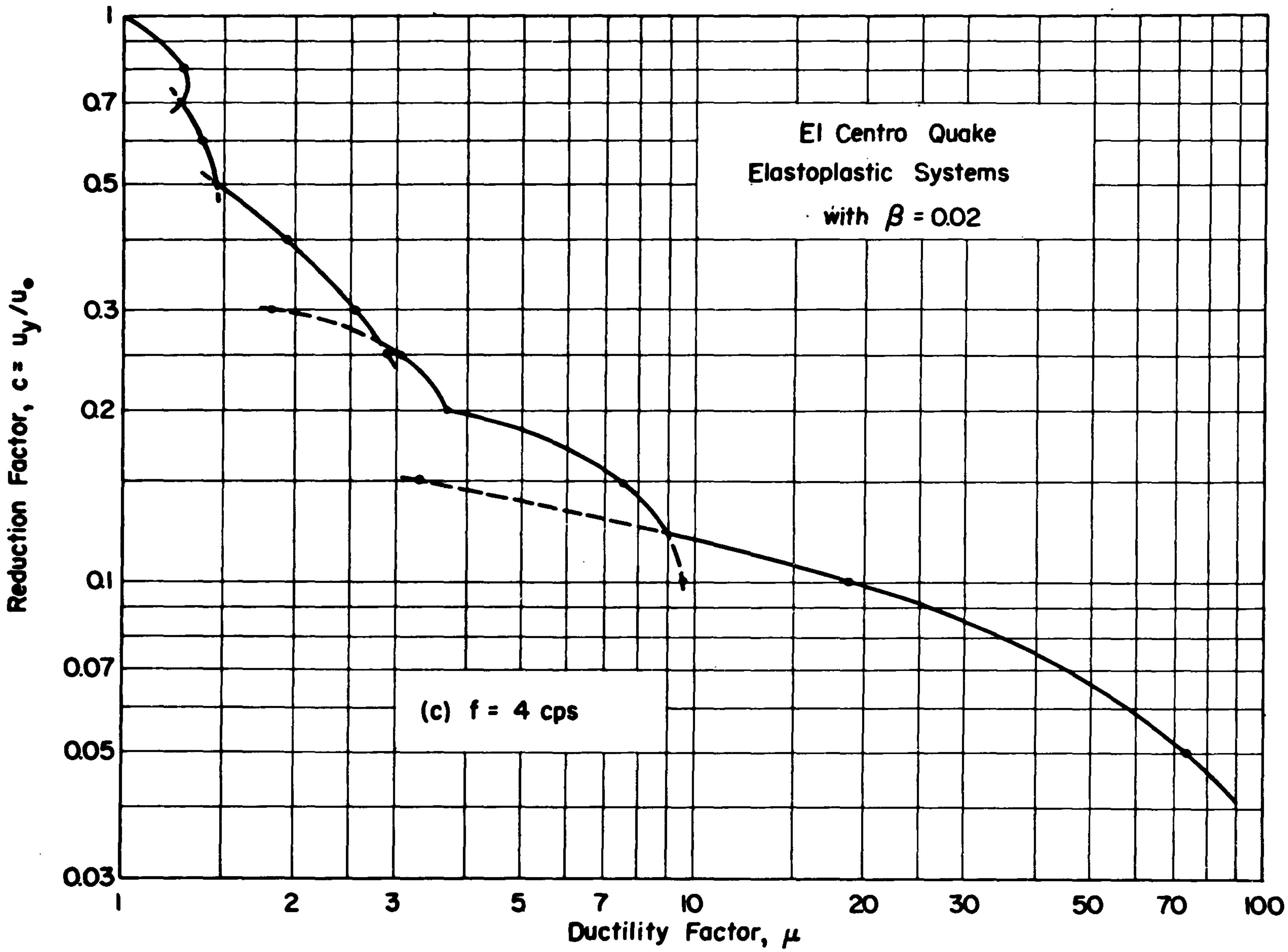


FIG. 3.8 (Continued)

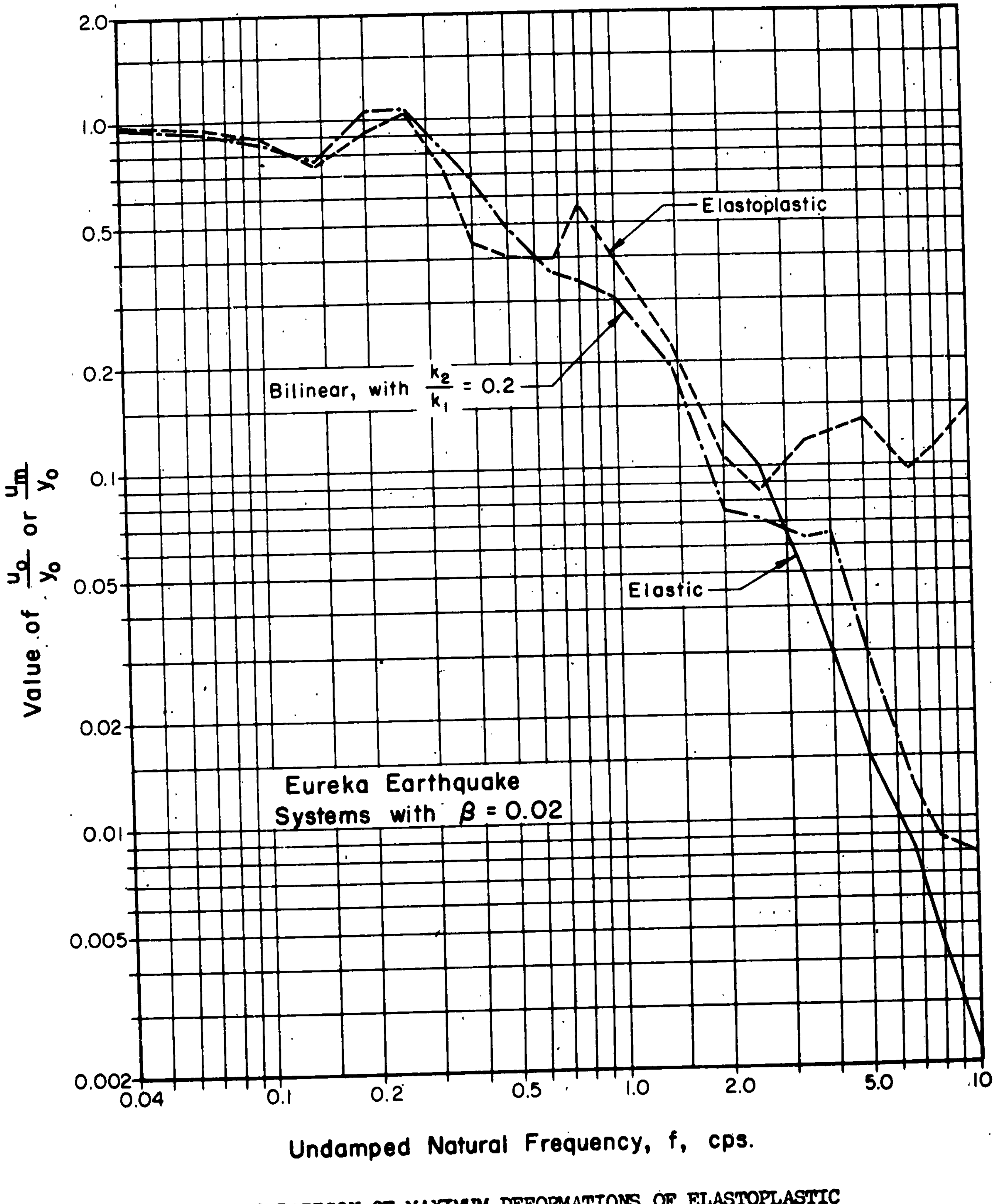


FIG. 3.9 COMPARISON OF MAXIMUM DEFORMATIONS OF ELASTOPLASTIC AND BILINEAR SYSTEMS WITH $u_y = 0.25\ u_o$--Systems with 2 Percent Critical Damping Subjected to the Eureka Quake

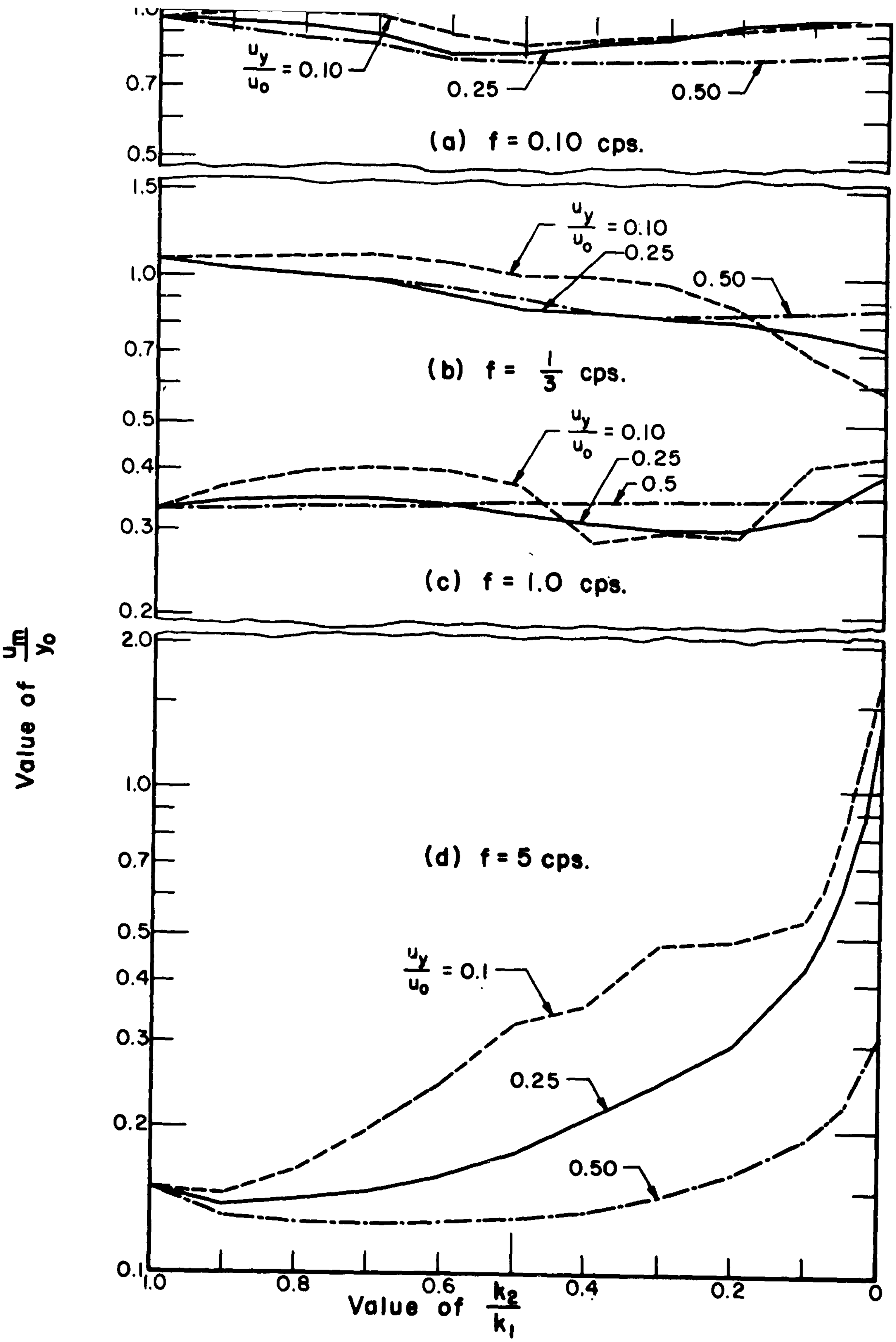


FIG. 3.10 EFFECT OF STIFFNESS PARAMETER k_2/k_1 ON MAXIMUM DEFORMATION OF BILINEAR SYSTEMS WITH TWO PERCENT CRITICAL DAMPING SUBJECTED TO THE EUREKA QUAKE

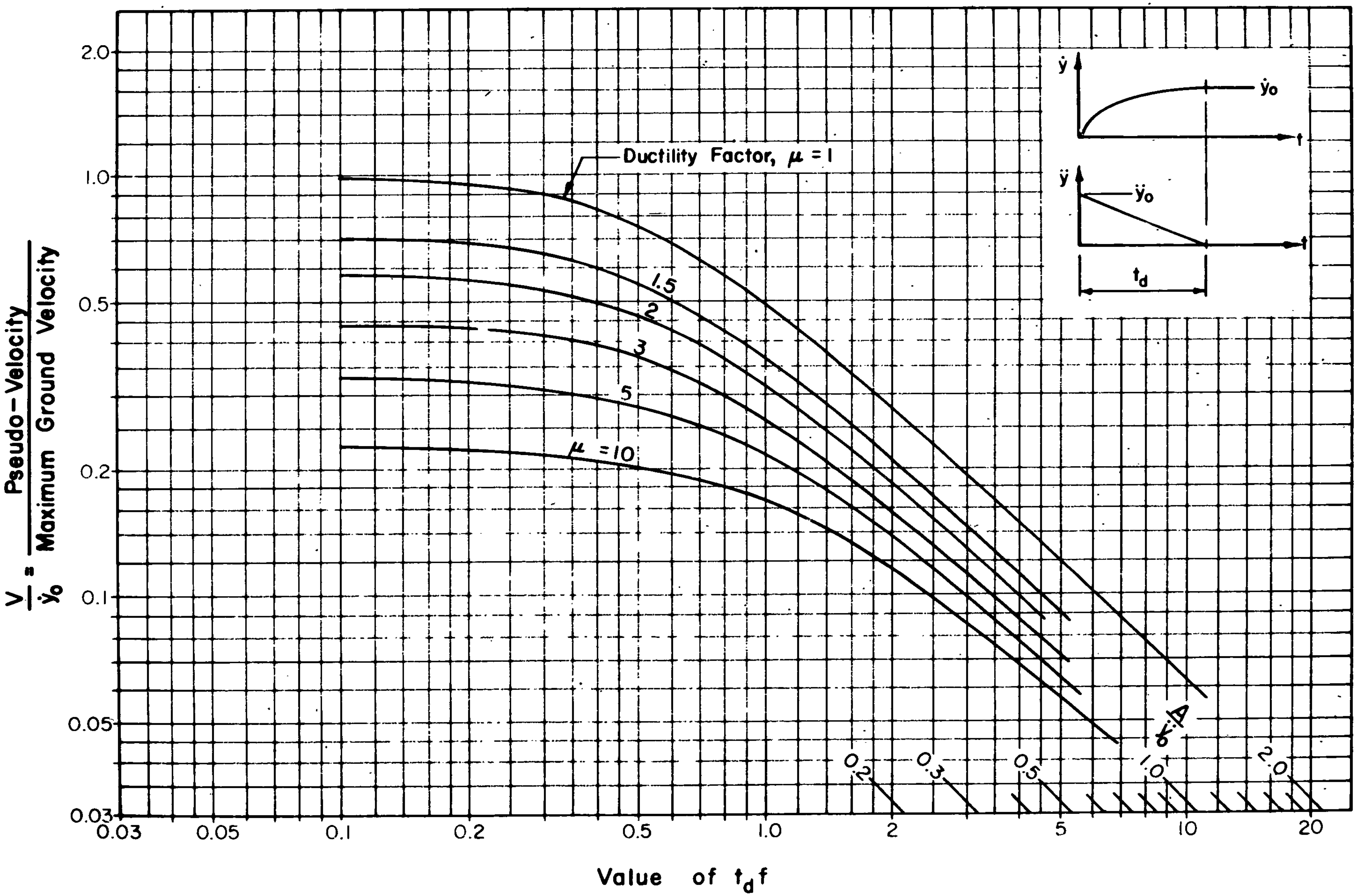


FIG. 3.11 DEFORMATION SPECTRA FOR UNDAMPED ELASTOPLASTIC SYSTEMS SUBJECTED TO A TRIANGULAR ACCELERATION PULSE

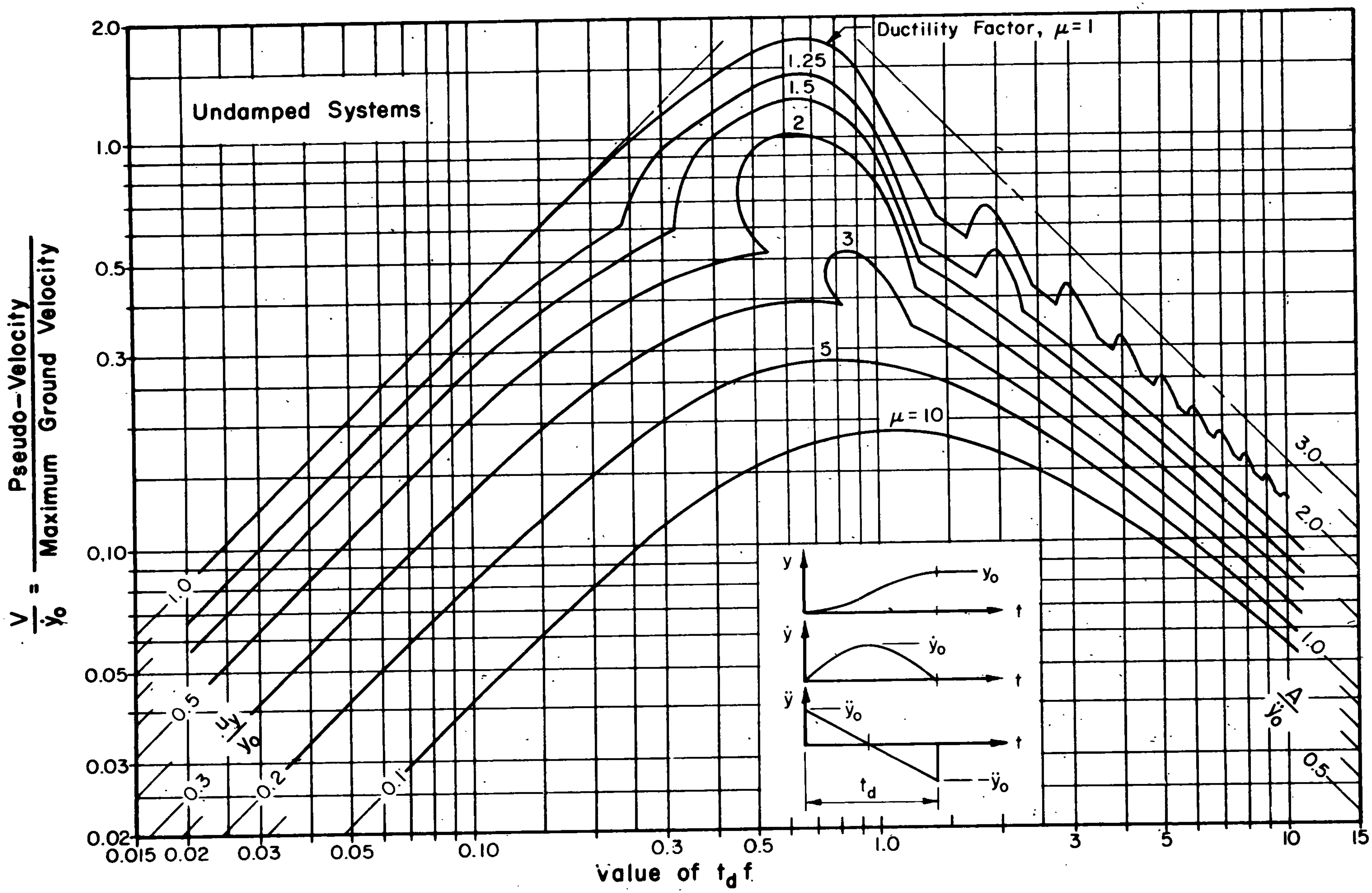


FIG. 3.12a DEFORMATION SPECTRA FOR UNDAMPED, ELASTOPLASTIC SYSTEMS SUBJECTED TO A HALF-CYCLE PARABOLIC VELOCITY PULSE

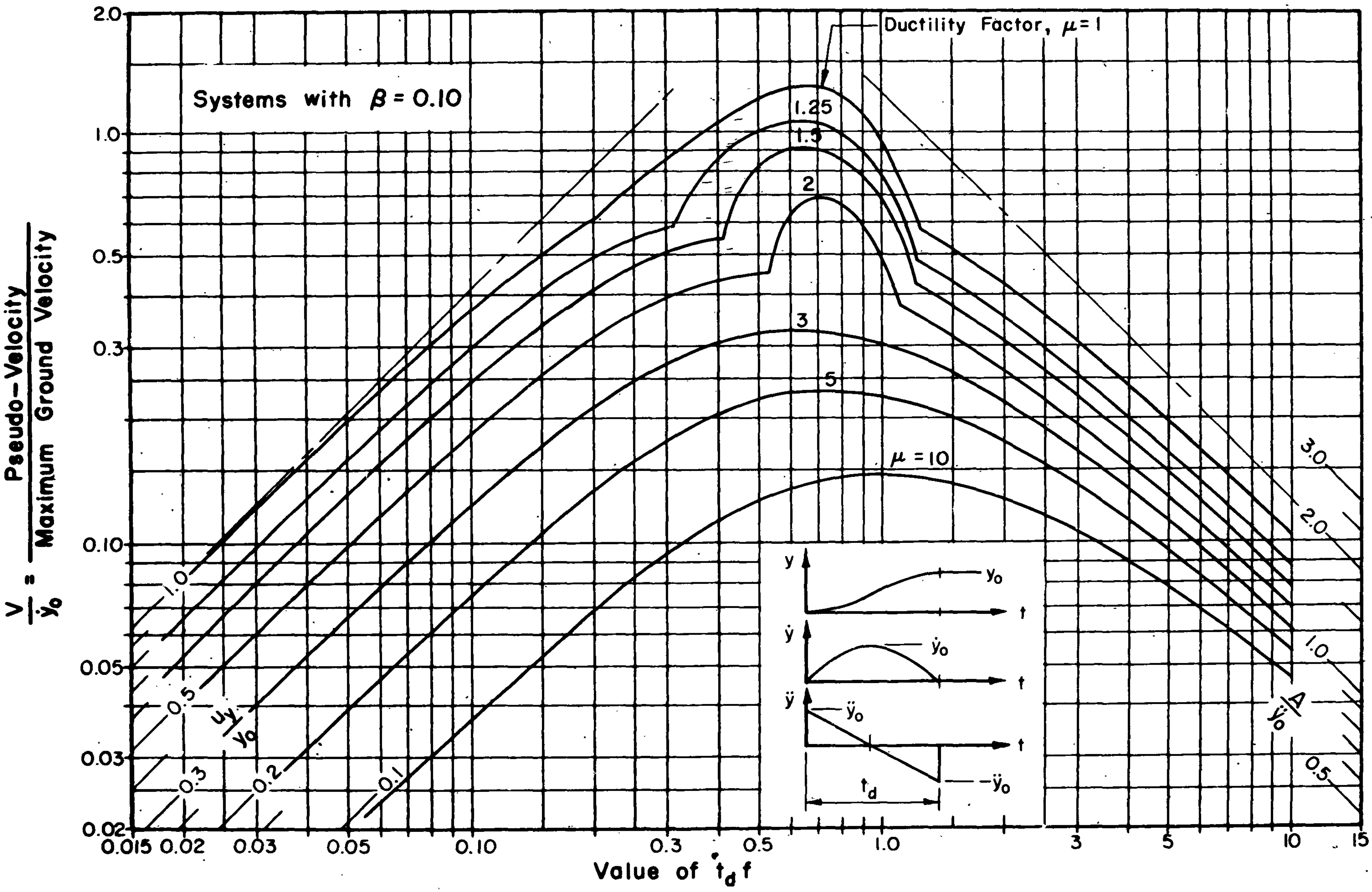


FIG. 3.12b DEFORMATION SPECTRA FOR ELASTOPLASTIC SYSTEMS WITH TEN PERCENT CRITICAL DAMPING SUBJECTED TO A HALF-CYCLE PARABOLIC VELOCITY PULSE

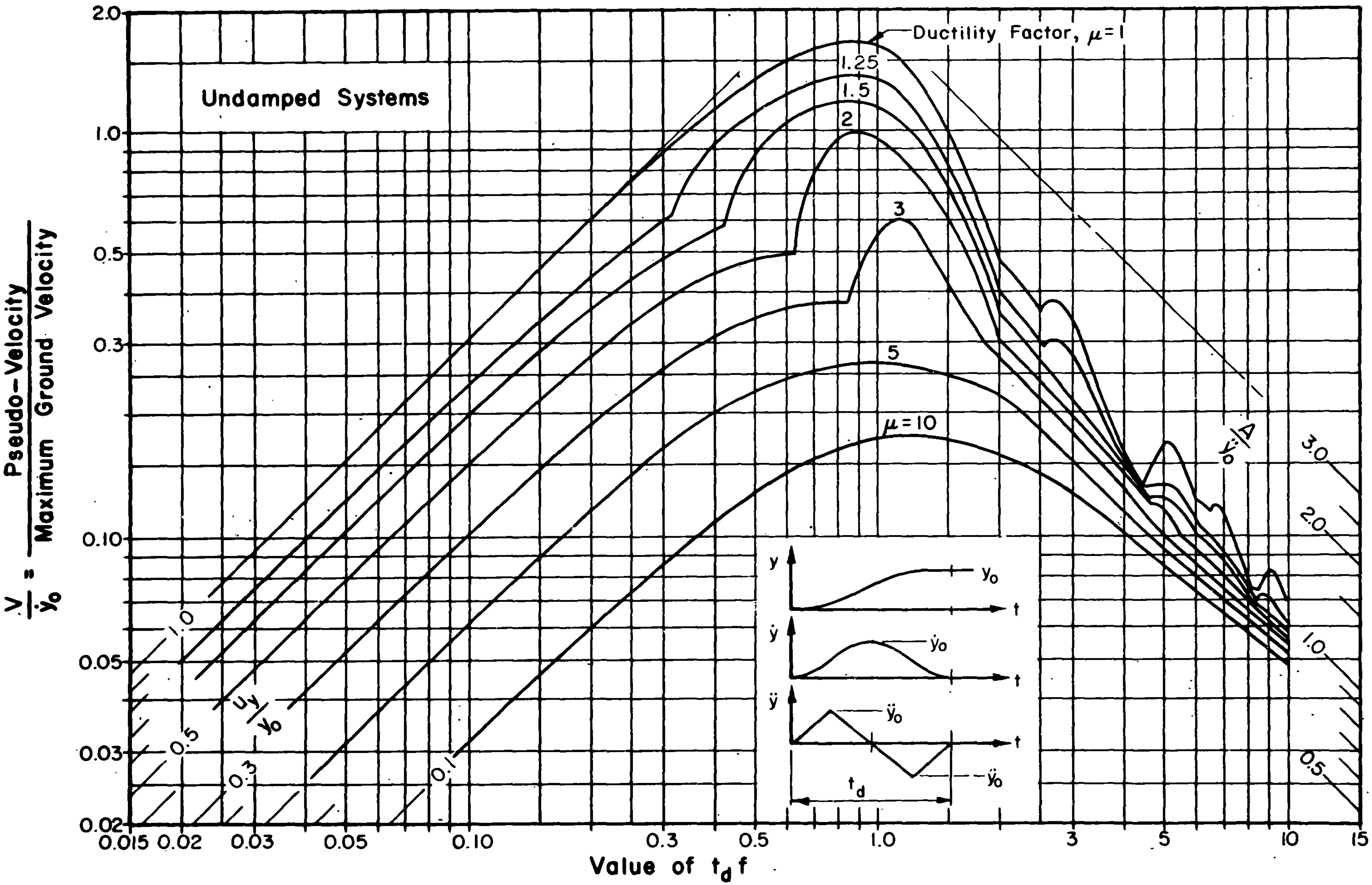


FIG. 3.13a DEFORMATION SPECTRA FOR UNDAMPED, ELASTOPLASTIC SYSTEMS SUBJECTED TO A HALF-CYCLE VELOCITY PULSE

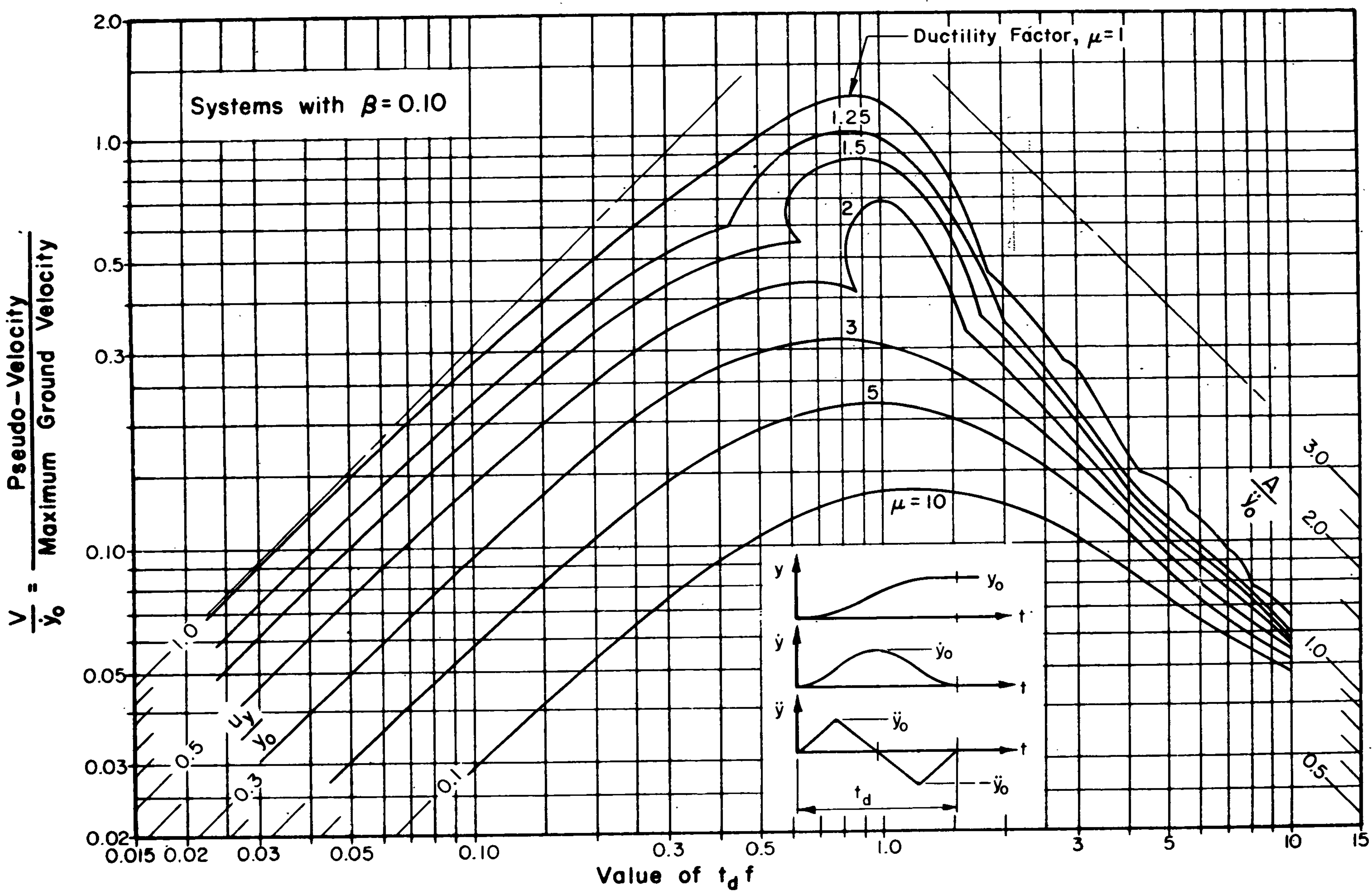


FIG. 3.13b DEFORMATION SPECTRA FOR ELASTOPLASTIC SYSTEMS WITH TEN PERCENT CRITICAL DAMPING SUBJECTED TO A HALF-CYCLE VELOCITY PULSE

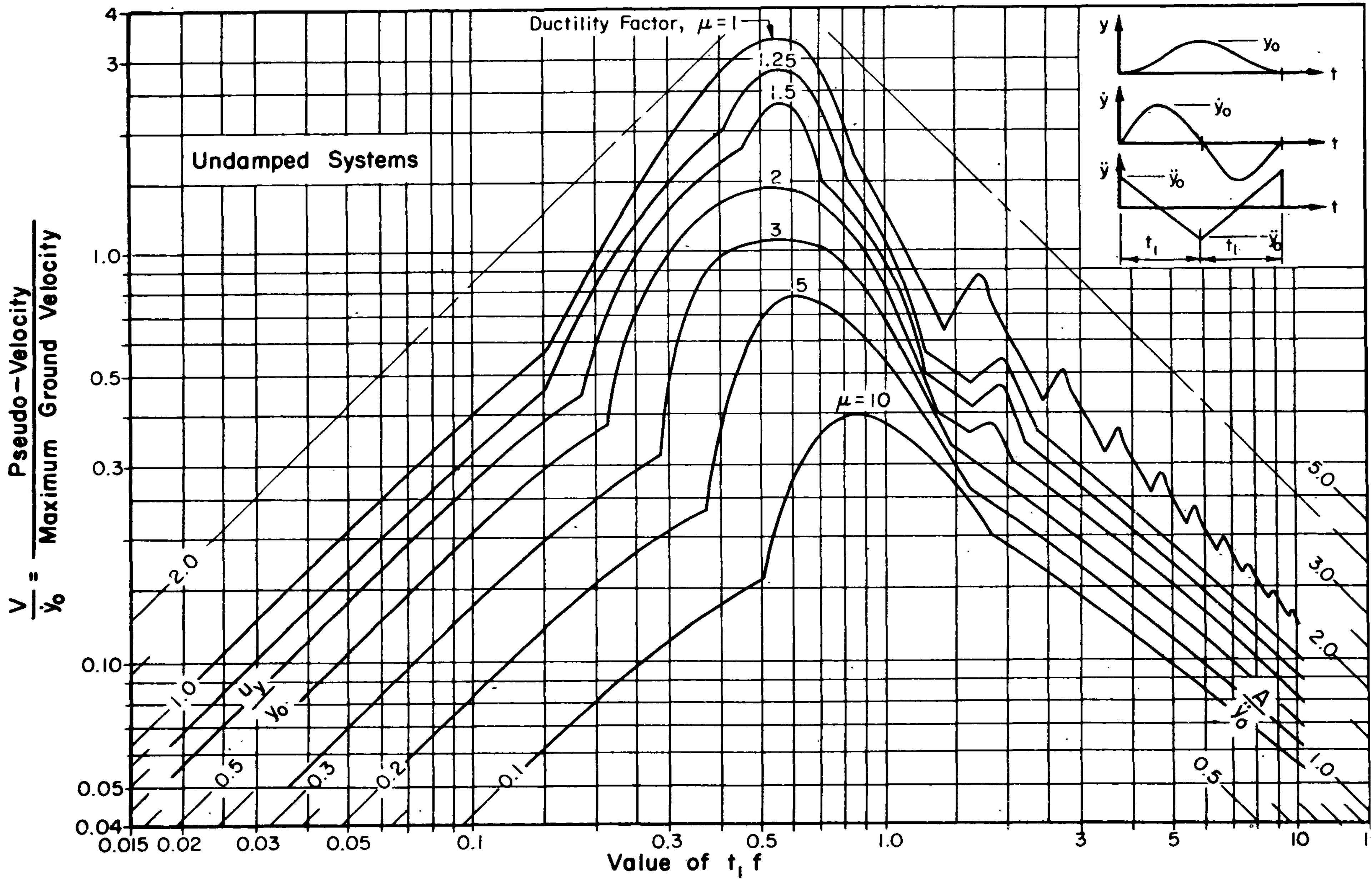


FIG. 3.14a DEFORMATION SPECTRA FOR UNDAMPED, ELASTOPLASTIC SYSTEMS SUBJECTED TO A FULL-CYCLE PARABOLIC VELOCITY PULSE

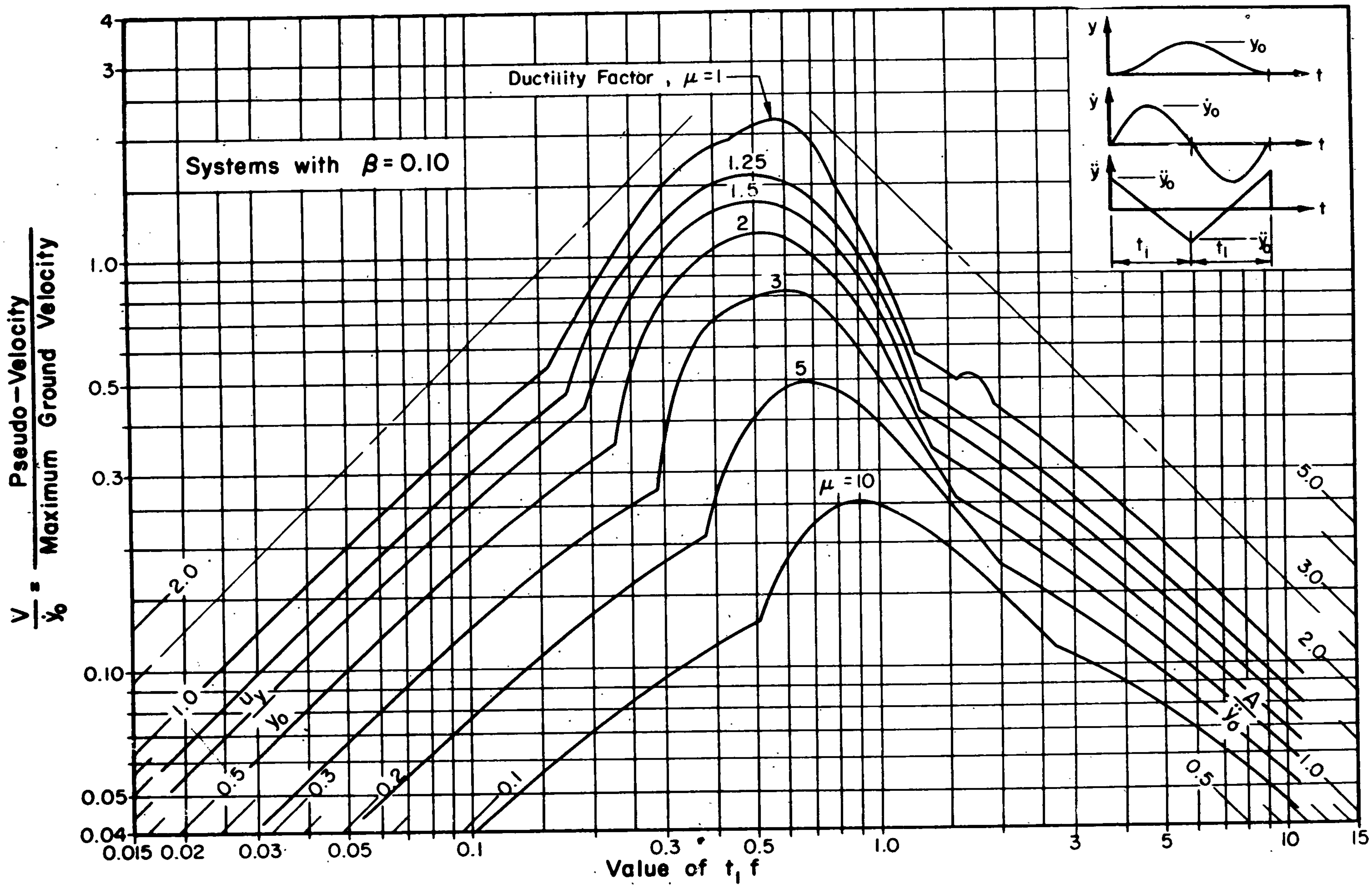


FIG. 3.14b DEFORMATION SPECTRA FOR ELASTOPLASTIC SYSTEMS WITH TEN PERCENT CRITICAL DAMPING SUBJECTED TO A FULL-CYCLE PARABOLIC VELOCITY PULSE

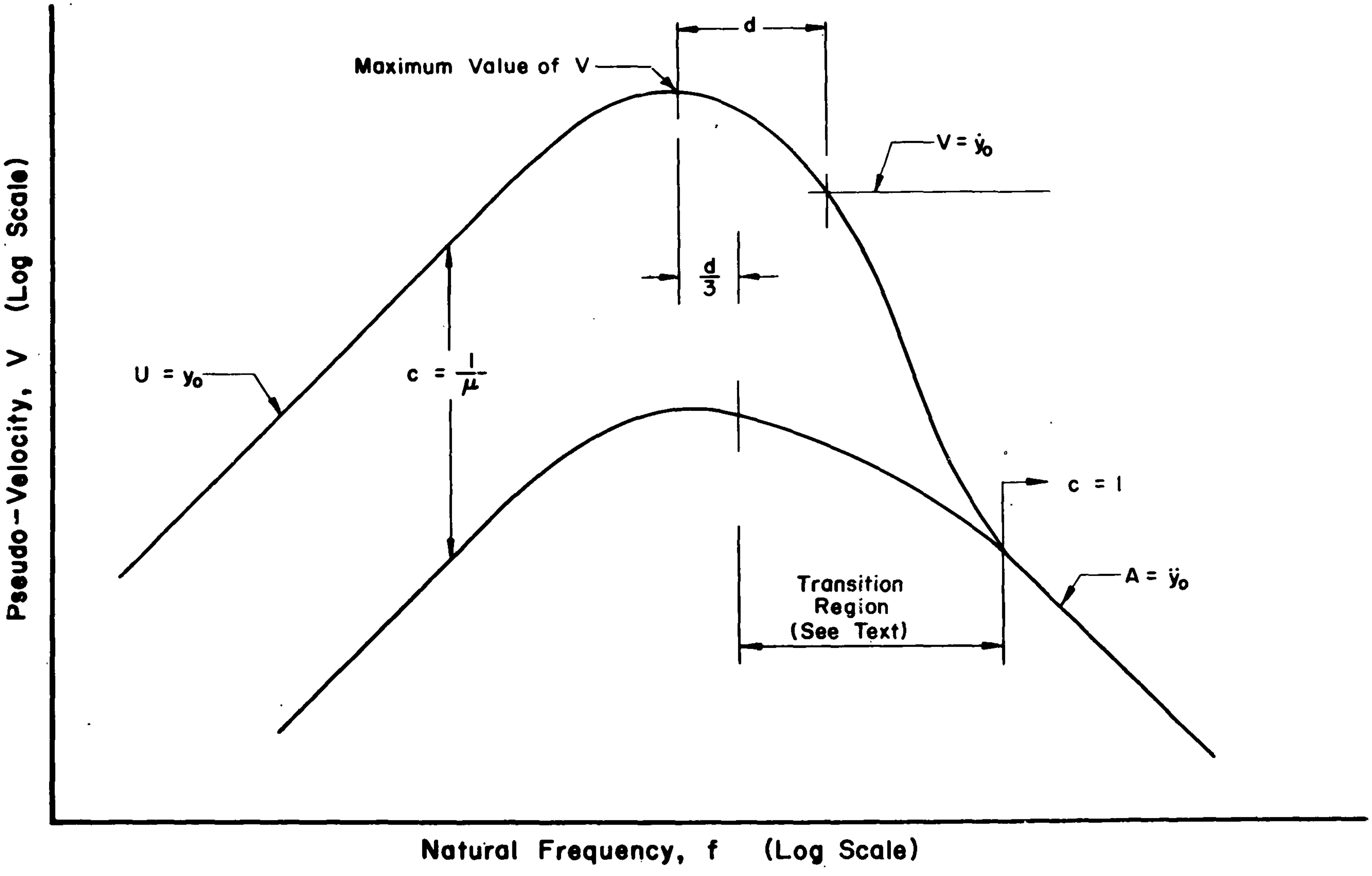


FIG. 3.15 APPROXIMATE DESIGN RULE FOR CONSTRUCTION OF DEFORMATION SPECTRA FOR ELASTOPLASTIC SYSTEMS--See Text for Limitations

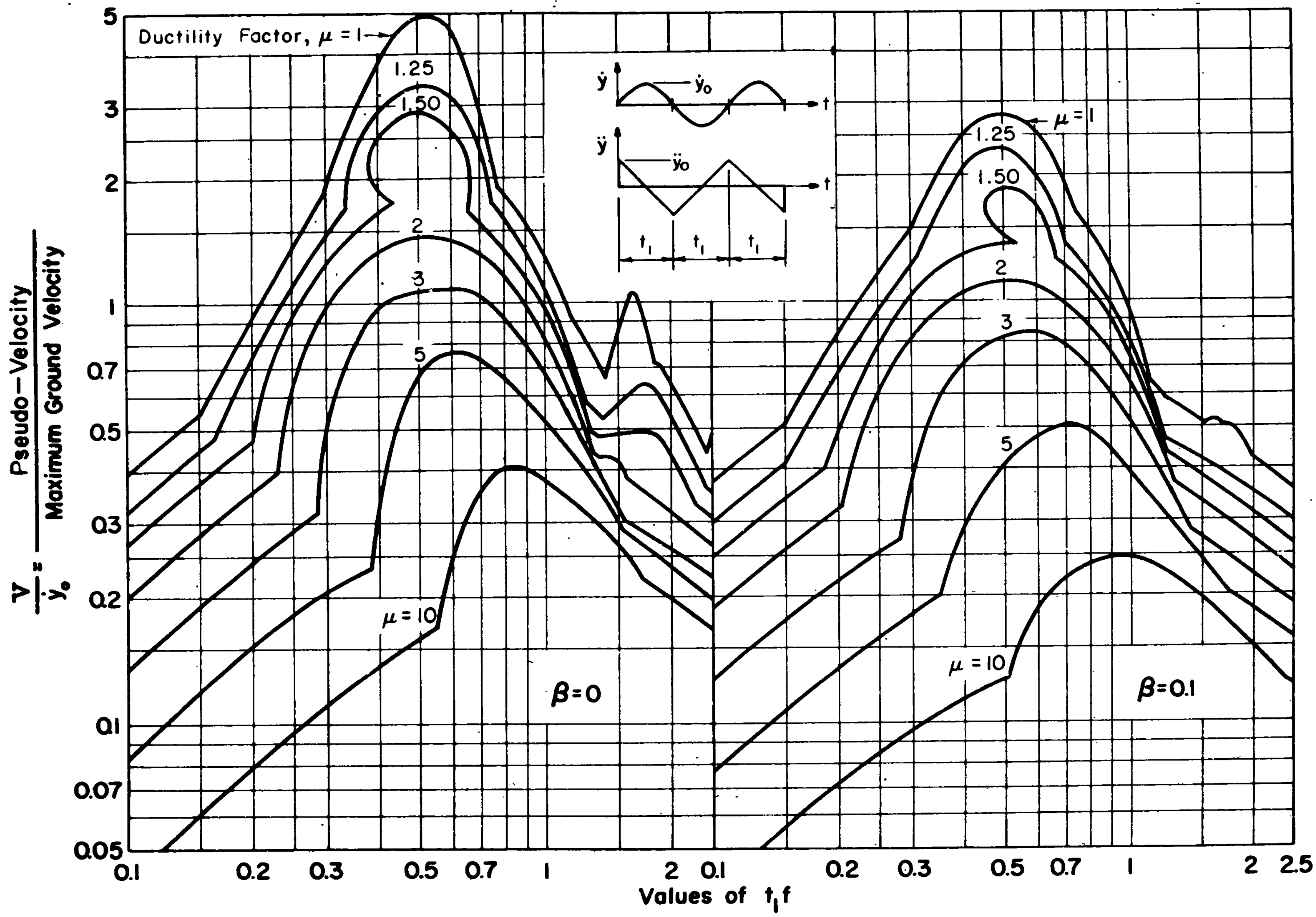


FIG. 3.16 DEFORMATION SPECTRA FOR ELASTOPLASTIC SYSTEMS SUBJECTED TO A PARABOLIC VELOCITY PULSE WITH THREE HALF-CYCLES

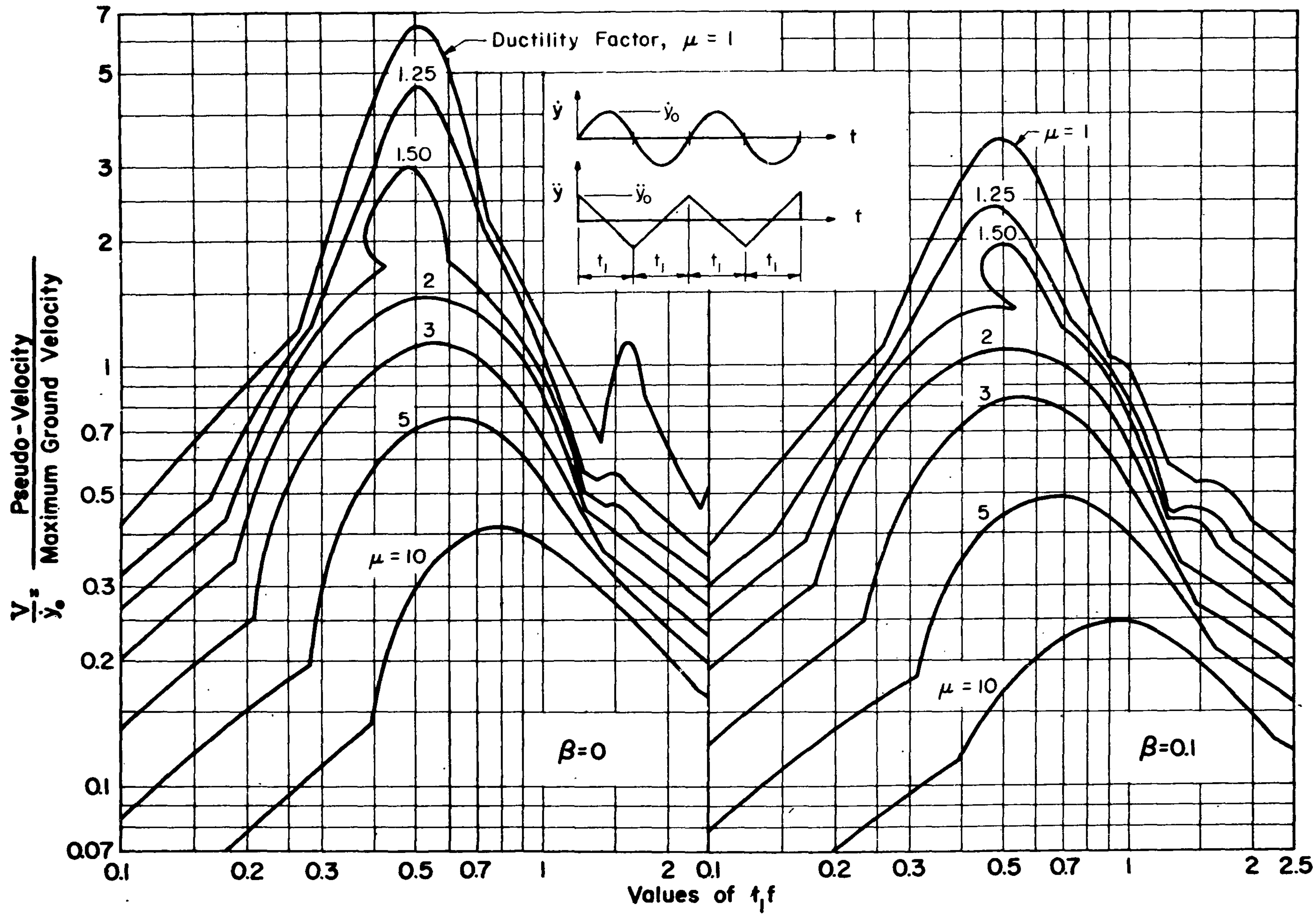


FIG. 3.17 DEFORMATION SPECTRA FOR ELASTOPLASTIC SYSTEMS SUBJECTED TO A PARABOLIC VELOCITY PULSE WITH TWO FULL-CYCLES

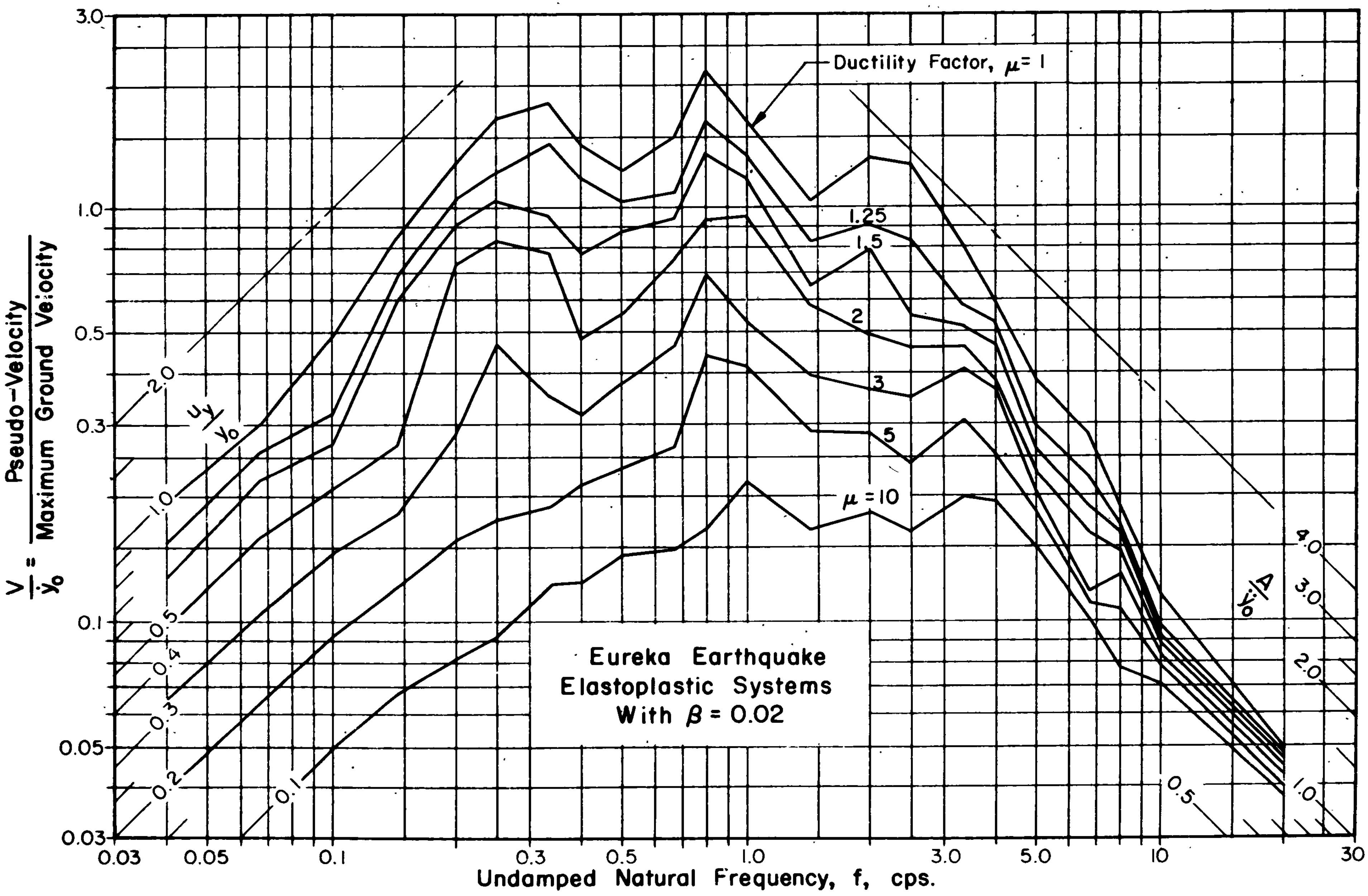


FIG. 3.18 DEFORMATION SPECTRA FOR ELASTOPLASTIC SYSTEMS WITH TWO PERCENT CRITICAL DAMPING SUBJECTED TO THE EUREKA QUAKE

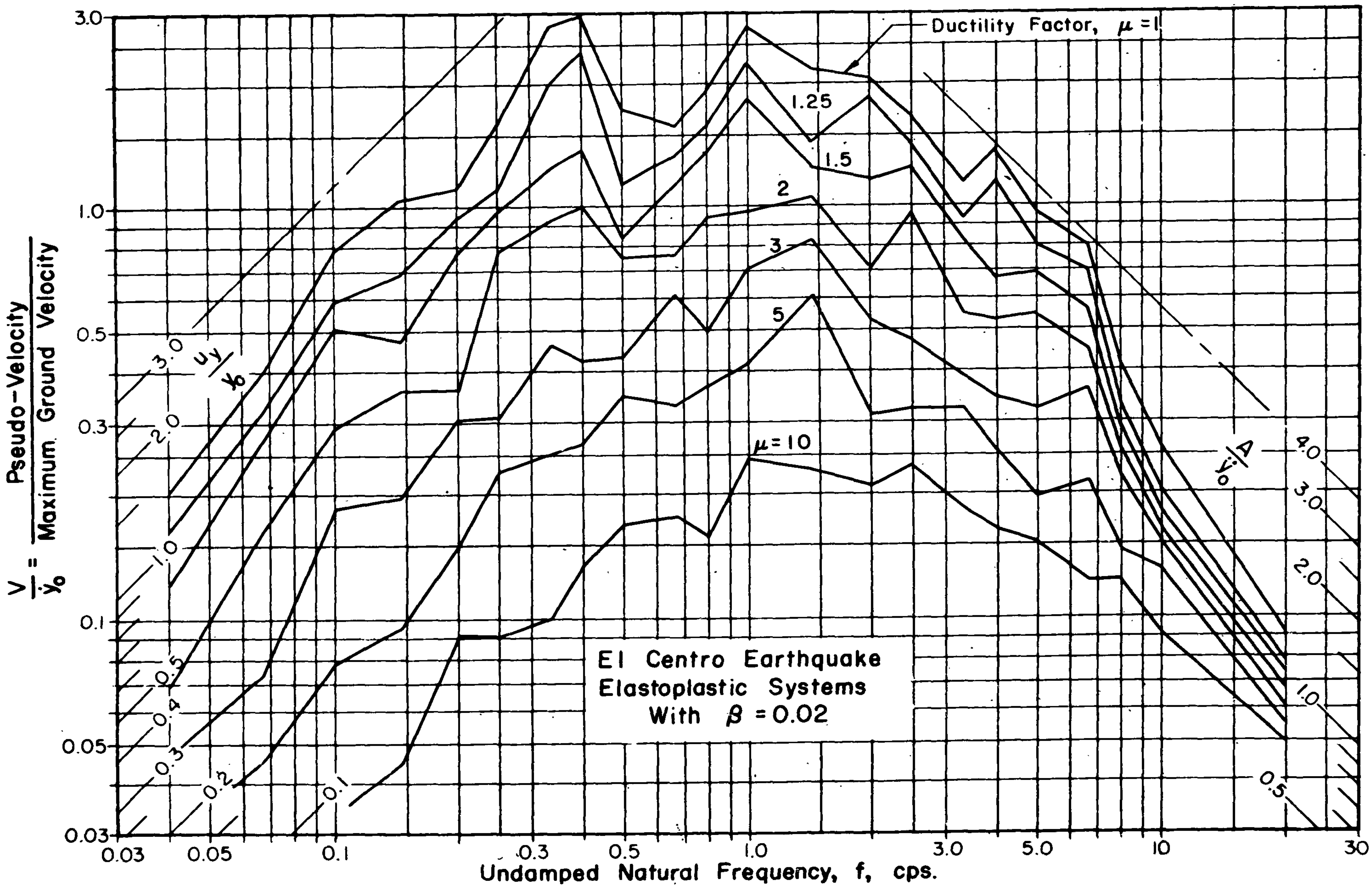


FIG. 3.19 DEFORMATION SPECTRA FOR ELASTOPLASTIC SYSTEMS WITH TWO PERCENT CRITICAL DAMPING SUBJECTED TO THE EL CENTRO QUAKE

APPENDIX A

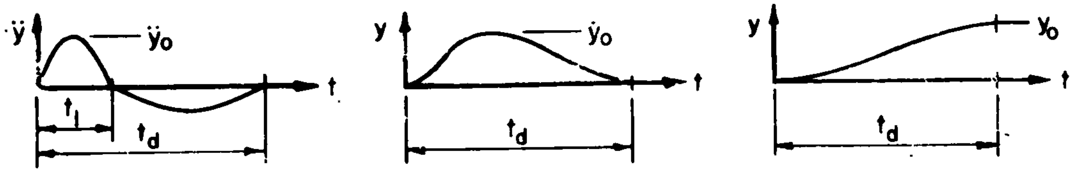
TABULATION OF NUMERICAL SOLUTIONS

In this Appendix is tabulated a portion of the numerical data used to construct the response spectra presented in the body of the report. These data were evaluated on the ILLIAC using the iterative scheme of numerical integration described in Ref. 11, with the acceleration of the mass assumed to vary linearly within each step of integration. A description of the computer program used is available in Ref. 12. The data for the remaining spectra were obtained analytically by a formal solution of the governing differential equation of motion.

For the elastic systems, the response quantities evaluated were the absolute displacement, absolute velocity and absolute acceleration of the mass, and the relative values of the displacement, velocity and acceleration between the mass and the ground. Both the maximum positive and the maximum negative values of these quantities were evaluated together with their respective times of occurrence. Some of the deformation spectra presented in this report were determined from these results by application of the analogies described in Section 2.4. For example, the deformation spectra in Figs. 2.38 through 2.40 were determined from the values of $\dot{u}$ presented in Table A.1b by noting that the values of $\dot{u}/\dot{y}_o$ in this table are also equal to the values of u/y_o for an input motion for which the shape of the displacement diagram is identical to that of the velocity diagram shown below the table heading. Similarly, the spectra in Figs. 2.43 were determined from the values of $\ddot{u}$ given in Table A.1c. For the inelastic systems, only the absolute displacement of the mass and the spring deformation were computed.

For the pulse-like excitations, the time interval of integration used in these computations was 1/50th of the undamped natural period of the system, whereas for the earthquake motions it ranged between 1/30th and 1/70th of the natural period. In addition to these time intervals, the response of the system was evaluated at the times for which the ground acceleration was an extremum. It should be noted that the tabulated data may not be accurate to the number of significant figures reported. One of the sources of inaccuracy is the fact that the response of the system is evaluated at discrete times. Since the maximum response may occur between the times at which the response is evaluated, in the absence of any other inaccuracies, the computed maxima must be numerically smaller than the actual maxima. The detailed data follow.

Elastic System Subjected to a Sequence of Two Half-Sine Ground Acceleration Pulses as Shown



$t_1 f$	β = 0 $pu_{max}/\ddot{y}_o$	β = 0 $pu_{min}/\ddot{y}_o$	β = 0.05 $pu_{max}/\ddot{y}_o$	β = 0.05 $pu_{min}/\ddot{y}_o$	β = 0.10 $pu_{max}/\ddot{y}_o$	β = 0.10 $pu_{min}/\ddot{y}_o$	β = 0.20 $pu_{max}/\ddot{y}_o$	β = 0.20 $pu_{min}/\ddot{y}_o$	β = 0.50 $pu_{max}/\ddot{y}_o$	β = 0.50 $pu_{min}/\ddot{y}_o$	β = 1.00 $pu_{max}/\ddot{y}_o$	β = 1.00 $pu_{min}/\ddot{y}_o$
(a) Ratio of $t_1/t_d = 1/2$												
0.025	0.157*	-0.157*	0.135*	-0.153	0.117*	-0.151	0.090*	-0.147	0.047*	-0.137	0.021*	-0.123
0.05	0.312*	-0.312*	0.268*	-0.293	0.232*	-0.286	0.178*	-0.273	0.093*	-0.240	0.042*	-0.201
0.075	0.464*	-0.464*	0.399*	-0.415	0.345*	-0.401	0.265*	-0.377	0.139*	-0.318	0.063*	-0.252
0.1	0.612*	-0.612*	0.526*	-0.518	0.455*	-0.498	0.350*	-0.461	0.183*	-0.376	0.083*	-0.287
0.125	0.754*	-0.754*	0.648*	-0.603	0.561*	-0.576	0.431*	-0.527	0.225*	-0.419	0.102*	-0.310
0.15	0.889*	-0.889*	0.763*	-0.672	0.661*	-0.637	0.508*	-0.578	0.265*	-0.450	0.120*	-0.325
0.2	1.132*	-1.132*	0.972*	-0.831*	0.842*	-0.719	0.647*	-0.644	0.338*	-0.486	0.154*	-0.340
0.25	1.333*	-1.333*	1.145*	-0.978*	0.992*	-0.758	0.762*	-0.673	0.399*	-0.498	0.182*	-0.342
0.3	1.485*	-1.485*	1.276*	-1.090*	1.106*	-0.806*	0.850*	-0.678	0.446*	-0.496	0.205*	-0.338
0.35	1.586*	-1.586*	1.362*	-1.164*	1.180*	-0.861*	0.908*	-0.668	0.479*	-0.486	0.223*	-0.329
0.4	1.633*	-1.633*	1.402*	-1.198*	1.216*	-0.886*	0.936*	-0.648	0.497*	-0.470	0.235*	-0.319
0.45	1.626*	-1.626*	1.397*	-1.194*	1.212*	-0.884*	0.935*	-0.623	0.502*	-0.452	0.243*	-0.308
0.5	1.571	-1.571*	1.349	-1.153*	1.172	-0.854*	0.907	-0.595	0.495	-0.433	0.247	-0.297
0.6	1.351	-1.336*	1.163	-0.982*	1.014	-0.730*	0.795	-0.536	0.457	-0.394	0.245	-0.275
0.7	1.082	-0.991*	0.937	-0.731*	0.822	-0.548*	0.658	-0.479	0.407	-0.357	0.236	-0.255
0.8	0.827	-0.610*	0.720	-0.514	0.639	-0.481	0.527	-0.427	0.358	-0.323	0.225	-0.236
0.9	0.610	-0.488	0.535	-0.456	0.482	-0.428	0.416	-0.382	0.314	-0.294	0.212	-0.219
1.0	0.433	-0.433	0.385	-0.405	0.357	-0.381	0.330	-0.342	0.278	-0.268	0.200	-0.205
1.1	0.295	-0.385	0.268	-0.361	0.262	-0.340	0.270	-0.307	0.249	-0.245	0.188	-0.191
1.2	0.240	-0.343	0.217	-0.323	0.222	-0.305	0.237	-0.277	0.225	-0.225	0.177	-0.179
1.3	0.270	-0.308	0.236	-0.290	0.224	-0.275	0.219	-0.251	0.206	-0.207	0.168	-0.169
1.4	0.269	-0.277	0.234	-0.261	0.217	-0.248	0.204	-0.228	0.189	-0.192	0.158	-0.159

* Denotes extremum occurring during free vibration.

TABLE A.1a(Continued) VALUES OF ABSOLUTE MAXIMUM AND ABSOLUTE MINIMUM PSEUDO-VELOCITIES

$t_1 f$	$\beta = 0$		$\beta = 0.05$		$\beta = 0.10$		$\beta = 0.20$		$\beta = 0.50$		$\beta = 1.00$	
	$\frac{pu_{max}}{\dot{y}_o}$	$\frac{pu_{min}}{\dot{y}_o}$	$\frac{pu_{max}}{\dot{y}_o}$	$\frac{pu_{min}}{\dot{y}_o}$	$\frac{pu_{max}}{\dot{y}_o}$	$\frac{pu_{min}}{\dot{y}_o}$	$\frac{pu_{max}}{\dot{y}_o}$	$\frac{pu_{min}}{\dot{y}_o}$	$\frac{pu_{max}}{\dot{y}_o}$	$\frac{pu_{min}}{\dot{y}_o}$	$\frac{pu_{max}}{\dot{y}_o}$	$\frac{pu_{min}}{\dot{y}_o}$
1.5	0.250	-0.250	0.218	-0.237	0.202	-0.225	0.189	-0.208	0.175	-0.178	0.150	-0.151
1.6	0.222	-0.227	0.195	-0.215	0.182	-0.205	0.174	-0.190	0.164	-0.166	0.142	-0.143
1.7	0.190	-0.206	0.169	-0.196	0.162	-0.187	0.160	-0.175	0.153	-0.155	0.135	-0.136
1.8	0.157	-0.188	0.143	-0.179	0.144	-0.172	0.148	-0.161	0.144	-0.146	0.129	-0.129
1.9	0.154	-0.173	0.138	-0.164	0.137	-0.158	0.140	-0.149	0.136	-0.137	0.123	-0.123
2.0	0.158	-0.158	0.139	-0.151	0.134	-0.146	0.132	-0.138	0.129	-0.130	0.118	-0.118
2.1	0.155	-0.146	0.136	-0.140	0.129	-0.135	0.126	-0.129	0.122	-0.123	0.113	-0.113
2.2	0.147	-0.135	0.129	-0.129	0.123	-0.125	0.120	-0.120	0.117	-0.117	0.108	-0.108
2.3	0.135	-0.125	0.120	-0.120	0.116	-0.116	0.113	-0.113	0.111	-0.111	0.104	-0.104
2.4	0.122	-0.116	0.111	-0.111	0.108	-0.108	0.107	-0.107	0.106	-0.106	0.100	-0.100
2.5	0.108	-0.108	0.099	-0.104	0.102	-0.101	0.104	-0.100	0.102	-0.102	0.096	-0.096
3.0	0.098	-0.098	0.088	-0.092	0.086	-0.088	0.086	-0.086	0.084	-0.084	0.081	-0.081
3.5	0.083	-0.083	0.075	-0.079	0.073	-0.076	0.073	-0.074	0.072	-0.072	0.070	-0.070
4.0	0.070	-0.070	0.064	-0.067	0.063	-0.065	0.063	-0.064	0.063	-0.063	0.062	-0.062
4.5	0.060	-0.060	0.056	-0.057	0.056	-0.056	0.056	-0.056	0.056	-0.056	0.055	-0.055
5.0	0.055	-0.055	0.051	-0.052	0.051	-0.051	0.050	-0.050	0.050	-0.050	0.050	-0.050
(b) Ratio of $t_1/t_d = 1/4$												
0.015	0.188*	-0.188*	---	---	---	---	0.108*	-0.171	---	---	0.026*	-0.136
0.025	0.312*	-0.312*	0.268*	-0.288	0.232*	-0.280	0.178*	-0.266	0.093*	-0.232	0.042	-0.192
0.05	0.611*	-0.611*	0.525*	-0.502	0.455*	-0.480	0.349*	-0.443	0.182*	-0.359	0.083	-0.273
0.075	0.885*	-0.885*	0.760*	-0.649*	0.658*	-0.612	0.505*	-0.555	0.263*	-0.432	0.119	-0.313
0.1	1.123*	-1.123*	0.964*	-0.824*	0.835*	-0.696	0.641*	-0.624	0.333*	-0.473	0.151	-0.334
0.125	1.317*	-1.317*	1.130*	-0.965*	0.977*	-0.748	0.749*	-0.665	0.388*	-0.497	0.176	-0.345
0.15	1.461*	-1.461*	1.251*	-1.069*	1.081*	-0.788*	0.826*	-0.689	0.427*	-0.509	0.194	-0.350
0.2	1.586*	-1.586*	1.351	-1.154*	1.162	-0.847*	0.881	-0.708	0.451	-0.517	0.211	-0.351
0.25	1.527	-1.508*	1.291	-1.085*	1.103	-0.802	0.827	-0.705	0.422	-0.512	0.208	-0.346
0.3	1.399	-1.286*	1.174	-0.903*	0.996	-0.788	0.738	-0.692	0.374	-0.501	0.198	-0.339
0.35	1.267	-1.017*	1.055	-0.823	0.887	-0.766	0.648	-0.672	0.324	-0.487	0.185	-0.330

TABLE A.1a(Continued) VALUES OF ABSOLUTE MAXIMUM AND ABSOLUTE MINIMUM PSEUDO-VELOCITIES

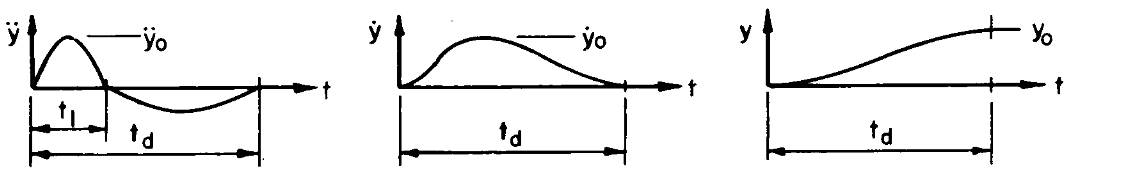
$t_1 f$	$\beta = 0$		$\beta = 0.05$		$\beta = 0.10$		$\beta = 0.20$		$\beta = 0.50$		$\beta = 1.00$	
	$\frac{pu_{max}}{\dot{y}_o}$	$\frac{pu_{min}}{\dot{y}_o}$	$\frac{pu_{max}}{\dot{y}_o}$	$\frac{pu_{min}}{\dot{y}_o}$	$\frac{pu_{max}}{\dot{y}_o}$	$\frac{pu_{min}}{\dot{y}_o}$	$\frac{pu_{max}}{\dot{y}_o}$	$\frac{pu_{min}}{\dot{y}_o}$	$\frac{pu_{max}}{\dot{y}_o}$	$\frac{pu_{min}}{\dot{y}_o}$	$\frac{pu_{max}}{\dot{y}_o}$	$\frac{pu_{min}}{\dot{y}_o}$
0.4	1.147	-0.858	0.948	-0.795	0.792	-0.740	0.569	-0.649	0.278	-0.470	0.172	-0.319
0.45	1.044	-0.823	0.857	-0.763	0.711	-0.710	0.504	-0.623	0.278	-0.452	0.160	-0.308
0.5	0.953	-0.785	0.778	-0.728	0.642	-0.677	0.449	-0.595	0.204	-0.433	0.148	-0.297
0.6	0.799	-0.726*	0.648	-0.653	0.529	-0.609	0.363	-0.536	0.153	-0.393	0.128	-0.275
0.7	0.704	-0.625	0.538	-0.580	0.438	-0.542	0.296	-0.479	0.118	-0.357	0.112	-0.255
0.8	0.607	-0.552	0.441	-0.514	0.358	-0.481	0.241	-0.427	0.106	-0.323	0.100	-0.236
0.9	0.503	-0.488	0.354	-0.455	0.287	-0.428	0.194	-0.382	0.095	-0.294	0.089	-0.219
1.0	0.402	-0.433	0.274	-0.405	0.223	-0.381	0.153	-0.342	0.085	-0.268	0.081	-0.205
1.1	0.307	-0.385	0.208	-0.361	0.166	-0.340	0.118	-0.307	0.077	-0.245	0.074	-0.191
1.2	0.230	-0.343	0.203	-0.323	0.188	-0.305	0.088	-0.277	0.070	-0.225	0.068	-0.179
1.3	0.163	-0.308	0.109	-0.290	0.089	-0.275	0.070	-0.251	0.065	-0.207	0.063	-0.169
1.4	0.104	-0.277	0.081	-0.261	0.072	-0.248	0.063	-0.228	0.060	-0.192	0.059	-0.159
1.5	0.060	-0.250	0.065	-0.237	0.064	-0.225	0.059	-0.208	0.056	-0.178	0.055	-0.151
1.6	0.084	-0.227	0.066	-0.215	0.060	-0.205	0.055	-0.190	0.052	-0.166	0.052	-0.143
1.7	0.102	-0.206	0.069	-0.196	0.058	-0.187	0.051	-0.175	0.049	-0.155	0.049	-0.136
1.8	0.110	-0.188	0.070	-0.179	0.055	-0.172	0.048	-0.161	0.046	-0.146	0.046	-0.129
1.9	0.110	-0.173	0.067	-0.164	0.052	-0.158	0.044	-0.149	0.044	-0.137	0.044	-0.123
2.0	0.105	-0.158	0.0623	-0.151	0.0483	-0.146	0.0421	-0.138	0.0418	-0.130	0.0414	-0.118
2.5	0.0357	-0.108	0.0360	-0.104	0.0346	-0.101	0.0334	-0.100	0.0334	-0.102	0.0332	-0.096
3.0	0.0548	-0.097	0.0326	-0.092	0.0288	-0.088	0.0279	-0.086	0.0278	-0.084	0.0277	-0.081
3.5	0.0248	-0.083	0.0250	-0.079	0.0242	-0.076	0.0239	-0.074	0.0238	-0.072	0.0238	-0.070
4.0	0.0358	-0.070	0.0223	-0.067	0.0210	-0.065	0.0209	-0.064	0.0208	-0.063	0.0208	-0.062
4.5	0.0143	-0.059	0.0190	-0.057	0.0186	-0.056	0.0185	-0.056	0.0185	-0.056	0.0184	-0.055
5.0	0.0262	-0.055	0.0172	-0.052	0.0167	-0.051	0.0167	-0.050	0.0167	-0.050	0.0166	-0.049
(c) Ratio of t_1/t_d = 1/8												
0.01	0.251*	-0.251*	---	---	---	---	0.143*	-0.219	---	---	0.034*	-0.163
0.015	0.378*	-0.378*	---	---	---	---	0.218*	-0.308	---	---	0.056*	-0.213
0.025	0.610*	-0.610*	0.523*	-0.493	0.453*	0.471	0.348*	-0.434	0.182*	-0.350	0.082*	-0.266

TABLE A.1a (Continued) VALUES OF ABSOLUTE MAXIMUM AND ABSOLUTE MINIMUM PSEUDO-VELOCITIES

$t_1 f$	β = 0		β = 0.05		β = 0.10		β = 0.20		β = 0.50		β = 1.00	
	$\frac{pu_{max}}{\bar{y}_o}$	$\frac{pu_{min}}{\bar{y}_o}$	$\frac{pu_{max}}{\bar{y}_o}$	$\frac{pu_{min}}{\bar{y}_o}$	$\frac{pu_{max}}{\bar{y}_o}$	$\frac{pu_{min}}{\bar{y}_o}$	$\frac{pu_{max}}{\bar{y}_o}$	$\frac{pu_{min}}{\bar{y}_o}$	$\frac{pu_{max}}{\bar{y}_o}$	$\frac{pu_{min}}{\bar{y}_o}$	$\frac{pu_{max}}{\bar{y}_o}$	$\frac{pu_{min}}{\bar{y}_o}$
0.05	1.113*	-1.118*	0.955*	-0.816*	0.826*	-0.679	0.634*	-0.609	0.329*	-0.462	0.148*	-0.327
0.075	1.430*	-1.430*	1.223*	-1.045*	1.055*	-0.769*	0.804*	-0.680	0.413*	-0.504	0.187*	-0.348
0.1	1.529*	-1.529*	1.298	-1.108*	1.112	-0.810*	0.837	-0.711	0.422	-0.521	0.197	-0.356
0.125	1.477	-1.433*	1.243	-1.021*	1.056	-0.822	0.783	-0.724	0.387	-0.527	0.191	-0.358
0.15	1.387	-1.255*	1.157	-0.888	0.974	-0.828	0.711	-0.728	0.341	-0.529	0.179	-0.358
0.2	1.227	-0.955*	1.008	-0.885	0.836	-0.825	0.589	-0.726	0.259	-0.524	0.153	-0.354
0.25	1.115	-1.005*	0.907	-0.871	0.743	-0.811	0.509	-0.711	0.200	-0.514	0.130	-0.347
0.3	1.035	-0.937*	0.836	-0.850	0.679	-0.791	0.456	-0.694	0.162	-0.502	0.112	-0.339
0.35	0.996	-0.889	0.781	-0.824	0.631	-0.767	0.418	-0.673	0.136	-0.487	0.098	-0.330
0.4	0.954	-0.880	0.734	-0.795	0.591	-0.740	0.388	-0.649	0.118	-0.470	0.086	-0.319
0.45	0.906	-0.823	0.692	-0.763	0.555	-0.710	0.362	-0.623	0.105	-0.452	0.077	-0.308
0.5	0.856	-0.785	0.651	-0.728	0.522	-0.677	0.338	-0.595	0.095	-0.433	0.070	-0.297
0.6	0.758	-0.704	0.573	-0.653	0.458	-0.609	0.295	-0.536	0.079	-0.394	0.059	-0.275
0.7	0.661	-0.625	0.494	-0.580	0.395	-0.542	0.255	-0.479	0.067	-0.357	0.050	-0.255
0.8	0.559	-0.552	0.416	-0.514	0.333	-0.481	0.215	-0.427	0.056	-0.323	0.044	-0.236
0.9	0.461	-0.488	0.340	-0.456	0.272	-0.428	0.177	-0.382	0.048	-0.294	0.039	-0.219
1.0	0.366	-0.433	0.267	-0.405	0.215	-0.381	0.142	-0.342	0.040	-0.268	0.036	-0.205
1.1	0.278	-0.385	0.199	-0.361	0.161	-0.340	0.110	-0.307	0.034	-0.245	0.032	-0.191
1.2	0.199	-0.343	0.137	-0.323	0.114	-0.305	0.083	-0.277	0.0298	-0.225	0.0296	-0.179
1.3	0.129	-0.308	0.085	-0.290	0.075	-0.275	0.062	-0.251	0.0275	-0.207	0.0274	-0.169
1.4	0.071	-0.277	0.044	-0.261	0.047	-0.248	0.046	-0.228	0.0255	-0.192	0.0254	-0.159
1.5	0.025	-0.250	0.0287	-0.237	0.0329	-0.225	0.0374	-0.208	0.0238	-0.178	0.0238	-0.151
1.6	0.055	-0.227	0.0309	-0.215	0.0329	-0.205	0.0330	-0.190	0.0223	-0.166	0.0223	-0.143
1.7	0.076	-0.206	0.0427	-0.196	0.0368	-0.187	0.0308	-0.175	0.0210	-0.155	0.0210	-0.136
1.8	0.087	-0.188	0.049	-0.179	0.0392	-0.172	0.0292	-0.161	0.0199	-0.146	0.0198	-0.129
2.0	0.084	-0.158	0.048	-0.151	0.0362	-0.146	0.0252	-0.138	0.0179	-0.130	0.0178	-0.118

TABLE A.1b MAXIMUM AND MINIMUM VALUES OF RELATIVE VELOCITY, $\dot{u}$

Elastic Systems Subjected to a Sequence of Two Half-Sine Ground Acceleration Pulses as Shown



$t_1 f$	β = 0 $\frac{\dot{u}_{max}}{\dot{y}_o}$	β = 0 $\frac{\dot{u}_{min}}{\dot{y}_o}$	β = 0.05 $\frac{\dot{u}_{max}}{\dot{y}_o}$	β = 0.05 $\frac{\dot{u}_{min}}{\dot{y}_o}$	β = 0.10 $\frac{\dot{u}_{max}}{\dot{y}_o}$	β = 0.10 $\frac{\dot{u}_{min}}{\dot{y}_o}$	β = 0.20 $\frac{\dot{u}_{max}}{\dot{y}_o}$	β = 0.20 $\frac{\dot{u}_{min}}{\dot{y}_o}$	β = 0.50 $\frac{\dot{u}_{max}}{\dot{y}_o}$	β = 0.50 $\frac{\dot{u}_{min}}{\dot{y}_o}$	β = 1.00 $\frac{\dot{u}_{max}}{\dot{y}_o}$	β = 1.00 $\frac{\dot{u}_{min}}{\dot{y}_o}$
(a) Ratio of t_1/t_d = 1/2												
0.025	0.157*	-0.996	0.146*	-0.988	0.138*	-0.981	0.129*	-0.966	0.155	-0.924	0.249	-0.861
0.05	0.312*	-0.985	0.291*	-0.970	0.275*	-0.956	0.256*	-0.928	0.299	-0.853	0.396	-0.752
0.075	0.464*	-0.968	0.432*	-0.947	0.409*	-0.926	0.381*	-0.887	0.424	-0.788	0.478	-0.663
0.1	0.612*	-0.945	0.570*	-0.918	0.539*	-0.893	0.502*	-0.845	0.526	-0.729	0.519	-0.591
0.125	0.754*	-0.918	0.702*	-0.887	0.664*	-0.857	0.619*	-0.803	0.603	-0.675	0.533	-0.530
0.15	0.889*	-0.889*	0.828*	-0.852	0.782*	-0.820	0.730*	-0.761	0.625	-0.625	0.531	-0.478
0.2	1.132*	-1.132*	1.054*	-0.901*	0.996*	-0.745	0.911	-0.681	0.711	-0.539	0.501	-0.396
0.25	1.333*	-1.333*	1.235	-1.061*	1.148	-0.856*	1.003	-0.607	0.710	-0.468	0.457	-0.334
0.3	1.485*	-1.485*	1.317	-1.182*	1.204	-0.954*	1.020	-0.643*	0.676	-0.409	0.411	-0.286
0.35	1.585*	-1.585*	1.315	-1.262*	1.188	-1.018*	0.985	-0.687*	0.625	-0.359	0.367	-0.248
0.4	1.632*	-1.632*	1.256	-1.299*	1.125	-1.048*	0.921	-0.708*	0.568	-0.318	0.327	-0.217
0.45	1.625*	-1.625*	1.165	-1.294*	1.037	-1.045*	0.841	-0.707*	0.511	-0.282	0.292	-0.191
0.5	1.570*	-1.570*	1.068*	-1.250*	0.939	-1.010*	0.757	-0.686*	0.456	-0.271	0.261	-0.170
0.6	1.335*	-1.335*	0.910*	-1.064*	0.743	-0.863*	0.595	-0.594*	0.359	-0.248*	0.210	-0.137
0.7	0.991*	-0.991*	0.677*	-0.792*	0.573	-0.648*	0.459	-0.460*	0.282	-0.214*	0.171	-0.113
0.8	0.610*	-0.624*	0.497	-0.498	0.437	-0.417	0.351	-0.321*	0.222	-0.178*	0.142	-0.095
0.9	0.436	-0.358	0.378	-0.276	0.332	-0.234	0.267	-0.209*	0.177	-0.146*	0.118	-0.081

* Denotes extremum occurring during free vibration.

TABLE A.1b MAXIMUM AND MINIMUM VALUES OF RELATIVE VELOCITY, $\dot{u}$ (CONTINUED)

$t_1 f$	$\beta = 0$		$\beta = 0.05$		$\beta = 0.10$		$\beta = 0.20$		$\beta = 0.50$		$\beta = 1.00$	
	$\frac{\dot{u}_{max}}{\dot{y}_o}$	$\frac{\dot{u}_{min}}{\dot{y}_o}$	$\frac{\dot{u}_{max}}{\dot{y}_o}$	$\frac{\dot{u}_{min}}{\dot{y}_o}$	$\frac{\dot{u}_{max}}{\dot{y}_o}$	$\frac{\dot{u}_{min}}{\dot{y}_o}$	$\frac{\dot{u}_{max}}{\dot{y}_o}$	$\frac{\dot{u}_{min}}{\dot{y}_o}$	$\frac{\dot{u}_{max}}{\dot{y}_o}$	$\frac{\dot{u}_{min}}{\dot{y}_o}$	$\frac{\dot{u}_{max}}{\dot{y}_o}$	$\frac{\dot{u}_{min}}{\dot{y}_o}$
1.0	0.333	-0.188	0.287	-0.173	0.252	-0.161	0.204	-0.145	0.142	-0.121	0.100	-0.069
1.1	0.256	-0.161	0.220	-0.149	0.192	-0.139	0.156	-0.121	0.115	-0.101*	0.0857	-0.0607*
1.2	0.199*	-0.199*	0.168	-0.145*	0.146	-0.121*	0.119	-0.105	0.0951	-0.0853*	0.0740	-0.0550*
1.3	0.166*	-0.171	0.130	-0.124	0.112	-0.106	0.0913	-0.0924	0.0796	-0.0730*	0.0644	-0.0497*
1.4	0.121	-0.132	0.101	-0.100	0.0861	-0.0931	0.0700	-0.0815	0.0676	-0.0631*	0.0566	-0.0451*
1.5	0.0961	-0.0965	0.0788	-0.0890	0.0663	-0.0827	0.0543	-0.0724	0.0583	-0.0551	0.0500	-0.0410*
1.6	0.0935	-0.0858	0.0700	-0.0794	0.0583	-0.0738	0.0511	-0.0648	0.0509	-0.0486	0.0445	-0.0372
1.7	0.0897*	-0.0897*	0.0669	-0.0713	0.0558	-0.0663	0.0476	-0.0582	0.0449	-0.0431	0.0398	-0.0339
1.8	0.0812	-0.0830	0.0621	-0.0643	0.0519	-0.0600	0.0436	-0.0525	0.0400	-0.0387	0.0358	-0.0310
1.9	0.0738	-0.0702	0.0566	-0.0583	0.0474	-0.0543	0.0396	-0.0476	0.0358	-0.0352	0.0324	-0.0285
2.0	0.0666	-0.0575	0.0511	-0.0530	0.0427	-0.0494	0.0357	-0.0434	0.0322	-0.0321	0.0294	-0.0261
2.1	0.0600	-0.0524	0.0457	-0.0485	0.0381	-0.0452	0.0319	-0.0397	0.0292	-0.0294	0.0268	-0.0241
2.2	0.0534	-0.0516*	0.0406	-0.0445	0.0338	-0.0414	0.0284	-0.0364	0.0265	-0.0270	0.0246	-0.0223
2.3	0.0476	-0.0494	0.0360	-0.0410	0.0298	-0.0381	0.0252	-0.0335	0.0242	-0.0249	0.0226	-0.0206
2.4	0.0425	-0.0438	0.0318	-0.0378	0.0262	-0.0352	0.0224	-0.0310	0.0222	-0.0230	0.0208	-0.0191
2.5	0.0378	-0.0378	0.0280	-0.0350	0.0229	-0.0326	0.0199	-0.0287	0.0204	-0.0213	0.0192	-0.0178
3.0	0.0286	-0.0267	0.0198	-0.0247	0.0164	-0.0230	0.0145	-0.0203	0.0141	-0.0152	0.0135	-0.0128
3.5	0.0198	-0.0198	0.0134	-0.0184	0.0110	-0.0171	0.0103	-0.0151	0.0103	-0.0113	0.0100	-0.0096
4.0	0.0159	-0.0153	0.0102	-0.0141	0.0086	-0.0132	0.0080	-0.0116	0.0079	-0.0088	0.0077	-0.0075
4.5	0.0121	-0.0121	0.0076	-0.0112	0.0064	-0.0105	0.0062	-0.0092	0.0062	-0.0070	0.0061	-0.0059
5.0	0.0101	-0.0099	0.0061	-0.0091	0.0053	-0.0085	0.0051	-0.0075	0.0050	-0.0057	0.0050	-0.0049
(b) Ratio of t_1/t_d = 1/4												
0.015	0.188*	-0.998	--	--	--	--	0.154*	-0.980	--	--	0.269	-0.912
0.025	0.312*	-0.996	0.291*	-0.988	0.275*	-0.981	0.256*	-0.966	0.293	-0.924	0.362	-0.861

TABLE A.1b MAXIMUM AND MINIMUM VALUES OF RELATIVE VELOCITY, $\hat{u}$ (CONTINUED)

$t_1 f$	$\beta = 0$		$\beta = 0.05$		$\beta = 0.10$		$\beta = 0.20$		$\beta = 0.50$		$\beta = 1.00$	
	$\frac{\hat{u}_{max}}{\dot{y}_o}$	$\frac{\hat{u}_{min}}{\dot{y}_o}$	$\frac{\hat{u}_{max}}{\dot{y}_o}$	$\frac{\hat{u}_{min}}{\dot{y}_o}$	$\frac{\hat{u}_{max}}{\dot{y}_o}$	$\frac{\hat{u}_{min}}{\dot{y}_o}$	$\frac{\hat{u}_{max}}{\dot{y}_o}$	$\frac{\hat{u}_{min}}{\dot{y}_o}$	$\frac{\hat{u}_{max}}{\dot{y}_o}$	$\frac{\hat{u}_{min}}{\dot{y}_o}$	$\frac{\hat{u}_{max}}{\dot{y}_o}$	$\frac{\hat{u}_{min}}{\dot{y}_o}$
0.05	0.611*	-0.985	0.569*	-0.970	0.538*	-0.956	0.501*	-0.928	0.494	-0.853	0.445	-0.752
0.075	0.885*	-0.968	0.824*	-0.947	0.779*	-0.926	0.721	-0.887	0.591	-0.788	0.437	-0.663
0.1	1.123*	-1.125*	1.043	-0.918	0.973	-0.898	0.856	-0.845	0.616	-0.729	0.402	-0.591
0.125	1.317*	-1.317*	1.170	-1.047*	1.069	-0.857	0.906	-0.803	0.601	-0.675	0.362	-0.530
0.15	1.461*	-1.461*	1.211	-1.159*	1.092	-0.932*	0.903	-0.761	0.568	-0.625	0.324	-0.478
0.2	1.586*	-1.586*	1.164	-1.252*	1.034	-1.002*	0.832	-0.681	0.490	-0.539	0.261	-0.396
0.25	1.508*	-1.508*	1.070	-1.177*	0.942	-0.932*	0.746	-0.609*	0.422	-0.468	0.216	-0.334
0.3	1.286*	-1.287	0.979	-0.981	0.858	-0.758	0.673	-0.541	0.371	-0.409	0.184	-0.286
0.35	1.042	-1.086	0.901	-0.809	0.787	-0.608	0.613	-0.482	0.333	-0.359	0.162	-0.248
0.4	0.967	-0.953	0.835	-0.698	0.727	-0.515	0.565	-0.431	0.304	-0.318	0.145	-0.217
0.45	0.901	-0.861	0.777	-0.625	0.676	-0.455	0.523	-0.386	0.280	-0.282	0.133	-0.191
0.5	0.841	-0.789	0.724	-0.570	0.629	-0.413	0.486	-0.347	0.259	-0.252	0.123	-0.170
0.6	0.729	-0.731	0.627	-0.487	0.545	-0.351	0.421	-0.283	0.225	-0.204	0.108	-0.137
0.7	0.624	-0.618	0.536	-0.416	0.466	-0.300	0.360	-0.233	0.195	-0.168	0.096	-0.113
0.8	0.523	-0.511	0.450	-0.349	0.391	-0.252	0.304	-0.195	0.169	-0.140	0.086	-0.095
0.9	0.425	-0.425	0.366	-0.283	0.319	-0.205	0.250	-0.165	0.144	-0.119	0.077	-0.081
1.0	0.333	-0.331	0.287	-0.221	0.252	-0.161	0.200	-0.141	0.123	-0.102	0.0689	-0.069
1.1	0.256	-0.245	0.220	-0.163	0.192	-0.139	0.156	-0.121	0.104	-0.0877	0.0619	-0.0604
1.2	0.198	-0.172	0.168	-0.130	0.146	-0.121	0.119	-0.105	0.0877	-0.0765	0.0557	-0.0530
1.3	0.154	-0.123	0.130	-0.114	0.112	-0.106	0.0913	-0.0924	0.0744	-0.0672	0.0502	-0.0469
1.4	0.121	-0.108	0.101	-0.100	0.0861	-0.0931	0.0700	-0.0815	0.0639	-0.0595	0.0454	-0.0418
1.5	0.0961	-0.0962	0.0788	-0.0889	0.0663	-0.0827	0.0537	-0.0725	0.0554	-0.0530	0.0411	-0.0375
1.6	0.0770	-0.0858	0.0618	-0.0794	0.0511	-0.0738	0.0461	-0.0648	0.0484	-0.0475	0.0373	-0.0338
1.7	0.0622	-0.0770	0.0489	-0.0713	0.0427	-0.0663	0.0431	-0.0582	0.0429	-0.0427	0.0340	-0.0306
1.8	0.0683	-0.0695	0.0529	-0.0643	0.0457	-0.0599	0.0409	-0.0525	0.0383	-0.0387	0.0311	-0.0279
1.9	0.0708	-0.0707	0.0543	-0.0583	0.0456	-0.0543	0.0385	-0.0476	0.0345	-0.0352	0.0284	-0.0255

TABLE A.1b MAXIMUM AND MINIMUM VALUES OF RELATIVE VELOCITY, $\dot{u}$ (CONTINUED)

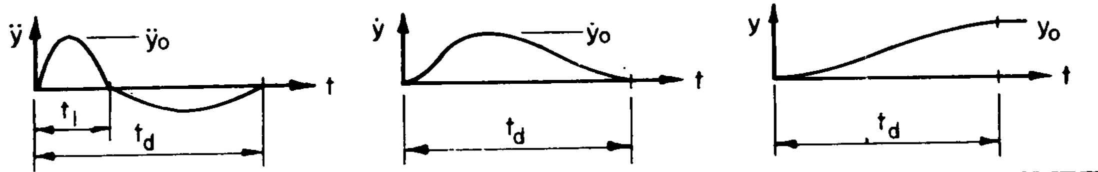
$t_1 f$	β = 0		β = 0.05		β = 0.10		β = 0.20		β = 0.50		β = 1.00	
	$\dot{u}_{max}/\dot{y}_o$	$\dot{u}_{min}/\dot{y}_o$	$\dot{u}_{max}/\dot{y}_o$	$\dot{u}_{min}/\dot{y}_o$	$\dot{u}_{max}/\dot{y}_o$	$\dot{u}_{min}/\dot{y}_o$	$\dot{u}_{max}/\dot{y}_o$	$\dot{u}_{min}/\dot{y}_o$	$\dot{u}_{max}/\dot{y}_o$	$\dot{u}_{min}/\dot{y}_o$	$\dot{u}_{max}/\dot{y}_o$	$\dot{u}_{min}/\dot{y}_o$
2.0	0.0667	-0.0665	0.0511	-0.0530	0.0426	-0.0494	0.0354	-0.0434	0.0312	-0.0321	0.0261	-0.0234
2.1	0.0598	-0.0570	0.0457	-0.0485	0.0381	-0.0452	0.0319	-0.0397	0.0283	-0.0294	0.0241	-0.0215
2.2	0.0534	-0.0481	0.0406	-0.0445	0.0338	-0.0414	0.0284	-0.0364	0.0258	-0.0270	0.0223	-0.0199
2.3	0.0476	-0.0443	0.0360	-0.0410	0.0299	-0.0381	0.0252	-0.0355	0.0236	-0.0249	0.0206	-0.0184
2.4	0.0425	-0.0409	0.0318	-0.0378	0.0262	-0.0352	0.0224	-0.0310	0.0217	-0.0230	0.0191	-0.0171
2.5	0.0378	-0.0378	0.0280	-0.0350	0.0229	-0.0326	0.0199	-0.0287	0.0200	-0.0213	0.0178	-0.0159
3.0	0.0286	-0.0285	0.0198	-0.0247	0.0164	-0.0230	0.0145	-0.0203	0.0139	-0.0152	0.0128	-0.0116
3.5	0.0198	-0.0198	0.0135	-0.0184	0.0110	-0.0171	0.0103	-0.0151	0.0102	-0.0113	0.0096	-0.0088
4.0	0.0159	-0.0159	0.0102	-0.0142	0.0086	-0.0132	0.0080	-0.0116	0.0078	-0.0088	0.0075	-0.0069
4.5	0.0121	-0.0121	0.0077	-0.0112	0.0064	-0.0105	0.0062	-0.0092	0.0062	-0.0070	0.0059	-0.0055
5.0	0.0101	-0.0101	0.0061	-0.0091	0.0053	-0.0085	0.0050	-0.0078	0.0050	-0.0057	0.0048	-0.0045
					(c) Ratio of t_1/t_d = 1/8							
0.01	0.250*	-0.999	--	--	--	--	0.205*	-0.987	--	--	0.311	-0.940
0.015	0.373*	-0.998	--	--	--	--	0.306*	-0.980	--	--	0.356	-0.912
0.025	0.610*	-0.996	0.568*	-0.988	0.536*	-0.981	0.500*	-0.966	0.478	-0.924	0.414	-0.861
0.05	1.113*	-1.113*	1.023	-0.970	0.947	-0.956	0.821	-0.928	0.574	-0.853	0.366	-0.752
0.075	1.430*	-1.430*	1.152	-1.133*	1.035	-0.926	0.851	-0.887	0.527	-0.788	0.296	-0.663
0.1	1.529*	-1.529*	1.118	-1.202*	0.991	-0.958	0.795	-0.845	0.463	-0.729	0.243	-0.591
0.125	1.433*	-1.433*	1.056	-1.107*	0.929	-0.867*	0.735	-0.803	0.414	-0.675	0.208	-0.530
0.15	1.225*	-1.250	1.001	-0.941	0.877	-0.820	0.688	-0.761	0.379	-0.625	0.185	-0.478
0.2	1.068	-1.056	0.922	-0.780	0.804	-0.745	0.624	-0.681	0.335	-0.539	0.158	-0.396
0.25	1.009	-1.005*	0.860	-0.710	0.756	-0.671	0.584	-0.607	0.310	-0.468	0.144	-0.334
0.3	0.962	-0.964	0.828	-0.675	0.719	-0.604	0.554	-0.541	0.293	-0.409	0.135	-0.286
0.35	0.920	-0.915	0.791	-0.647	0.686	-0.542	0.529	-0.482	0.278	-0.359	0.128	-0.248
0.4	0.880	-0.880	0.755	-0.621	0.655	-0.487	0.504	-0.431	0.265	-0.318	0.122	-0.217
0.45	0.837	-0.837	0.720	-0.595	0.624	-0.437	0.480	-0.386	0.253	-0.282	0.117	-0.191

TABLE A.1b MAXIMUM AND MINIMUM VALUES OF RELATIVE VELOCITY, $\dot{u}$ (CONTINUED)

$t_1 f$	β = 0 $\frac{\dot{u}_{max}}{\dot{y}_o}$	β = 0 $\frac{\dot{u}_{min}}{\dot{y}_o}$	β = 0.05 $\frac{\dot{u}_{max}}{\dot{y}_o}$	β = 0.05 $\frac{\dot{u}_{min}}{\dot{y}_o}$	β = 0.10 $\frac{\dot{u}_{max}}{\dot{y}_o}$	β = 0.10 $\frac{\dot{u}_{min}}{\dot{y}_o}$	β = 0.20 $\frac{\dot{u}_{max}}{\dot{y}_o}$	β = 0.20 $\frac{\dot{u}_{min}}{\dot{y}_o}$	β = 0.50 $\frac{\dot{u}_{max}}{\dot{y}_o}$	β = 0.50 $\frac{\dot{u}_{min}}{\dot{y}_o}$	β = 1.00 $\frac{\dot{u}_{max}}{\dot{y}_o}$	β = 1.00 $\frac{\dot{u}_{min}}{\dot{y}_o}$
0.5	0.795	-0.795	0.683	-0.567	0.592	-0.416	0.456	-0.347	0.240	-0.252	0.112	-0.170
0.6	0.707	-0.707	0.607	-0.507	0.526	-0.372	0.406	-0.283	0.216	-0.204	0.102	-0.137
0.7	0.614	-0.614	0.528	-0.441	0.458	-0.325	0.354	-0.233	0.191	-0.168	0.093	-0.113
0.8	0.519	-0.519	0.446	-0.374	0.387	-0.276	0.301	-0.195	0.166	-0.140	0.0837	-0.0948
0.9	0.424	-0.424	0.365	-0.306	0.318	-0.227	0.249	-0.165	0.143	-0.119	0.0786	-0.0806
1.0	0.333	-0.333	0.287	-0.241	0.251	-0.179	0.200	-0.141	0.122	-0.102	0.0681	-0.0694
1.1	0.256	-0.246	0.220	-0.179	0.192	-0.139	0.156	-0.121	0.103	-0.0877	0.0614	-0.0604
1.2	0.198	-0.171	0.168	-0.130	0.146	-0.121	0.119	-0.105	0.0875	-0.0764	0.0553	-0.0530
1.3	0.154	-0.123	0.130	-0.114	0.112	-0.106	0.0913	-0.0924	0.0744	-0.0672	0.0500	-0.0469
1.4	0.121	-0.108	0.101	-0.100	0.0861	-0.0931	0.0700	-0.0815	0.0639	-0.0595	0.0452	-0.0418
1.5	0.0961	-0.0962	0.0788	-0.0890	0.0663	-0.0827	0.0537	-0.0725	0.0553	-0.0530	0.0409	-0.0375
1.6	0.0770	-0.0858	0.0618	-0.0794	0.0511	-0.0738	0.0460	-0.0648	0.0484	-0.0475	0.0373	-0.0338
1.7	0.0622	-0.0770	0.0488	-0.0713	0.0424	-0.0663	0.0430	-0.0582	0.0429	-0.0427	0.0340	-0.0306
1.8	0.0676	-0.0695	0.0525	-0.0643	0.0455	-0.0599	0.0408	-0.0525	0.0383	-0.0387	0.0310	-0.0279
1.9	0.0706	-0.0706	0.0543	-0.0583	0.0455	-0.0543	0.0385	-0.0476	0.0344	-0.0352	0.0284	-0.0255
2.0	0.0667	-0.0666	0.0511	-0.0530	0.0426	-0.0494	0.0354	-0.0434	0.0311	-0.0321	0.0261	-0.0234
2.1	0.0599	-0.0572	0.0456	-0.0485	0.0381	-0.0452	0.0319	-0.0397	0.0283	-0.0294	0.0241	-0.0215
2.2	0.0534	-0.0481	0.0406	-0.0445	0.0338	-0.0414	0.0284	-0.0364	--	--	--	--

TABLE A.1c MAXIMUM AND MINIMUM VALUES OF RELATIVE ACCELERATION, $\ddot{u}$

Elastic Systems Subjected to a Sequence of Two Half-Sine Ground Acceleration Pulses as Shown



$t_1 f$	β = 0 $\ddot{u}_{max}/\ddot{y}_o$	β = 0 $\ddot{u}_{min}/\ddot{y}_o$	β = 0.05 $\ddot{u}_{max}/\ddot{y}_o$	β = 0.05 $\ddot{u}_{min}/\ddot{y}_o$	β = 0.10 $\ddot{u}_{max}/\ddot{y}_o$	β = 0.10 $\ddot{u}_{min}/\ddot{y}_o$	β = 0.20 $\ddot{u}_{max}/\ddot{y}_o$	β = 0.20 $\ddot{u}_{min}/\ddot{y}_o$	β = 0.50 $\ddot{u}_{max}/\ddot{y}_o$	β = 0.50 $\ddot{u}_{min}/\ddot{y}_o$	β = 1.00 $\ddot{u}_{max}/\ddot{y}_o$	β = 1.00 $\ddot{u}_{min}/\ddot{y}_o$
(a) Ratio of $t_1/t_d = 1/2$												
0.025	1.014	-0.999	1.018	-0.994	1.023	-0.989	1.031	-0.979	1.050	-0.951	1.067	-0.908
0.050	1.056	-0.994	1.063	-0.985	1.070	-0.975	1.081	-0.956	1.096	-0.905	1.087	-0.831
0.075	1.123	-0.987	1.130	-0.973	1.135	-0.959	1.142	-0.933	1.135	-0.862	1.076	-0.765
0.1	1.210	-0.978	1.213	-0.959	1.214	-0.942	1.209	-0.908	1.164	-0.821	1.048	-0.708
0.125	1.313	-0.966	1.307	-0.944	1.298	-0.922	1.275	-0.883	1.181	-0.783	1.009	-0.659
0.150	1.424	-0.953	1.405	-0.927	1.384	-0.903	1.337	-0.857	1.188	-0.746	0.966	-0.615
0.20	1.644	-0.921	1.589	-0.889	1.535	-0.860	1.433	-0.807	1.173	-0.682	0.873	-0.617
0.25	1.824	-1.333*	1.727	-1.150*	1.637	-1.012*	1.480	-0.828*	1.129	-0.717	0.783	-0.667
0.3	1.937	-1.783*	1.804	-1.539*	1.685	-1.354*	1.483	-1.108*	1.067	-0.899	0.700	-0.672
0.35	2.220*	-2.220*	1.822	-1.916*	1.681	-1.686*	1.448	-1.381*	0.995	-1.015	0.626	-0.648
0.4	2.611*	-2.611*	1.926*	-2.254*	1.637	-1.984*	1.387	-1.627*	0.919	-1.056	0.561	-0.607
0.45	2.926*	-2.926*	2.159*	-2.527*	1.623*	-2.225*	1.309	-1.789	0.844	-1.030	0.504	-0.557
0.5	3.141*	-3.141*	2.318*	-2.693	1.743*	-2.325	1.224	-1.776	0.772	-0.953	0.454	-0.505
0.6	3.205*	-3.205*	2.368*	-2.320	1.788*	-1.950	1.077*	-1.442	0.642	-0.760	0.372	-0.412
0.7	2.774*	-2.774*	2.057*	-1.793	1.567*	-1.490	0.975*	-1.087	0.533	-0.591	0.309	-0.339
0.8	1.953*	-1.953*	1.464*	-1.323	1.150*	-1.083	0.777*	-0.782	0.443	-0.456	0.259	-0.281
0.9	1.053	-1.227	0.915	-0.955	0.802	-0.764	0.632	-0.542	0.370	-0.353	0.220	-0.236
1.0	0.912	-0.913	0.787	-0.684	0.686	-0.528	0.537	-0.362	0.311	-0.279	0.188	-0.200
1.1	0.796	-0.691	0.683	-0.495	0.593	-0.363	0.460	-0.315	0.263	-0.232	0.163	-0.171

* Denotes extremum occurring during free vibration.

TABLE A.1c MAXIMUM AND MINIMUM VALUES OF RELATIVE ACCELERATION, $\ddot{u}$ (CONTINUED)

$t_1 f$	β = 0		β = 0.05		β = 0.10		β = 0.20		β = 0.50		β = 1.00	
	$\ddot{u}_{max}/\ddot{y}_o$	$\ddot{u}_{min}/\ddot{y}_o$	$\ddot{u}_{max}/\ddot{y}_o$	$\ddot{u}_{min}/\ddot{y}_o$	$\ddot{u}_{max}/\ddot{y}_o$	$\ddot{u}_{min}/\ddot{y}_o$	$\ddot{u}_{max}/\ddot{y}_o$	$\ddot{u}_{min}/\ddot{y}_o$	$\ddot{u}_{max}/\ddot{y}_o$	$\ddot{u}_{min}/\ddot{y}_o$	$\ddot{u}_{max}/\ddot{y}_o$	$\ddot{u}_{min}/\ddot{y}_o$
1.2	0.957*	-0.957*	0.648*	-0.554*	0.517	-0.366*	0.397	-0.293	0.224	-0.215	0.142	-0.148
1.3	0.862*	-0.862*	0.585*	-0.500*	0.458*	-0.334*	0.346	-0.273	0.208*	-0.200	0.124	-0.137
1.4	0.555	-0.548	0.470	-0.343	0.402	-0.370	0.305*	-0.256	0.194*	-0.187	0.110	-0.127
1.5	0.500	-0.500	0.422	-0.310	0.360	-0.273	0.273*	-0.240	0.182*	-0.176	0.0977	-0.120
1.6	0.454	-0.450	0.381	-0.276	0.323	-0.257	0.253*	-0.226	0.170*	-0.165	0.0879*	-0.112
1.7	0.611*	-0.611*	0.380*	-0.326*	0.299*	-0.243	0.241*	-0.214	0.160*	-0.156	0.0851*	-0.106
1.8	0.575*	-0.575*	0.360*	-0.308*	0.284*	-0.231	0.227*	-0.203	0.152*	-0.148	0.0826*	-0.100
1.9	0.353	-0.342	0.294	-0.235	0.246	-0.219	0.212*	-0.193	0.143*	-0.140	0.0797*	-0.0954
2.0	0.328	-0.328	0.272	-0.224	0.227	-0.209	0.200*	-0.184	0.137*	-0.134	0.0773*	-0.0909
2.1	0.307	-0.312	0.253	-0.214	0.211	-0.200	0.188*	-0.175	0.130*	-0.128	0.0747*	-0.0864
2.2	0.455*	-0.455*	0.266*	-0.227*	0.215*	-0.191	0.180*	-0.168	0.124*	-0.122	0.0723*	-0.0826
2.3	0.435*	-0.435*	0.256*	-0.219*	0.207*	-0.183	0.172*	-0.161	0.119*	-0.117	0.0700*	-0.0792
2.4	0.261*	-0.261*	0.210	-0.189	0.187*	-0.176	0.164*	-0.154	0.114*	-0.112	0.0678*	-0.0760
2.5	0.242	-0.242	0.198	-0.182	0.172*	-0.169	0.157*	-0.149	0.109*	-0.108	0.0657*	-0.0729
3.0	0.199	-0.199	0.155	-0.152	0.144*	-0.142	0.129*	-0.124	0.0910*	-0.0903	0.0567*	-0.0610
3.5	0.167	-0.167	0.128	-0.131	0.124*	-0.122	0.110*	-0.107	0.0782*	-0.0775	0.0496*	-0.0524
4.0	0.143	-0.143	0.109	-0.115	0.109*	-0.107	0.0960*	-0.0939	0.0684*	-0.0679	0.0440*	-0.0460
4.5	0.124	-0.124	0.0980*	-0.102	0.0966*	-0.0953	0.0851*	-0.0836	0.0609*	-0.0605	0.0396*	-0.0408
5.0	0.111	-0.111	0.0896*	-0.0920	0.0868*	-0.0858	0.0764*	-0.0755	0.0548*	-0.0547	0.0359*	-0.0370
(b) Ratio of t_1/t_d = 1/4												
0.025	0.360	-0.999	0.364	-0.994	0.367	-0.989	0.372	-0.979	0.383	-0.951	0.387	-0.908
0.050	0.434	-0.994	0.436	-0.985	0.436	-0.975	0.436	-0.956	0.425	-0.905	0.402	-0.831
0.075	0.537	-0.987	0.529	-0.973	0.520	-0.959	0.503	-0.933	0.457	-0.862	0.428	-0.765

TABLE A.1c MAXIMUM AND MINIMUM VALUES OF RELATIVE ACCELERATION, $\ddot{u}$ (CONTINUED)

$t_1 f$	$\beta = 0$		$\beta = 0.05$		$\beta = 0.10$		$\beta = 0.20$		$\beta = 0.50$		$\beta = 1.00$	
	$\frac{\ddot{u}_{max}}{\ddot{y}_o}$	$\frac{\ddot{u}_{min}}{\ddot{y}_o}$	$\frac{\ddot{u}_{max}}{\ddot{y}_o}$	$\frac{\ddot{u}_{min}}{\ddot{y}_o}$	$\frac{\ddot{u}_{max}}{\ddot{y}_o}$	$\frac{\ddot{u}_{min}}{\ddot{y}_o}$	$\frac{\ddot{u}_{max}}{\ddot{y}_o}$	$\frac{\ddot{u}_{min}}{\ddot{y}_o}$	$\frac{\ddot{u}_{max}}{\ddot{y}_o}$	$\frac{\ddot{u}_{min}}{\ddot{y}_o}$	$\frac{\ddot{u}_{max}}{\ddot{y}_o}$	$\frac{\ddot{u}_{min}}{\ddot{y}_o}$
0.1	0.646	-0.978	0.622	-0.959	0.600	-0.942	0.562	-0.908	0.485	-0.821	0.503	-0.708
0.125	0.745	-0.966	0.705	-0.944	0.671	-0.922	0.614	-0.883	0.517	-0.783	0.560	-0.659
0.150	0.876*	-0.953	0.777	-0.927	0.732	-0.903	0.660	-0.858	0.556	-0.746	0.601	-0.615
0.20	1.268*	-1.268*	0.928*	-1.081	0.840	-0.922	0.750	-0.807	0.662	-0.682	0.640	-0.542
0.25	1.508*	-1.508*	1.091*	-1.125	0.943	-0.918	0.842	-0.759	0.753	-0.625	0.642	-0.483
0.3	1.543*	-1.543*	1.119	-1.124	1.042	-0.914	0.935	-0.712	0.810	-0.575	0.621	-0.435
0.35	1.424*	-1.497	1.220	-1.153	1.137	-0.929	1.025	-0.669	0.835	-0.531	0.586	-0.395
0.4	1.422	-1.503	1.310	-1.193	1.223	-0.958	1.104	-0.637	0.832	-0.493	0.544	-0.362
0.45	1.506	-1.561	1.388	-1.236	1.298	-0.990	1.148	-0.657	0.806	-0.459	0.499	-0.333
0.5	1.574	-1.612	1.452	-1.275	1.342	-1.021	1.152	-0.676	0.761	-0.429	0.454	-0.308
0.6	1.742*	-1.742*	1.432	-1.326	1.285	-1.062	1.049	-0.705	0.642	-0.378	0.372	-0.268
0.7	1.660	-1.682	1.246	-1.327	1.105	-1.064	0.888	-0.709	0.533	-0.337	0.309	-0.236
0.8	1.622*	-1.622*	1.068	-1.273	0.940	-1.023	0.749	-0.686	0.443	-0.303	0.259	-0.211
0.9	1.508*	-1.508*	0.984	-1.165	0.802	-0.939	0.632	-0.636	0.370	-0.275	0.220	-0.191
1.0	1.273	-1.286	0.858	-1.014	0.686	-0.821	0.537	-0.565	0.311	-0.252	0.188	-0.173
1.1	1.073*	-1.073*	0.703	-0.830	0.593	-0.677	0.460	-0.481	0.263	-0.232	0.163	-0.159
1.2	0.796*	-0.796*	0.598	-0.623	0.517	-0.520	0.397	-0.392	0.228	-0.215	0.142	-0.147
1.3	0.621	-0.514	0.528	-0.412	0.454	-0.364	0.346	-0.310	0.192	-0.200	0.124	-0.137
1.4	0.555	-0.364	0.470	-0.310	0.402	-0.290	0.304	-0.256	0.166	-0.187	0.110	-0.127
1.5	0.500	-0.315	0.422	-0.292	0.360	-0.273	0.269	-0.240	0.144	-0.176	0.0977	-0.120
1.6	0.454	-0.296	0.381	-0.276	0.323	-0.257	0.240	-0.226	0.125	-0.165	0.0873	-0.112
1.7	0.415	-0.362	0.347	-0.261	0.293	-0.243	0.216	-0.214	0.110	-0.156	0.0784	-0.106
1.8	0.469*	-0.469*	0.318	-0.324	0.268	-0.258	0.195	-0.203	0.0973	-0.148	0.0707	-0.100
1.9	0.528*	-0.528*	0.303	-0.356	0.246	-0.274	0.178	-0.199	0.0864	-0.140	0.0641	-0.0954

TABLE A.1c MAXIMUM AND MINIMUM VALUES OF RELATIVE ACCELERATION, $\ddot{u}$ (CONTINUED)

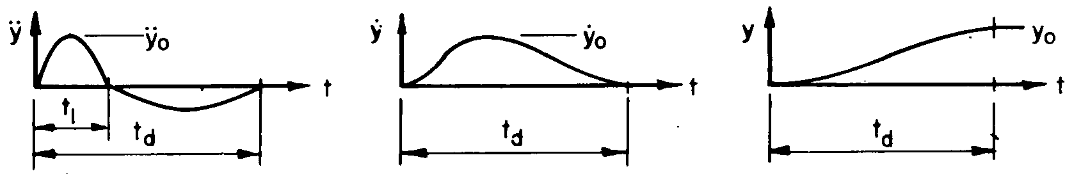
$t_1 f$	β = 0		β = 0.05		β = 0.10		β = 0.20		β = 0.50		β = 1.00	
	$\frac{\ddot{u}_{max}}{\ddot{y}_o}$	$\frac{\ddot{u}_{min}}{\ddot{y}_o}$	$\frac{\ddot{u}_{max}}{\ddot{y}_o}$	$\frac{\ddot{u}_{min}}{\ddot{y}_o}$	$\frac{\ddot{u}_{max}}{\ddot{y}_o}$	$\frac{\ddot{u}_{min}}{\ddot{y}_o}$	$\frac{\ddot{u}_{max}}{\ddot{y}_o}$	$\frac{\ddot{u}_{min}}{\ddot{y}_o}$	$\frac{\ddot{u}_{max}}{\ddot{y}_o}$	$\frac{\ddot{u}_{min}}{\ddot{y}_o}$	$\frac{\ddot{u}_{max}}{\ddot{y}_o}$	$\frac{\ddot{u}_{min}}{\ddot{y}_o}$
2.0	0.505	-0.508	0.301	-0.353	0.227	-0.271	0.163	-0.193	0.0772	-0.134	0.0584	-0.0909
2.1	0.471*	-0.471*	0.273	-0.320	0.211	-0.249	0.150	-0.183	0.0692	-0.128	0.0533	-0.0864
2.2	0.377*	-0.377*	0.237	-0.264	0.197	-0.215	0.139	-0.169	0.0625	-0.122	0.0489	-0.0826
2.3	0.271	-0.259	0.222	-0.197	0.184	-0.183	0.130	-0.161	0.0567	-0.117	0.0449	-0.0792
2.4	0.256	-0.209	0.210	-0.189	0.173	-0.176	0.121	-0.154	0.0517	-0.112	0.0415	-0.0760
2.5	0.242	-0.196	0.198	-0.182	0.163	-0.169	0.114	-0.149	0.0473	-0.108	0.0384	-0.0729
3.0	0.324	-0.325	0.174	-0.204	0.127	-0.154	0.0866	-0.124	0.0322	-0.0903	0.0270	-0.0610
3.5	0.167	-0.141	0.128	-0.131	0.104	-0.122	0.0698	-0.107	0.0238	-0.0775	0.0200	-0.0524
4.0	0.240	-0.241	0.118	-0.138	0.0884	-0.107	0.0586	-0.0939	0.0186	-0.0679	0.0154	-0.0460
4.5	0.124	-0.110	0.0954	-0.102	0.0769	-0.0954	0.0505	-0.0836	0.0152	-0.0605	0.0122	-0.0408
5.0	0.191	-0.191	0.0877	-0.103	0.0680	-0.0858	0.0445	-0.0755	0.0128	-0.0547	0.0099	-0.0370
(c) Ratio of t_1/t_d = 1/8												
0.01	0.151	-1.000	--	--	--	--	0.157	-0.992	--	--	0.169	-0.961
0.015	0.161	-0.999	--	--	--	--	0.168	-0.988	--	--	0.177	-0.943
0.025	0.193	-0.999	0.193	-0.994	0.193	-0.989	0.193	-0.979	0.191	-0.951	0.193	-0.908
0.05	0.296	-0.994	0.285	-0.985	0.275	-0.975	0.259	-0.956	0.236	-0.905	0.316	-0.831
0.075	0.392	-0.987	0.368	-0.973	0.349	-0.959	0.320	-0.933	0.299	-0.862	0.423	-0.765
0.1	0.611	-0.978	0.446	-0.959	0.419	-0.942	0.382	-0.908	0.380	-0.821	0.503	-0.708
0.125	0.717	-0.966	0.521	-0.944	0.490	-0.922	0.447	-0.883	0.462	-0.783	0.560	-0.659
0.15	0.735*	-0.953	0.598	-0.927	0.562	-0.903	0.514	-0.858	0.537	-0.746	0.601	-0.615
0.2	0.811	-0.921	0.753	-0.889	0.708	-0.860	0.651	-0.807	0.662	-0.682	0.640	-0.542
0.25	1.005*	-1.005*	0.902	-0.849	0.849	-0.817	0.783	-0.759	0.753	-0.625	0.642	-0.483
0.3	1.125*	-1.125*	1.042	-0.898	0.981	-0.774	0.908	-0.712	0.810	-0.575	0.621	-0.435
0.35	1.259	-1.265	1.170	-1.004	1.103	-0.808	1.021	-0.669	0.835	-0.531	0.586	-0.395
0.4	1.408	-1.408	1.283	-1.099	1.210	-0.885	1.104	-0.629	0.832	-0.493	0.544	-0.362
0.45	1.485	-1.489	1.381	-1.181	1.298	-0.951	1.148	-0.638	0.805	-0.459	0.499	-0.333

TABLE A.1c MAXIMUM AND MINIMUM VALUES OF RELATIVE ACCELERATION, $\ddot{u}$ (CONTINUED)

$t_1 f$	β = 0		β = 0.05		β = 0.10		β = 0.20		β = 0.50		β = 1.00	
	$\frac{\ddot{u}_{max}}{\ddot{Y}_o}$	$\frac{\ddot{u}_{min}}{\ddot{Y}_o}$	$\frac{\ddot{u}_{max}}{\ddot{Y}_o}$	$\frac{\ddot{u}_{min}}{\ddot{Y}_o}$	$\frac{\ddot{u}_{max}}{\ddot{Y}_o}$	$\frac{\ddot{u}_{min}}{\ddot{Y}_o}$	$\frac{\ddot{u}_{max}}{\ddot{Y}_o}$	$\frac{\ddot{u}_{min}}{\ddot{Y}_o}$	$\frac{\ddot{u}_{max}}{\ddot{Y}_o}$	$\frac{\ddot{u}_{min}}{\ddot{Y}_o}$	$\frac{\ddot{u}_{max}}{\ddot{Y}_o}$	$\frac{\ddot{u}_{min}}{\ddot{Y}_o}$
0.5	1.571	-1.571	1.452	-1.249	1.342	-1.006	1.152	-0.675	0.761	-0.429	0.454	-0.308
0.6	1.680	-1.680	1.432	-1.336	1.285	-1.077	1.049	-0.724	0.642	-0.378	0.372	-0.268
0.7	1.706	-1.706	1.246	-1.356	1.105	-1.094	0.888	-0.739	0.533	-0.337	0.309	-0.236
0.8	1.661*	-1.661*	1.120	-1.311	0.940	-1.060	0.749	-0.720	0.443	-0.303	0.259	-0.211
0.9	1.518	-1.518	1.032	-1.208	0.802	-0.979	0.632	-0.670	0.370	-0.275	0.220	-0.191
1.0	1.324	-1.325	0.902	-1.055	0.686	-0.859	0.537	-0.598	0.311	-0.262	0.188	-0.174
1.1	1.083	-1.083	0.741	-0.867	0.593	-0.711	0.460	-0.512	0.263	-0.244	0.163	-0.159
1.2	0.820*	-0.820*	0.598	-0.654	0.517	-0.550	0.397	-0.420	0.224	-0.226	0.142	-0.147
1.3	0.621	-0.530	0.528	-0.438	0.454	-0.391	0.346	-0.336	0.192	-0.208	0.124	-0.137
1.4	0.555	-0.364	0.470	-0.310	0.402	-0.290	0.304	-0.272	0.166	-0.192	0.110	-0.127
1.5	0.500	-0.315	0.422	-0.292	0.360	-0.273	0.269	-0.240	0.144	-0.178	0.0977	-0.120
1.6	0.454	-0.296	0.381	-0.276	0.323	-0.257	0.240	-0.226	0.125	-0.166	0.0872	-0.122
1.7	0.415	-0.373	0.347	-0.279	0.293	-0.244	0.216	-0.217	0.110	-0.156	0.0784	-0.106
1.8	0.486*	-0.486*	0.318	-0.346	0.268	-0.278	0.195	-0.219	0.0973	-0.148	0.0707	-0.100
1.9	0.532	-0.532	0.323	-0.378	0.246	-0.294	0.178	-0.217	0.0864	-0.140	0.0641	-0.0954
2.0	0.529	-0.529	0.320	-0.374	0.227	-0.290	0.163	-0.210	0.0772	-0.134	0.0584	-0.0909
2.1	0.476	-0.476	0.290	-0.340	0.211	-0.267	0.150	-0.199	0.069	-0.128	0.053	-0.086
2.2	0.390*	-0.390*	0.240	-0.281	0.197	-0.232	0.139	-0.185	--	--	--	--

TABLE A.1d MAXIMUM AND MINIMUM VALUES OF ABSOLUTE VELOCITY OF MASS, $\dot{x}$

Elastic Systems Subjected to a Sequence of Two Half-Sine Ground Acceleration Pulses as Shown



$t_1 f$	β = 0		β = 0.05		β = 0.10		β = 0.20		β = 0.50		β = 1.00	
	$\frac{\dot{x}_{max}}{\dot{y}_o}$	$\frac{\dot{x}_{min}}{\dot{y}_o}$	$\frac{\dot{x}_{max}}{\dot{y}_o}$	$\frac{\dot{x}_{min}}{\dot{y}_o}$	$\frac{\dot{x}_{max}}{\dot{y}_o}$	$\frac{\dot{x}_{min}}{\dot{y}_o}$	$\frac{\dot{x}_{max}}{\dot{y}_o}$	$\frac{\dot{x}_{min}}{\dot{y}_o}$	$\frac{\dot{x}_{max}}{\dot{y}_o}$	$\frac{\dot{x}_{min}}{\dot{y}_o}$	$\frac{\dot{x}_{max}}{\dot{y}_o}$	$\frac{\dot{x}_{min}}{\dot{y}_o}$
(a) Ratio of t_1/t_d = 1/2												
0.025	0.157*	-0.157*	0.146*	-0.125*	0.138*	-0.101*	0.129*	-0.0678*	0.155	-0.0256*	0.260	-0.0078*
0.050	0.312*	-0.312*	0.291*	-0.248*	0.275*	-0.200*	0.256*	-0.135*	0.301	-0.0509*	0.445	-0.0155*
0.075	0.464*	-0.464*	0.432*	-0.369*	0.409*	-0.298*	0.381*	-0.201*	0.435	-0.0757*	0.584	-0.0231*
0.1	0.612*	.612*	0.570*	-0.487*	0.539*	-0.392*	0.502*	-0.265*	0.554	-0.0998*	0.689	-0.0305*
0.125	0.754*	0.754*	0.702*	-0.600*	0.664*	-0.484*	0.619*	-0.326*	0.658	-0.123*	0.771	-0.0376*
0.15	0.889*	-0.889*	0.828*	-0.707*	0.782*	-0.570*	0.730*	-0.384*	0.748	-0.145*	0.836	-0.0443*
0.20	1.132*	-1.132*	1.054*	-0.901*	0.996*	-0.726*	0.927	-0.489*	0.893	-0.185*	0.928	-0.0566*
0.25	1.333	-1.333*	1.241	-1.061*	1.172*	-0.856*	1.084	-0.576*	0.998	-0.218*	0.988	-0.0670*
0.3	1.486	-1.485*	1.384	-1.182*	1.305	-0.954*	1.199	-0.643*	1.072	-0.244*	1.026	-0.0755*
0.35	1.592	-1.585*	1.483	-1.262*	1.398	-1.018*	1.280	-0.687*	1.123	-0.261*	1.052	-0.0820*
0.4	1.659	-1.631*	1.547	-1.299*	1.459	-1.048*	1.333	-0.708*	1.157	-0.272*	1.068	-0.0866*
0.45	1.694	-1.625*	1.582	-1.294*	1.493	-1.045*	1.365	-0.707*	1.177	-0.274*	1.078	-0.0984*
0.5	1.704	-1.570*	1.594	-1.250*	1.507	-1.010*	1.379	-0.686	1.189	-0.271*	1.083	-0.0907*
0.6	1.674	-1.335*	1.574	-1.064*	1.493	-0.863*	1.374	-0.594*	1.192	-0.248*	1.087	-0.0899*
0.7	1.605	-0.991*	1.517	-0.792*	1.447	-0.648*	1.342	-0.460*	1.180	-0.214*	1.084	-0.0859*
0.8	1.517	-0.610*	1.443	-0.494	1.385	-0.416	1.297	-0.321*	1.162	-0.178*	1.079	-0.0800*
0.9	1.424	-0.263*	1.365	-0.228	1.317	-0.219	1.248	-0.209	1.141	-0.146*	1.073	-0.0736*
1.0	1.333	-0.0013	1.287	-0.0775	1.251	-0.118	1.200	-0.145*	1.121	-0.121*	1.066	-0.0671*
1.1	1.248	-0.152*	1.214	-0.121*	1.189	-0.117*	1.155	-0.119*	1.103	-0.101*	1.060	-0.0608*

* Denotes extremum occurring during free vibration.

TABLE A.1d MAXIMUM AND MINIMUM VALUES OF ABSOLUTE VELOCITY OF MASS, $\dot{x}$ (CONTINUED)

$t_1 f$	β = 0		β = 0.05		β = 0.10		β = 0.20		β = 0.50		β = 1.00	
	$\dot{x}_{max}/\dot{y}_o$	$\dot{x}_{min}/\dot{y}_o$	$\dot{x}_{max}/\dot{y}_o$	$\dot{x}_{min}/\dot{y}_o$	$\dot{x}_{max}/\dot{y}_o$	$\dot{x}_{min}/\dot{y}_o$	$\dot{x}_{max}/\dot{y}_o$	$\dot{x}_{min}/\dot{y}_o$	$\dot{x}_{max}/\dot{y}_o$	$\dot{x}_{min}/\dot{y}_o$	$\dot{x}_{max}/\dot{y}_o$	$\dot{x}_{min}/\dot{y}_o$
1.2	1.169	-0.199*	1.148	-0.145*	1.134	-0.121*	1.117	-0.103*	1.087	-0.0858*	1.055	-0.0550*
1.3	1.100	-0.165*	1.091	-0.121	1.088	-0.102	1.086	-0.0875*	1.074	-0.0730*	1.049	-0.0497*
1.4	1.041	-0.0870*	1.046	-0.0725	1.054	-0.0724	1.065	-0.0719	1.064	-0.0631*	1.045	-0.0451*
1.5	1.000	-0.0016*	1.023	-0.0381	1.038	-0.0528	1.052	-0.0600	1.055	-0.0549*	1.040	-0.0410*
1.6	1.030	-0.0629*	1.032	-0.0498*	1.037	-0.0506	1.046	-0.0523	1.048	-0.0486	1.037	-0.0372
1.7	1.054	-0.0897*	1.044	-0.0603	1.042	-0.0510	1.043	-0.0467	1.043	-0.0431	1.034	-0.0339
1.8	1.067	-0.0798	1.052	-0.0539	1.045	-0.0457	1.041	-0.0417	1.038	-0.0384	1.031	-0.0310
1.9	1.070	-0.0444*	1.054	-0.0365	1.045	-0.0369	1.038	-0.0369	1.034	-0.0345	1.028	-0.0285
2.0	1.066	-0.0009	1.051	-0.0238	1.042	-0.0306	1.035	-0.0328	1.031	-0.0311	1.026	-0.0261
2.1	1.057	-0.0349*	1.044	-0.0278*	1.037	-0.0289	1.032	-0.0296	1.028	-0.0283	1.024	-0.0241
2.2	1.044	-0.0516*	1.035	-0.0325	1.031	-0.0283	1.028	-0.0270	1.026	-0.0258	1.022	-0.0223
2.3	1.029	-0.0473*	1.025	-0.0300	1.025	-0.0261	1.025	-0.0247	1.023	-0.0236	1.021	-0.0206
2.4	1.013	-0.0271*	1.016	-0.0222	1.019	-0.0226	1.022	-0.0225	1.022	-0.0217	1.019	-0.0191
2.5	1.000	-0.0006*	1.011	-0.0165	1.016	-0.0199	1.020	-0.0207	1.020	-0.0200	1.018	-0.0178
3.0	1.028	-0.0003*	1.020	-0.0121	1.016	-0.0140	1.014	-0.0142	1.014	-0.0139	1.013	-0.0128
3.5	1.000	-0.0004*	1.007	-0.0093	1.009	-0.0103	1.010	-0.0104	1.010	-0.0102	1.009	-0.0096
4.0	1.016	-0.0004*	1.010	-0.0073	1.009	-0.0079	1.008	-0.0079	1.008	-0.0078	1.007	-0.0074
4.5	1.000	-0.0005*	1.005	-0.0058	1.006	-0.0062	1.006	-0.0062	1.006	-0.0062	1.006	-0.0059
5.0	1.010	-0.0005*	1.006	-0.0048	1.005	-0.0050	1.005	-0.0050	1.005	-0.0050	1.005	-0.0048
(b) Ratio of $t_1 t_d$ = 1/4												
0.025	0.312*	-0.312*	0.290*	-0.249*	0.274*	-0.201*	0.255*	-0.135*	0.298	-0.0512*	0.428	-0.0158*
0.050	0.611*	-0.611*	0.569*	-0.486*	0.537*	-0.392*	0.501*	-0.264*	0.539	-0.0998*	0.655	-0.0307*
0.075	0.885*	-0.885*	0.824*	-0.704*	0.778*	-0.568*	0.725	-0.382*	0.721	-0.144*	0.794	-0.0442*

TABLE A.1d MAXIMUM AND MINIMUM VALUES OF ABSOLUTE VELOCITY OF MASS, $\dot{x}$ (CONTINUED)

$t_1 f$	$\beta = 0$		$\beta = 0.05$		$\beta = 0.10$		$\beta = 0.20$		$\beta = 0.50$		$\beta = 1.00$	
	$\frac{\dot{x}_{max}}{\dot{y}_o}$	$\frac{\dot{x}_{min}}{\dot{y}_o}$	$\frac{\dot{x}_{max}}{\dot{y}_o}$	$\frac{\dot{x}_{min}}{\dot{y}_o}$	$\frac{\dot{x}_{max}}{\dot{y}_o}$	$\frac{\dot{x}_{min}}{\dot{y}_o}$	$\frac{\dot{x}_{max}}{\dot{y}_o}$	$\frac{\dot{x}_{min}}{\dot{y}_o}$	$\frac{\dot{x}_{max}}{\dot{y}_o}$	$\frac{\dot{x}_{min}}{\dot{y}_o}$	$\frac{\dot{x}_{max}}{\dot{y}_o}$	$\frac{\dot{x}_{min}}{\dot{y}_o}$
0.1	1.123	-1.123*	1.045	-0.894*	0.987*	-0.720*	0.913	-0.485*	0.857	-0.182*	0.884	-0.0557*
0.125	1.132	-1.132*	1.225	-1.047*	1.154	-0.843*	1.059	-0.567*	0.957	-0.212*	0.946	-0.0649*
0.15	1.468	-1.461*	1.364	-1.159*	1.283	-0.933*	1.171	-0.625*	1.032	-0.234*	0.990	-0.0716*
0.20	1.664	-1.586*	1.546	-1.252*	1.452	-1.002*	1.318	-0.666*	1.130	-0.247*	1.044	-0.0777*
0.25	1.766	-1.508*	1.642	-1.177*	1.543	-0.933*	1.398	-0.609*	1.184	-0.224*	1.074	-0.0762*
0.3	1.813	-1.286*	1.688	-0.979*	1.588	-0.756	1.439	-0.473	1.213	-0.177*	1.090	-0.0702*
0.35	1.828	-1.017*	1.704	-0.730	1.604	-0.532	1.456	-0.295	1.228	-0.125*	1.099	-0.0624*
0.4	1.821	-0.0822*	1.701	-0.490	1.603	-0.311	1.458	-0.113	1.232	-0.0845*	1.102	-0.0545*
0.45	1.801	-0.0766*	1.685	-0.384*	1.591	-0.188*	1.451	-0.0411	1.231	-0.0626*	1.103	-0.0473*
0.5	1.772	-0.0785*	1.661	-0.434*	1.571	-0.235*	1.437	-0.0884	1.226	-0.0522*	1.102	-0.0411*
0.6	1.698	-0.0726*	1.599	-0.401	1.518	-0.231	1.398	-0.0930	1.209	-0.0394*	1.097	-0.0313*
0.7	1.610	-0.0567*	1.524	-0.241	1.454	-0.101	1.351	-0.0181	1.188	-0.0208	1.090	-0.0244*
0.8	1.518	-0.0507*	1.445	-0.203*	1.386	-0.0855	1.306	-0.0252*	1.160	-0.0218	1.082	-0.0194*
0.9	1.424	-0.0418*	1.365	-0.170	1.318	-0.0754	1.248	-0.0264	1.142	-0.0174	1.075	-0.0158
1.0	1.333	-0.305*	1.287	-0.0794	1.251	-0.0162*	1.200	-0.0114	1.121	-0.0142	1.067	-0.0131
1.1	1.248	-0.244*	1.214	-0.0744*	1.189	-0.0264*	1.156	-0.0115*	1.103	-0.0117	1.061	-0.0110
1.2	1.160	-0.166*	1.148	-0.0520	1.134	-0.0218*	1.117	-0.0113	1.087	-0.0099	1.055	-0.0094
1.3	1.100	-0.0903*	1.091	-0.0140*	1.088	-0.0033	1.086	-0.0084	1.074	-0.0085	1.050	-0.0089
1.4	1.041	-0.0508*	1.046	-0.0148*	1.054	-0.0076*	1.065	-0.0072	1.064	-0.0074	1.045	-0.0071
1.5	1.000	-0.0013*	1.023	-0.0107*	1.038	-0.0084	1.052	-0.0066	1.055	-0.0064	1.041	-0.0062
1.6	1.032	-0.0370	1.032	-0.0105	1.038	-0.0063	1.046	-0.0057	1.048	-0.0055	1.037	-0.0054
1.7	1.055	-0.0483*	1.045	-0.0033*	1.042	-0.0033	1.043	-0.0049	1.043	-0.0049	1.034	-0.0048
1.8	1.067	-0.0653*	1.052	-0.0112*	1.045	-0.0051*	1.041	-0.0044	1.038	-0.0044	1.031	-0.0043
1.9	1.070	-0.0606*	1.054	-0.0118	1.045	-0.0050	1.038	-0.0040	1.034	-0.0040	1.028	-0.0039

TABLE A.1d MAXIMUM AND MINIMUM VALUES OF ABSOLUTE VELOCITY OF MASS, $\dot{x}$ (CONTINUED)

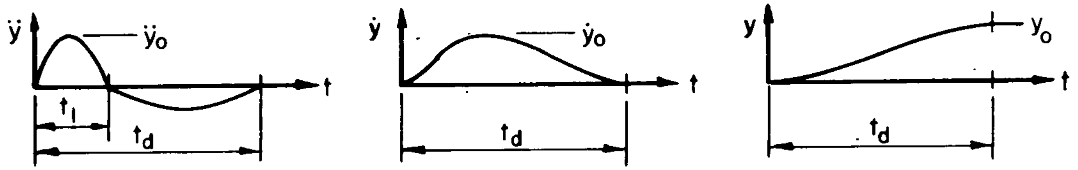
$t_1 f$	$\beta = 0$		$\beta = 0.05$		$\beta = 0.10$		$\beta = 0.20$		$\beta = 0.50$		$\beta = 1.00$	
	$\frac{\dot{x}_{max}}{\dot{y}_o}$	$\frac{\dot{x}_{min}}{\dot{y}_o}$	$\frac{\dot{x}_{max}}{\dot{y}_o}$	$\frac{\dot{x}_{min}}{\dot{y}_o}$	$\frac{\dot{x}_{max}}{\dot{y}_o}$	$\frac{\dot{x}_{min}}{\dot{y}_o}$	$\frac{\dot{x}_{max}}{\dot{y}_o}$	$\frac{\dot{x}_{min}}{\dot{y}_o}$	$\frac{\dot{x}_{max}}{\dot{y}_o}$	$\frac{\dot{x}_{min}}{\dot{y}_o}$	$\frac{\dot{x}_{max}}{\dot{y}_o}$	$\frac{\dot{x}_{min}}{\dot{y}_o}$
2.0	1.067	-0.0598*	1.051	-0.0034*	1.043	-0.0028	1.035	-0.0036	1.031	-0.0036	1.026	-0.0035
2.1	1.057	-0.0562*	1.044	-0.0074*	1.037	-0.0034*	1.032	-0.0032	1.028	-0.0032	1.024	-0.0032
2.2	1.044	-0.0429*	1.035	-0.0064	1.031	-0.0033	1.028	-0.0030	1.026	-0.0029	1.022	-0.0029
2.3	1.029	-0.0255*	1.025	-0.0005	1.025	-0.0025	1.025	-0.0027	1.024	-0.0027	1.021	-0.0027
2.4	1.013	-0.0160*	1.016	-0.0029*	1.019	-0.0024	1.022	-0.0025	1.022	-0.0025	1.019	-0.0024
2.5	1.000	-0.0005*	1.011	-0.0032	1.016	-0.0024	1.020	-0.0023	1.020	-0.0023	1.018	-0.0023
3.0	1.028	-0.0255*	1.020	-0.0005	1.016	-0.0015	1.014	-0.0016	1.014	-0.0016	1.013	-0.0016
3.5	1.000	-0.0003*	1.007	-0.0014	1.009	-0.0012	1.010	-0.0016	1.010	-0.0012	1.010	-0.0012
4.0	1.016	-0.0141*	1.010	-0.0007	1.008	-0.0009	1.008	-0.0009	1.008	-0.0009	1.007	-0.0009
4.5	1.000	-0.0003*	1.005	-0.0008	1.006	-0.0007	1.006	-0.0007	1.006	-0.0007	1.006	-0.0007
5.0	1.010	-0.0090*	1.006	-0.0005	1.005	-0.0006	1.009	-0.0006	1.005	-0.0006	1.005	-0.0006
				(c) Ratio of $t_1 t_d$ = 1/8								
0.01	0.250*	-0.250*	--	--	--	--	0.205*	-0.108*	--	--	0.358	-0.0128*
0.015	0.373*	-0.373*	--	--	--	--	0.306	-0.162*	--	--	0.474	-0.0189*
0.025	0.609*	-0.609*	0.567*	-0.485*	0.536*	-0.391*	0.500*	-0.264*	0.531	-0.0996*	0.638	-0.0306*
0.050	1.113*	-1.113*	1.035	-0.885*	0.976	-0.713*	0.900	-0.479*	0.836	-0.0188*	0.859	-0.0549*
0.075	1.444	-1.430*	1.341	-1.134*	1.260	-0.910*	1.148	-0.609*	1.006	-0.226*	0.964	-0.0690*
0.1	1.640	-1.529*	1.522	-1.202*	1.428	-0.958*	1.294	-0.632*	1.105	-0.229*	1.022	-0.0725*
0.125	1.754	-1.433*	1.629	-1.107*	1.528	-0.867*	1.381	-0.553*	1.165	-0.195*	1.057	-0.0686*
0.15	1.822	-1.225*	1.692	-0.913	1.588	-0.687	1.434	-0.402*	1.202	-0.140*	1.078	-0.0610*
0.2	1.885	-0.950*	1.753	-0.508	1.646	-0.303	1.487	-0.0614	1.240	-0.0566*	1.101	-0.0449*
0.25	1.901	-1.006*	1.770	-0.554	1.663	-0.315	1.504	-0.118*	1.255	-0.0402*	1.111	-0.0327*
0.3	1.895	-0.937*	1.766	-0.493	1.662	-0.261	1.505	-0.0776	1.259	-0.0295*	1.115	-0.0244*
0.35	1.877	-0.884*	1.751	-0.346	1.650	-0.128*	1.497	-0.0173*	1.257	-0.0210*	1.115	-0.0187*

TABLE A.1d MAXIMUM AND MINIMUM VALUES OF ABSOLUTE VELOCITY OF MASS, $\dot{x}$ (CONTINUED)

$t_1 f$	β = 0		β = 0.05		β = 0.10		β = 0.20		β = 0.50		β = 1.00	
	$\frac{\dot{x}_{max}}{\dot{y}_o}$	$\frac{\dot{x}_{min}}{\dot{y}_o}$	$\frac{\dot{x}_{max}}{\dot{y}_o}$	$\frac{\dot{x}_{min}}{\dot{y}_o}$	$\frac{\dot{x}_{max}}{\dot{y}_o}$	$\frac{\dot{x}_{min}}{\dot{y}_o}$	$\frac{\dot{x}_{max}}{\dot{y}_o}$	$\frac{\dot{x}_{min}}{\dot{y}_o}$	$\frac{\dot{x}_{max}}{\dot{y}_o}$	$\frac{\dot{x}_{min}}{\dot{y}_o}$	$\frac{\dot{x}_{max}}{\dot{y}_o}$	$\frac{\dot{x}_{min}}{\dot{y}_o}$
0.4	1.850	-0.880*	1.729	-0.354	1.631	-0.149	1.484	-0.0365*	1.251	-0.0162*	1.114	-0.0148*
0.45	1.818	-0.816*	1.702	-0.300	1.608	-0.106	1.467	-0.0141	1.243	-0.0129	1.111	-0.0120*
0.5	1.782	-0.785*	1.671	-0.225*	1.581	-0.0659*	1.447	-0.0103	1.234	-0.0105*	1.107	-0.0099*
0.6	1.701	-0.693*	1.602	-0.173	1.521	-0.0336	1.402	-0.0053	1.212	-0.0074	1.100	-0.0071
0.7	1.611	-0.609*	1.525	-0.126	1.455	-0.0271	1.352	-0.0062	1.189	-0.0055	1.091	-0.0053
0.8	1.518	-0.519	1.445	-0.0828	1.386	-0.0162	1.300	-0.0045	1.165	-0.0043	1.083	-0.0042
0.9	1.424	-0.421*	1.365	-0.0472*	1.318	-0.0059*	1.249	-0.0034	1.142	-0.0034	1.075	-0.0034
1.0	1.333	-0.328*	1.287	-0.0288	1.251	-0.0012	1.200	-0.0028	1.121	-0.0028	1.067	-0.0028
1.1	1.247	-0.246*	1.214	-0.0207	1.189	-0.0035	1.156	-0.0024	1.103	-0.0024	1.061	-0.0024
1.2	1.169	-0.171*	1.148	-0.0122*	1.134	-0.0026*	1.117	-0.0021*	1.087	-0.0021	1.055	-0.0020
1.3	1.100	-0.102*	1.091	-0.0046	1.088	-0.0016*	1.086	-0.0018	1.074	-0.0018	1.050	-0.0018
1.4	1.041	-0.0445*	1.046	-0.0016	1.054	-0.0016	1.065	-0.0016	1.063	-0.0016	1.045	-0.0016
1.5	1.000	-0.0012*	1.023	-0.0022	1.038	-0.0015	1.052	-0.0014	1.055	-0.0014	1.041	-0.0014
1.6	1.033	-0.0319*	1.032	-0.0006*	1.037	-0.0011	1.046	-0.0011	1.048	-0.0011	1.037	-0.0011
1.7	1.055	-0.0550*	1.045	-0.0015	1.042	-0.0010	1.043	-0.0010	1.043	-0.0010	1.034	-0.0010
1.8	1.067	-0.0675*	1.052	-0.0019	1.045	-0.0009	1.041	-0.0009	1.038	-0.0009	1.031	-0.0009
1.9	1.070	-0.0700*	1.063	-0.0116*	1.054	-0.0011*	1.045	-0.0008	1.038	-0.0008	1.034	-0.0008
2.0	1.067	-0.0654*	1.051	-0.0003	1.043	-0.0008	1.035	-0.0008	1.031	-0.0008	1.026	-0.0008
2.1	1.057	-0.0567*	1.044	-0.0009	1.037	-0.0007	1.032	-0.0007	1.028	-0.0007	1.024	-0.0007
2.2	1.044	-0.0444*	1.035	-0.0009*	1.031	-0.0006	1.028	-0.0006	--	--	--	--

TABLE A.1e MAXIMUM AND MINIMUM VALUES OF ABSOLUTE ACCELERATION OF MASS, $\ddot{x}$

Elastic Systems Subjected to a Sequence of Two Half-Sine Ground Acceleration Pulses as Shown



$t_1 f$	β = 0 $\ddot{x}_{max}/\ddot{y}_o$	β = 0 $\ddot{x}_{min}/\ddot{y}_o$	β = 0.05 $\ddot{x}_{max}/\ddot{y}_o$	β = 0.05 $\ddot{x}_{min}/\ddot{y}_o$	β = 0.10 $\ddot{x}_{max}/\ddot{y}_o$	β = 0.10 $\ddot{x}_{min}/\ddot{y}_o$	β = 0.20 $\ddot{x}_{max}/\ddot{y}_o$	β = 0.20 $\ddot{x}_{min}/\ddot{y}_o$	β = 0.50 $\ddot{x}_{max}/\ddot{y}_o$	β = 0.50 $\ddot{x}_{min}/\ddot{y}_o$	β = 1.00 $\ddot{x}_{max}/\ddot{y}_o$	β = 1.00 $\ddot{x}_{min}/\ddot{y}_o$
(a) Ratio of $t_1/t_d = 1/2$												
0.025	0.0157*	-0.0157*	0.0198	-0.0135*	0.0284	-0.0119*	0.0467	-0.0097*	0.0996	-0.0086*	0.179	-0.0381*
0.050	0.0624*	-0.0624*	0.0636	-0.0539*	0.0758	-0.0474*	0.107	-0.0387*	0.302	-0.0341*	0.322	-0.123*
0.075	0.139*	-0.139*	0.130	-0.120*	0.142	-0.106*	0.178	-0.0864*	0.291	-0.0761*	0.440	-0.232
0.1	0.245*	-0.245*	0.213	-0.211*	0.222	-0.186*	0.258	-0.152*	0.381	-0.134*	0.537	-0.345
0.125	0.377*	-0.377*	0.308	-0.325*	0.313	-0.286*	0.343	-0.234*	0.464	-0.206*	0.618	-0.453
0.15	0.533*	-0.533*	0.410	-0.460*	0.408	-0.405*	0.431	-0.331*	0.542	-0.291*	0.687	-0.553
0.20	0.905*	-0.905*	0.668*	-0.781*	0.603	-0.687*	0.604	-0.562*	0.680	-0.495*	0.794	-0.723
0.25	1.333*	-1.333*	0.983*	-1.150*	0.787	-1.012*	0.763	-0.828*	0.796	-0.720	0.872	-0.854
0.3	1.783*	-1.783*	1.315*	-1.539*	0.987*	-1.354*	0.903	-1.108*	0.890	-0.924	0.930	-0.951
0.35	2.220*	-2.220*	1.637*	-1.916*	1.230*	-1.686*	1.022	-1.381*	0.967	-1.093	0.974	-1.022
0.4	2.611*	-2.611*	1.926*	-2.254*	1.447*	-1.984*	1.121	-1.627*	1.028	-1.222	1.007	-1.072
0.45	2.926*	-2.926*	2.159*	-2.527*	1.623*	-2.225*	1.201	-1.820	1.078	-1.314	1.031	-1.107
0.5	3.140*	-3.140*	2.318*	-2.712	1.743*	-2.388	1.264	-1.944	1.116	-1.374	1.050	-1.130
0.6	3.205*	-3.205*	2.368*	-2.804	1.788*	-2.472	1.352	-2.017	1.169	-1.420	1.074	-1.153
0.7	2.774*	-3.031	2.057*	-2.631	1.567*	-2.331	1.400	-1.926	1.198	-1.402	1.087	-1.157
0.8	1.953*	-2.646	1.650	-2.312	1.556	-2.067	1.419	-1.749	1.210	-1.354	1.093	-1.152
0.9	1.758	-2.195	1.644	-1.931	1.554	-1.752	1.419	-1.542	1.213	-1.297	1.095	-1.142
1.0	1.732	-1.734	1.623	-1.543	1.536	-1.436	1.407	-1.348	1.207	-1.244	1.093	-1.132
1.1	1.693	-1.298	1.591	-1.180	1.509	-1.156	1.387	-1.206	1.198	-1.203	1.090	-1.120

* Denotes extremum occurring during free vibration.

TABLE A.1e MAXIMUM AND MINIMUM VALUES OF ABSOLUTE ACCELERATION OF MASS, $\ddot{x}$ (CONTINUED)

$t_1 f$	$\beta = 0$		$\beta = 0.05$		$\beta = 0.10$		$\beta = 0.20$		$\beta = 0.50$		$\beta = 1.00$	
	$\frac{\ddot{x}_{max}}{\ddot{y}_o}$	$\frac{\ddot{x}_{min}}{\ddot{y}_o}$	$\frac{\ddot{x}_{max}}{\ddot{y}_o}$	$\frac{\ddot{x}_{min}}{\ddot{y}_o}$	$\frac{\ddot{x}_{max}}{\ddot{y}_o}$	$\frac{\ddot{x}_{min}}{\ddot{y}_o}$	$\frac{\ddot{x}_{max}}{\ddot{y}_o}$	$\frac{\ddot{x}_{min}}{\ddot{y}_o}$	$\frac{\ddot{x}_{max}}{\ddot{y}_o}$	$\frac{\ddot{x}_{min}}{\ddot{y}_o}$	$\frac{\ddot{x}_{max}}{\ddot{y}_o}$	$\frac{\ddot{x}_{min}}{\ddot{y}_o}$
1.2	1.649	-1.150	1.551	-1.041	1.475	-1.069	1.360	-1.151	1.185	-1.170	1.086	-1.109
1.3	1.600	-1.404	1.510	-1.230	1.437	-1.168	1.331	-1.151	1.171	-1.145	1.081	-1.099
1.4	1.549	-1.506	1.465	-1.312	1.399	-1.219	1.301	-1.155	1.155	-1.126	1.076	-1.090
1.5	1.500	-1.499	1.422	-1.310	1.360	-1.216	1.269	-1.144	1.140	-1.110	1.071	-1.081
1.6	1.450	-1.419	1.377	-1.249	1.319	-1.171	1.237	-1.121	1.125	-1.097	1.067	-1.074
1.7	1.402	-1.290	1.334	-1.149	1.281	-1.103	1.205	-1.094	1.110	-1.086	1.062	-1.068
1.8	1.356	-1.133	1.292	-1.030	1.242	-1.037	1.175	-1.074	1.096	-1.077	1.057	-1.062
1.9	1.311	-1.173	1.252	-1.046	1.206	-1.041	1.145	-1.066	1.084	-1.069	1.054	-1.057
2.0	1.268	-1.266	1.213	-1.115	1.171	-1.073	1.117	-1.064	1.073	-1.062	1.050	-1.052
2.1	1.227	-1.303	1.176	-1.144	1.137	-1.088	1.090	-1.062	1.063	-1.056	1.046	-1.048
2.2	1.189	-1.292	1.140	-1.139	1.105	-1.085	1.065	-1.058	1.055	-1.051	1.043	-1.044
2.3	1.151	-1.247	1.106	-1.107	1.074	-1.066	1.040	-1.051	1.048	-1.047	1.040	-1.041
2.4	1.117	-1.174	1.074	-1.057	1.044	-1.040	1.018	-1.045	1.042	-1.043	1.038	-1.038
2.5	1.083	-1.084	1.043	-0.994	1.016	-1.022	0.999	-1.040	1.038	-1.040	1.035	-1.036
3.0	1.170	-1.170	1.101	-1.053	1.063	-1.032	1.032	-1.029	1.026	-1.028	1.025	-1.026
3.5	1.167	-1.167	1.105	-1.049	1.070	-1.026	1.037	-1.021	1.020	-1.020	1.019	-1.019
4.0	1.126	-1.124	1.073	-1.023	1.044	-1.015	1.022	-1.016	1.016	-1.016	1.015	-1.015
4.5	1.071	-1.072	1.026	-1.000	1.005	-1.011	1.006	-1.012	1.012	-1.012	1.012	-1.012
5.0	1.099	-1.10	1.043	-1.015	1.020	-1.010	1.009	-1.010	1.010	-1.010	1.010	-1.010
					(b) Ratio of $t_1 t_d$ = 1/4							
0.025	0.0312*	-0.0312*	0.0305	-0.0269*	0.0350	-0.0237*	0.0494	-0.0194*	0.0996	-0.0171*	0.179	-0.0579
0.050	0.122*	-0.122*	0.102	-0.105*	0.105	-0.0928*	0.120	-0.0758*	0.197	-0.0667*	0.323	-0.150
0.075	0.266*	-0.266*	0.196	-0.229*	0.194	-0.201*	0.203	-0.164*	0.291	-0.145*	0.440	-0.229

TABLE A.1e MAXIMUM AND MINIMUM VALUES OF ABSOLUTE ACCELERATION OF MASS, $\ddot{x}$ (CONTINUED)

$t_1 f$	β = 0		β = 0.05		β = 0.10		β = 0.20		β = 0.50		β = 1.00	
	$\frac{\ddot{x}_{max}}{\ddot{y}_o}$	$\frac{\ddot{x}_{min}}{\ddot{y}_o}$	$\frac{\ddot{x}_{max}}{\ddot{y}_o}$	$\frac{\ddot{x}_{min}}{\ddot{y}_o}$	$\frac{\ddot{x}_{max}}{\ddot{y}_o}$	$\frac{\ddot{x}_{min}}{\ddot{y}_o}$	$\frac{\ddot{x}_{max}}{\ddot{y}_o}$	$\frac{\ddot{x}_{min}}{\ddot{y}_o}$	$\frac{\ddot{x}_{max}}{\ddot{y}_o}$	$\frac{\ddot{x}_{min}}{\ddot{y}_o}$	$\frac{\ddot{x}_{max}}{\ddot{y}_o}$	$\frac{\ddot{x}_{min}}{\ddot{y}_o}$
0.1	0.449*	-0.449*	0.331*	-0.387*	0.290	-0.341*	0.291	-0.278*	0.381	-0.241	0.537	-0.290
0.125	0.659*	-0.659*	0.485*	-0.568*	0.387	-0.499*	0.379	-0.806*	0.464	-0.332	0.618	-0.333
0.15	0.876*	-0.876*	0.645*	-0.754*	0.483*	-0.662*	0.464	-0.538*	0.542	-0.408	0.687	-0.363
0.2	1.268*	-1.268*	0.928*	-0.086	0.692*	-0.947*	0.625	-0.757	0.680	-0.508	0.794	-0.396
0.25	1.508*	-1.508*	1.091*	-1.297	0.821	-1.121	0.772	-0.878	0.796	-0.554	0.872	-0.408
0.3	1.543*	-1.679	1.088*	-1.414	0.966	-1.213	0.904	-0.935	0.890	-0.566	0.930	-0.407
0.35	1.424*	-1.773	1.159	-1.482	1.095	-1.261	1.022	-0.956	0.967	-0.557	0.974	-0.401
0.4	1.372	-1.835	1.278	-1.523	1.208	-1.286	1.121	-0.961	1.028	-0.539	1.007	-0.392
0.45	1.481	-1.878	1.380	-1.549	1.303	-1.300	1.200	-0.958	1.078	-0.518	1.031	-0.383
0.5	1.570	-1.906	1.462	-1.563	1.379	-1.304	1.264	-0.950	1.116	-0.496	1.050	-0.374
0.6	1.742*	-1.917	1.573	-1.561	1.482	-1.291	1.352	-0.925	1.169	-0.457	1.074	-0.361
0.7	1.749	-1.970	1.630	-1.513	1.536	-1.246	1.400	-0.883	1.198	-0.423	1.087	-0.353
0.8	1.767	-1.943	1.650	-1.419	1.556	-1.166	1.419	-0.824	1.210	-0.391	1.093	-0.347
0.9	1.758	-1.812	1.645	-1.281	1.554	-1.052	1.419	-0.746	1.213	-0.361	1.095	-0.344
1.0	1.732	-1.608	1.623	-1.102	1.536	-0.909	1.407	-0.654	1.208	-0.348	1.093	-0.342
1.1	1.693	-1.351	1.591	-0.917	1.509	-0.746	1.387	-0.554	1.198	-0.343	1.090	-0.341
1.2	1.649	-1.105	1.552	-0.744	1.475	-0.574	1.360	-0.455	1.185	-0.340	1.086	-0.339
1.3	1.600	-0.845	1.510	-0.570	1.437	-0.469	1.331	-0.371	1.171	-0.338	1.081	-0.339
1.4	1.549	-0.585	1.465	-0.454	1.399	-0.403	1.301	-0.356	1.155	-0.338	1.076	-0.338
1.5	1.500	-0.359	1.422	-0.389	1.360	-0.384	1.269	-0.354	1.140	-0.337	1.071	-0.337
1.6	1.450	-0.540	1.377	-0.425	1.319	-0.387	1.237	-0.351	1.125	-0.337	1.067	-0.337
1.7	1.402	-0.604	1.334	-0.472	1.281	-0.396	1.205	-0.348	1.110	-0.337	1.062	-0.336
1.8	1.356	-0.709	1.292	-0.502	1.242	-0.400	1.175	-0.344	1.096	-0.336	1.057	-0.336
1.9	1.311	-0.835	1.252	-0.510	1.206	-0.397	1.145	-0.338	1.084	-0.336	1.053	-0.336

TABLE A.1e MAXIMUM AND MINIMUM VALUES OF ABSOLUTE ACCELERATION OF MASS, $\ddot{X}$ (CONTINUED)

$t_1 f$	$\beta = 0$		$\beta = 0.05$		$\beta = 0.10$		$\beta = 0.20$		$\beta = 0.50$		$\beta = 1.00$	
	$\frac{\ddot{x}_{max}}{\ddot{y}_o}$	$\frac{\ddot{x}_{min}}{\ddot{y}_o}$	$\frac{\ddot{x}_{max}}{\ddot{y}_o}$	$\frac{\ddot{x}_{min}}{\ddot{y}_o}$	$\frac{\ddot{x}_{max}}{\ddot{y}_o}$	$\frac{\ddot{x}_{min}}{\ddot{y}_o}$	$\frac{\ddot{x}_{max}}{\ddot{y}_o}$	$\frac{\ddot{x}_{min}}{\ddot{y}_o}$	$\frac{\ddot{x}_{max}}{\ddot{y}_o}$	$\frac{\ddot{x}_{min}}{\ddot{y}_o}$	$\frac{\ddot{x}_{max}}{\ddot{y}_o}$	$\frac{\ddot{x}_{min}}{\ddot{y}_o}$
2.0	1.268	-0.838	1.213	-0.499	1.171	-0.389	1.117	-0.337	1.073	-0.336	1.050	-0.336
2.1	1.227	-0.791	1.175	-0.470	1.137	-0.373	1.090	-0.339	1.063	-0.335	1.046	-0.335
2.2	1.189	-0.703	1.140	-0.437	1.105	-0.367	1.065	-0.339	1.055	-0.335	1.043	-0.335
2.3	1.151	-0.587	1.106	-0.408	1.074	-0.360	1.040	-0.338	1.048	-0.335	1.040	-0.335
2.4	1.117	-0.461	1.074	-0.377	1.044	-0.352	1.018	-0.336	1.042	-0.335	1.038	-0.335
2.5	1.083	-0.357	1.043	-0.361	1.016	-0.347	0.999	-0.334	1.038	-0.335	1.035	-0.335
3.0	1.170	-0.658	1.101	-0.391	1.064	-0.346	1.032	-0.335	1.026	-0.334	1.025	-0.334
3.5	1.166	-0.347	1.105	-0.350	1.070	-0.338	1.037	-0.334	1.020	-0.334	1.019	-0.334
4.0	1.126	-0.573	1.073	-0.356	1.044	-0.336	1.022	-0.334	1.016	-0.334	1.015	-0.334
4.5	1.071	-0.347	1.026	-0.342	1.005	-0.334	1.006	-0.334	1.012	-0.334	1.012	-0.334
5.0	1.099	-0.524	1.043	-0.345	1.020	-0.334	1.009	-0.334	1.010	-0.334	1.010	-0.334

(c) Ratio of $t_1 t_d = 1/8$

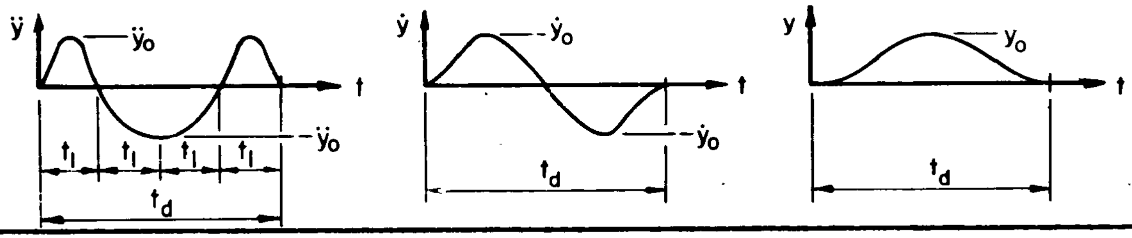
$t_1 f$	$\beta = 0$		$\beta = 0.05$		$\beta = 0.10$		$\beta = 0.20$		$\beta = 0.50$		$\beta = 1.00$	
	$\frac{\ddot{x}_{max}}{\ddot{y}_o}$	$\frac{\ddot{x}_{min}}{\ddot{y}_o}$	$\frac{\ddot{x}_{max}}{\ddot{y}_o}$	$\frac{\ddot{x}_{min}}{\ddot{y}_o}$	$\frac{\ddot{x}_{max}}{\ddot{y}_o}$	$\frac{\ddot{x}_{min}}{\ddot{y}_o}$	$\frac{\ddot{x}_{max}}{\ddot{y}_o}$	$\frac{\ddot{x}_{min}}{\ddot{y}_o}$	$\frac{\ddot{x}_{max}}{\ddot{y}_o}$	$\frac{\ddot{x}_{min}}{\ddot{y}_o}$	$\frac{\ddot{x}_{max}}{\ddot{y}_o}$	$\frac{\ddot{x}_{min}}{\ddot{y}_o}$
0.01	0.0100*	-0.0100*	--	--	--	--	0.0182	-0.0062*	--	--	0.0764	-0.0197
0.015	0.0224*	-0.0224*	--	--	--	--	0.0298	-0.0139*	--	--	0.112	-0.0363
0.025	0.0610*	-0.0610*	0.0502	-0.0526*	0.0510	-0.0463*	0.0578	-0.0378*	0.0986	-0.0333*	0.179	-0.0700
0.05	0.223*	-0.223*	0.164*	-0.192*	0.141	-0.169*	0.142	-0.138*	0.197	-0.117	0.323	-0.131
0.075	0.429*	-0.429*	0.315*	-0.369*	0.237	-0.323*	0.229	-0.262*	0.292	-0.190	0.440	-0.162
0.1	0.611*	-0.612	0.446*	-0.522	0.331*	-0.453*	0.332*	-0.358	0.381	-0.233	0.537	-0.175
0.125	0.717*	-0.739	0.513*	-0.624	0.421	-0.537	0.397	-0.415	0.464	-0.253	0.618	-0.179
0.15	0.735*	-0.832	0.536	-0.697	0.508	-0.594	0.478	-0.451	0.542	-0.261	0.687	-0.178
0.2	0.764	-0.981	0.712	-0.809	0.674	-0.679	0.631	-0.500	0.680	-0.263	0.793	-0.170
0.25	1.005*	-1.115	0.875	-0.911	0.828	-0.755	0.773	-0.542	0.796	-0.263	0.872	-0.160
0.3	1.125*	-1.242	1.025	-1.007	0.969	-0.829	0.904	-0.585	0.890	-0.268	0.930	-0.153
0.35	1.245	-1.394	1.160	-1.098	1.096	-0.899	1.022	-0.628	0.967	-0.275	0.973	-0.150

TABLE A.1e MAXIMUM AND MINIMUM VALUES OF ABSOLUTE ACCELERATION OF MASS, $\ddot{x}$ (CONTINUED)

$t_1 f$	$\beta = 0$		$\beta = 0.05$		$\beta = 0.10$		$\beta = 0.20$		$\beta = 0.50$		$\beta = 1.00$	
	$\frac{\ddot{x}_{max}}{\ddot{y}_o}$	$\frac{\ddot{x}_{min}}{\ddot{y}_o}$	$\frac{\ddot{x}_{max}}{\ddot{y}_o}$	$\frac{\ddot{x}_{min}}{\ddot{y}_o}$	$\frac{\ddot{x}_{max}}{\ddot{y}_o}$	$\frac{\ddot{x}_{min}}{\ddot{y}_o}$	$\frac{\ddot{x}_{max}}{\ddot{y}_o}$	$\frac{\ddot{x}_{min}}{\ddot{y}_o}$	$\frac{\ddot{x}_{max}}{\ddot{y}_o}$	$\frac{\ddot{x}_{min}}{\ddot{y}_o}$	$\frac{\ddot{x}_{max}}{\ddot{y}_o}$	$\frac{\ddot{x}_{min}}{\ddot{y}_o}$
0.4	1.408*	-1.526	1.279	-1.180	1.208	-0.963	1.121	-0.667	1.028	-0.283	1.007	-0.148
0.45	1.481	-1.632	1.380	-1.251	1.303	-1.018	1.201	-0.701	1.078	-0.292	1.031	-0.146
0.5	1.570	-1.712	1.462	-1.309	1.379	-1.064	1.264	-0.730	1.116	-0.299	1.050	-0.146
0.6	1.689	-1.819	1.573	-1.381	1.481	-1.121	1.352	-0.766	1.212	-0.310	1.074	-0.145
0.7	1.749	-1.850	1.630	-1.391	1.536	-1.127	1.400	-0.771	1.198	-0.312	1.087	-0.144
0.8	1.767	-1.788	1.650	-1.339	1.556	-1.087	1.419	-0.745	1.210	-0.307	1.093	-0.144
0.9	1.758	-1.659	1.645	-1.228	1.554	-1.000	1.419	-0.690	1.213	-0.294	1.095	-0.144
1.0	1.732	-1.466	1.623	-1.071	1.536	-0.874	1.407	-0.614	1.208	-0.277	1.093	-0.144
1.1	1.693	-1.223	1.591	-0.878	1.509	-0.724	1.387	-0.525	1.198	-0.258	1.090	-0.143
1.2	1.649	-0.953	1.551	-0.664	1.475	-0.559	1.360	-0.431	1.185	-0.237	1.086	-0.143
1.3	1.600	-0.670	1.510	-0.445	1.437	-0.399	1.331	-0.346	1.171	-0.219	1.081	-0.143
1.4	1.549	-0.399	1.465	-0.245	1.399	-0.267	1.301	-0.281	1.155	-0.201	1.076	-0.143
1.5	1.500	-0.151	1.422	-0.172	1.360	-0.201	1.269	-0.242	1.140	-0.187	1.071	-0.143
1.6	1.450	-0.353	1.377	-0.199	1.319	-0.214	1.237	-0.228	1.125	-0.175	1.067	-0.143
1.7	1.402	-0.515	1.334	-0.291	1.281	-0.255	1.205	-0.227	1.110	-0.164	1.062	-0.143
1.8	1.356	-0.624	1.292	-0.357	1.242	-0.289	1.175	-0.227	1.096	-0.155	1.057	-0.143
1.9	1.311	-0.674	1.252	-0.387	1.206	-0.303	1.145	-0.225	1.084	-0.147	1.053	-0.143
2.0	1.268	-0.670	1.213	-0.382	1.171	-0.298	1.171	-0.218	1.073	-0.143	1.050	-0.143
2.1	1.227	-0.619	1.176	-0.346	1.137	-0.274	1.090	-0.206	1.063	-0.143	1.046	-0.143
2.2	1.189	-0.528	1.168	-0.338	1.140	-0.286	1.105	-0.238	1.064	-0.191	--	--

TABLE A.2a MAXIMUM AND MINIMUM VALUES OF PSEUDO VELOCITY

Elastic Systems Subjected to a Sequence of Three Half-Sine Ground Acceleration Pulses as Shown



$t_1 f$	$\beta = 0$		$\beta = 0.05$		$\beta = 0.10$		$\beta = 0.20$		$\beta = 0.50$		$\beta = 1.00$	
	$\frac{pu_{max}}{\dot{y}_o}$	$\frac{pu_{min}}{\dot{y}_o}$	$\frac{pu_{max}}{\dot{y}_o}$	$\frac{pu_{min}}{\dot{y}_o}$	$\frac{pu_{max}}{\dot{y}_o}$	$\frac{pu_{min}}{\dot{y}_o}$	$\frac{pu_{max}}{\dot{y}_o}$	$\frac{pu_{min}}{\dot{y}_o}$	$\frac{pu_{max}}{\dot{y}_o}$	$\frac{pu_{min}}{\dot{y}_o}$	$\frac{pu_{max}}{\dot{y}_o}$	$\frac{pu_{min}}{\dot{y}_o}$
0.015	0.0191*	-0.1066	--	--	--	--	0.0156*	-0.0103	--	--	0.0286	-0.0908
0.025	0.0522*	-0.1763	0.0486*	-0.1738	0.0459*	-0.1714	0.0428*	-0.1667	0.0497	-0.1542	0.0672	-0.1369
0.05	0.204*	-0.340	0.190*	-0.331	0.179*	-0.323	0.167*	-0.3065	0.177	-0.2667	0.182	-0.2187
0.075	0.4457*	-0.483	0.415	-0.465	0.392*	-0.449	0.366*	-0.418	0.340	-0.347	0.289	-0.270
0.10	0.762*	-0.762*	0.710*	-0.606*	0.670*	-0.548	0.618	-0.504	0.504	-0.404	0.373	-0.302
0.125	1.131*	-1.131*	1.052	-0.900*	0.985	-0.726	0.874	-0.569	0.646	-0.444	0.433	-0.322
0.15	1.530*	-1.530*	1.387	-1.218*	1.279	-0.982*	1.100	-0.662*	0.756	-0.471	0.473	-0.334
0.20	2.312*	-2.312*	1.874	-1.840*	1.688	-1.485*	1.394	-1.002*	0.875	-0.498	0.509	-0.344
0.25	2.903*	-2.903*	2.053	-2.312*	1.826	-1.867*	1.476	-1.264*	0.892	-0.504	0.510	-0.344
0.30	3.151*	-3.151*	2.145*	-2.510*	1.777	-2.032*	1.424	-1.386*	0.854	-0.561*	0.495	-0.338
0.35	2.982*	-2.982*	2.034*	-2.381*	1.643	-1.937*	1.311	-1.346*	0.792	-0.586*	0.473	-0.330
0.40	2.426	-2.430	1.678	-1.948	1.474	-1.607	1.175	-1.167	0.721	-0.570	0.448	-0.319
0.45	1.714	-1.697	1.484	-1.358	1.301	-1.152	1.036	-0.921	0.651	-0.544	0.422	-0.309
0.50	1.508	-1.067	1.301	-0.796	1.137	-0.713	0.905	-0.703	0.585	-0.504	0.398	-0.297
0.60	1.156	-0.795*	0.989	-0.653	0.860	-0.609	0.682	-0.547	0.471	-0.434	0.354	-0.275
0.70	1.074*	-1.113	0.750	-0.807	0.648	-0.650	0.511	-0.514	0.386	-0.380	0.316	-0.255
0.80	0.675	-0.718	0.568	-0.529	0.488	-0.481	0.385	-0.427	0.325	-0.336	0.284	-0.236
0.90	0.576	-0.488	0.428	-0.455	0.366	-0.428	0.290	-0.382	0.285	-0.301	0.258	-0.219

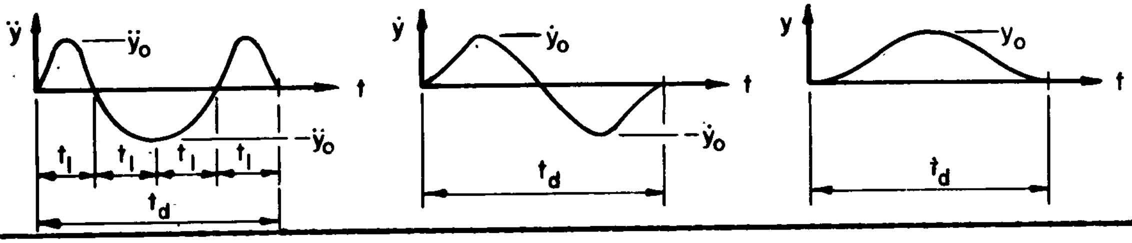
* Denotes extremum occurring during free vibration.

TABLE A.2a MAXIMUM AND MINIMUM VALUES OF PSEUDO VELOCITY (CONTINUED)

$t_1 f$	β = 0		β = 0.05		β = 0.10		β = 0.20		β = 0.50		β = 1.00	
	$\frac{pu_{max}}{\dot{y}_o}$	$\frac{pu_{min}}{\dot{y}_o}$	$\frac{pu_{max}}{\dot{y}_o}$	$\frac{pu_{min}}{\dot{y}_o}$	$\frac{pu_{max}}{\dot{y}_o}$	$\frac{pu_{min}}{\dot{y}_o}$	$\frac{pu_{max}}{\dot{y}_o}$	$\frac{pu_{min}}{\dot{y}_o}$	$\frac{pu_{max}}{\dot{y}_o}$	$\frac{pu_{min}}{\dot{y}_o}$	$\frac{pu_{max}}{\dot{y}_o}$	$\frac{pu_{min}}{\dot{y}_o}$
1.00	0.534*	-0.618	0.397	-0.437	0.332	-0.381	0.276	-0.342	0.251	-0.270	0.235	-0.205
1.10	0.435	-0.492	0.347	-0.373	0.298	-0.340	0.255	-0.307	0.232	-0.245	0.216	-0.191
1.20	0.353	-0.343	0.292	-0.323	0.259	-0.305	0.231	-0.277	0.213	-0.225	0.200	-0.179
1.30	0.277	-0.311	0.238	-0.290	0.221	-0.275	0.208	-0.251	0.196	-0.207	0.185	-0.169
1.40	0.210	-0.287	0.192	-0.261	0.190	-0.248	0.188	-0.228	0.182	-0.192	0.173	-0.159
1.50	0.195	-0.250	0.172	-0.237	0.173	-0.225	0.173	-0.208	0.169	-0.178	0.162	-0.151
1.60	0.195	-0.232	0.171	0.215	0.164	-0.205	0.160	-0.190	0.158	-0.166	0.153	-0.143
1.70	0.193	-0.211	0.165	-0.196	0.155	-0.187	0.149	-0.175	0.149	-0.155	0.144	-0.136
1.80	0.186	-0.188	0.156	-0.179	0.144	-0.172	0.139	-0.161	0.140	-0.146	0.136	-0.129
1.90	0.180	-0.196	0.146	-0.164	0.135	-0.158	0.133	-0.149	0.133	-0.137	0.129	-0.123
2.00	0.175	-0.193	0.142	-0.151	0.133	-0.146	0.127	-0.138	0.126	-0.130	0.123	-0.118
2.10	0.164	-0.146	0.135	-0.140	0.126	-0.135	0.121	-0.129	0.120	-0.123	0.117	-0.113
2.20	0.149	-0.140	0.127	-0.129	0.120	-0.125	0.116	-0.120	0.114	-0.117	0.112	-0.108
2.30	0.133	-0.139	0.117	-0.120	0.113	-0.116	0.111	-0.113	0.109	-0.111	0.107	-0.104
2.40	0.116	-0.116	0.107	-0.112	0.107	-0.108	0.106	-0.106	0.105	-0.106	0.103	-0.0998
2.50	0.110	-0.108	0.101	-0.104	0.102	-0.101	0.101	-0.102	0.101	-0.102	0.099	-0.0962
3.00	0.105	-0.112	0.0879	-0.0917	0.0846	-0.0885	0.0838	-0.0888	0.0836	-0.0844	0.0828	-0.0811
3.50	0.0765	-0.0833	0.0721	-0.0789	0.0721	-0.0763	0.0718	-0.0739	0.0716	-0.0722	0.0711	-0.0700
4.00	0.0744	-0.0784	0.0641	-0.0670	0.0629	-0.0652	0.0627	-0.0638	0.0626	-0.0630	0.0623	-0.0615
4.50	0.0586	-0.0595	0.0560	-0.570	0.0558	-0.0558	0.0557	-0.0560	0.0556	-0.0559	--	--

TABLE A.2b MAXIMUM AND MINIMUM VALUES OF RELATIVE VELOCITY, $\dot{u}$

Elastic Systems Subjected to a Sequence of Three Half-Sine Ground Acceleration Pulses as Shown



$t_1 f$	$\beta = 0$ $p\dot{u}_{max}/\ddot{y}_o$	$\beta = 0$ $p\dot{u}_{min}/\ddot{y}_o$	$\beta = 0.05$ $p\dot{u}_{max}/\ddot{y}_o$	$\beta = 0.05$ $p\dot{u}_{min}/\ddot{y}_o$	$\beta = 0.10$ $p\dot{u}_{max}/\ddot{y}_o$	$\beta = 0.10$ $p\dot{u}_{min}/\ddot{y}_o$	$\beta = 0.20$ $p\dot{u}_{max}/\ddot{y}_o$	$\beta = 0.20$ $p\dot{u}_{min}/\ddot{y}_o$	$\beta = 0.50$ $p\dot{u}_{max}/\ddot{y}_o$	$\beta = 0.50$ $p\dot{u}_{min}/\ddot{y}_o$	$\beta = 1.00$ $p\dot{u}_{max}/\ddot{y}_o$	$\beta = 1.00$ $p\dot{u}_{min}/\ddot{y}_o$
0.015	0.0610	-0.0599	--	--	--	--	0.0619	-0.0588	--	--	0.0634	-0.0547
0.025	0.1048	-0.0996	0.1053	-0.0988	0.1058	-0.0981	0.1066	-0.0966	0.1077	-0.0924	0.1063	-0.0861
0.05	0.236	-0.197	0.236	-0.194	0.236	-0.191	0.235	-0.186	0.226	-0.171	0.204	-0.150
0.075	0.409	-0.290	0.404	-0.284	0.398	-0.278	0.386	-0.266	0.345	-0.236	0.283	-0.199
0.10	0.624	-0.378	0.604	-0.367	0.585	-0.357	0.549	-0.338	0.455	-0.292	0.343	-0.236
0.125	0.863	-0.566*	0.820	-0.488*	0.781	-0.430*	0.710	-0.402	0.549	-0.337	0.386	-0.286
0.15	1.104	-0.918*	1.033	-0.792*	0.468	-0.697*	0.858	-0.570	0.627	-0.460	0.417	-0.356
0.20	1.849*	-1.849*	1.393	-1.596*	1.277	-1.405*	1.089	-1.151	0.732	-0.759	0.451	-0.461
0.25	2.903*	-2.903*	2.142*	-2.497	1.610*	-2.172	1.231	-1.692	0.785	-0.955	0.463	-0.522
0.30	3.781*	-3.780*	2.791*	-2.986	2.103*	-2.540	1.301	-1.910	0.801	-1.024	0.461	-0.550
0.35	4.175*	-4.175*	3.088	-2.888	2.339*	-2.428	1.425*	-1.808	0.796	-0.998	0.451	-0.557
0.40	3.881*	-3.881*	2.929*	-2.465*	2.216*	-2.005	1.412*	-1.498	0.778	-0.919	0.439	-0.550
0.45	2.875*	-2.874*	2.178*	-2.019	1.753*	-1.613	1.297	-1.116	0.755	-0.824	0.424	-0.536
0.50	2.095	-2.271	1.824	-1.729	1.602	-1.374	1.267	-0.883	0.730	-0.739	0.409	-0.519
0.60	2.009	-1.907*	1.739	-1.304	1.522	-0.952	1.195	-0.679	0.680	-0.622	0.380	-0.479
0.70	3.007*	-3.007*	2.003*	-1.711	1.495*	-1.029	1.114	-0.714	0.634	-0.552	0.354	-0.441
0.80	1.730	-1.744	1.492	-1.072	1.302	-0.751*	1.021	-0.624	0.591	-0.499	0.334	-0.407
0.90	1.545	-1.519	1.332	-0.918	1.163	-0.679	0.919	-0.593	0.548	-0.455	0.317	-0.377

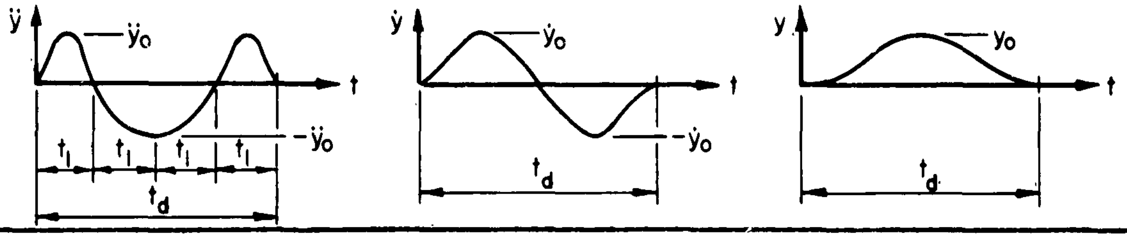
*Denotes extremum occurring during free vibration.

TABLE A.2b MAXIMUM AND MINIMUM VALUES OF RELATIVE VELOCITY, $\dot{u}$ (CONTINUED)

$t_1 f$	$\beta = 0$		$\beta = 0.05$		$\beta = 0.10$		$\beta = 0.20$		$\beta = 0.50$		$\beta = 1.00$	
	$\frac{p\dot{u}_{max}}{\dot{y}_o}$	$\frac{p\dot{u}_{min}}{\dot{y}_o}$	$\frac{p\dot{u}_{max}}{\dot{y}_o}$	$\frac{p\dot{u}_{min}}{\dot{y}_o}$	$\frac{p\dot{u}_{max}}{\dot{y}_o}$	$\frac{p\dot{u}_{min}}{\dot{y}_o}$	$\frac{p\dot{u}_{max}}{\dot{y}_o}$	$\frac{p\dot{u}_{min}}{\dot{y}_o}$	$\frac{p\dot{u}_{max}}{\dot{y}_o}$	$\frac{p\dot{u}_{min}}{\dot{y}_o}$	$\frac{p\dot{u}_{max}}{\dot{y}_o}$	$\frac{p\dot{u}_{min}}{\dot{y}_o}$
1.00	2.133*	-2.133*	1.298	-1.110*	1.007	-0.711*	0.805	-0.563	0.506	-0.419	0.303	-0.351
1.10	1.298	-1.301	0.966	-0.777	0.844	-0.610	0.686	-0.534	0.465	-0.390	0.292	-0.327
1.20	0.951	-0.871	0.808	-0.623	0.703	-0.579	0.572	-0.506	0.426	-0.367	0.281	-0.307
1.30	0.999	-0.889*	0.675	-0.591	0.583	-0.549	0.475	-0.480	0.391	-0.349	0.271	-0.290
1.40	0.688	-0.702	0.565	-0.561	0.482	-0.521	0.392	-0.457	0.360	-0.333	0.261	-0.274
1.50	0.577	-0.577	0.473	-0.534	0.398	-0.496	0.339	-0.435	0.333	-0.318	0.252	-0.260
1.60	0.564	-0.540	0.396	-0.508	0.327	-0.473	0.315	-0.414	0.312	-0.304	0.243	-0.247
1.70	0.623*	-0.664	0.340	-0.485	0.306	-0.451	0.304	-0.396	0.293	-0.291	0.234	-0.235
1.80	0.520	-0.528	0.398	-0.463	0.338	-0.431	0.297	-0.378	0.276	-0.278	0.226	-0.225
1.90	0.545	-0.543	0.418	-0.443	0.349	-0.412	0.293	-0.362	0.262	-0.267	0.218	-0.215
2.00	0.812	-0.813*	0.409	-0.424	0.341	-0.395	0.284	-0.347	0.250	-0.257	0.211	-0.206
2.10	0.608	-0.610	0.383	-0.408	0.320	-0.379	0.268	-0.333	0.238	-0.247	0.204	-0.198
2.20	0.470	-0.423	0.357	-0.392	0.297	-0.365	0.250	-0.320	0.227	-0.238	0.197	-0.191
2.30	0.511	-0.445*	0.331	-0.377	0.274	-0.351	0.232	-0.309	0.217	-0.229	0.191	-0.184
2.40	0.420	-0.424	0.305	-0.363	0.251	-0.338	0.215	-0.297	0.208	-0.221	0.184	-0.177
2.50	0.378	-0.378	0.280	-0.350	0.229	-0.326	0.203	-0.287	0.200	-0.214	0.179	-0.171
3.00	0.518	-0.517*	0.238	-0.277	0.197	-0.276	0.174	-0.243	0.167	-0.182	0.154	-0.147
3.50	0.278	-0.277	0.188	-0.257	0.154	-0.240	0.145	-0.211	0.143	-0.159	0.135	-0.128
4.00	0.382*	-0.382*	0.163	-0.226	0.137	-0.211	0.127	-0.186	0.125	-0.140	0.119	-0.114
4.50	0.219	-0.218	0.138	-0.202	0.116	-0.188	0.112	-0.166	0.111	-0.126	--	--

TABLE A.2c SPECTRAL VALUES OF ABSOLUTE DISPLACEMENT OF THE MASS, X

Elastic System Subjected to a Sequence of Three
Half-Sine Ground Acceleration Pulses as Shown



$t_1 f$	Value of $\dfrac{X}{p\left[\int y(\tau)\,d\tau\right]_o}$					
	β = 0	β = 0.05	β = 0.10	β = 0.20	β = 0.50	β = 1.00
0.015	1.00	--	--	0.812*	--	1.627
0.025	1.00	0.922*	0.871*	0.814*	0.964	1.452
0.05	0.990*	0.910*	0.860*	0.802*	0.894	1.138
0.075	0.957*	0.885*	0.837*	0.781*	0.815	0.929
0.10	0.918*	0.852*	0.805*	0.750	0.736	0.779
0.125	0.872*	0.810	0.765	0.710	0.664	0.667
0.15	0.818*	0.761	0.718	0.662	0.599	0.580
0.20	0.700	0.652	0.615	0.562	0.490	0.455
0.25	0.586	0.547	0.517	0.472	0.407	0.371
0.30	0.487	0.457	0.432	0.397	0.342	0.311
0.35	0.405	0.382	0.363	0.335	0.292	0.267
0.40	0.339	0.321	0.307	0.286	0.252	0.232
0.45	0.286	0.273	0.262	0.246	0.221	0.206
0.50	0.243	0.233	0.225	0.213	0.195	0.184
0.60	0.180	0.175	0.171	0.166	0.158	0.152
0.70	0.138	0.136	0.135	0.134	0.132	0.129
0.80	0.110	0.110	0.111	0.113	0.113	0.111
0.90	0.0975	0.0974	0.0979	0.0988	0.0994	0.0986
1.00	0.0903	0.0892	0.0888	0.0887	0.0887	0.0883
1.10	0.0822	0.0813	0.0808	0.0804	0.0802	0.0799
1.20	0.0744	0.0739	0.0737	0.0734	0.0732	0.0730
1.30	0.0675	0.0675	0.0675	0.0675	0.0674	0.0672
1.40	0.0621	0.0623	0.0624	0.0624	0.0624	0.0623
1.50	0.0581	0.0581	0.0581	0.0581	0.0581	0.0580
1.60	0.0545	0.0544	0.0544	0.0544	0.0544	0.0543
1.70	0.0511	0.0511	0.0511	0.0511	0.0511	0.0510
1.80	0.0480	0.0481	0.0482	0.0482	0.0482	0.0482
1.90	0.0455	0.0455	0.0456	0.0456	0.0456	0.0456

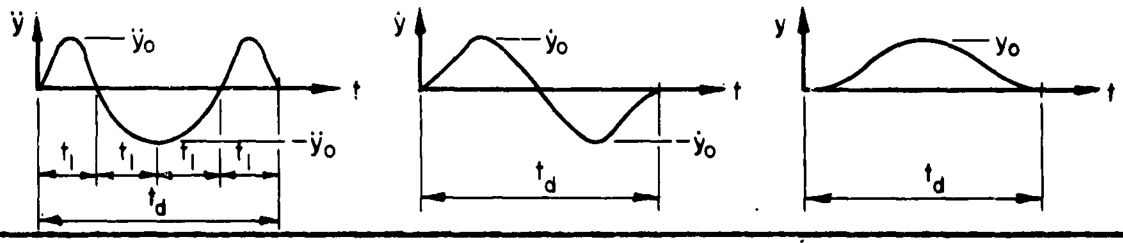
* Denotes extremum occurring during free vibration.

TABLE A.2c SPECTRAL VALUES OF ABSOLUTE DISPLACEMENT OF THE MASS, X (CONTINUED)

$t_1 f$	Value of $\frac{X}{p\left[\int y(\tau)\, d\tau\right]_o}$					
	$\beta = 0$	$\beta = 0.05$	$\beta = 0.10$	$\beta = 0.20$	$\beta = 0.50$	$\beta = 1.00$
2.00	0.0433	0.0433	0.0433	0.0433	0.0433	0.0433
2.10	0.0413	0.0412	0.0412	0.0412	0.0412	0.0412
2.20	0.0394	0.0393	0.0393	0.0393	0.0393	0.0393
2.30	0.0376	0.0376	0.0376	0.0376	0.0376	0.0375
2.40	0.0359	0.0360	0.0360	0.0360	0.0360	0.0360
2.50	0.0345	0.0345	0.0345	0.0345	0.0345	0.0345
3.00	0.0287	0.0287	0.0287	0.0287	0.0287	0.0287
3.50	0.0246	0.0246	0.0246	0.0246	0.0246	0.0246
4.00	0.0215	0.0215	0.0215	0.0215	0.0215	0.0215
4.50	0.0191	0.0191	0.0191	0.0191	0.0191	

TABLE A.2d SPECTRAL VALUES OF ABSOLUTE VELOCITY OF THE MASS, $\dot{x}$

Elastic Systems Subjected to a Sequence of Three Half-Sine Ground Acceleration Pulses as Shown



$t_1 f$	Value of $\frac{\dot{x}}{p y_o}$					
	β = 0	β = 0.05	β = 0.10	β = 0.20	β = 0.50	β = 1.00
0.015	0.183*	--	--	0.474	--	1.765
0.025	0.295*	0.302	0.366	0.524	0.988	1.637
0.05	0.572*	0.508	0.534	0.632	0.958	1.379
0.075	0.833*	0.719*	0.660	0.707	0.915	1.187
0.10	1.067*	0.921*	0.810*	0.749	0.867	1.038
0.125	1.268*	1.094*	0.963*	0.787*	0.817	0.919
0.15	1.429*	1.233*	1.085*	0.888*	0.768	0.839
0.20	1.618*	1.397*	1.230*	1.007	0.783	0.724
0.25	1.626	1.404	1.237	1.010	0.732	0.619
0.30	1.475	1.274	1.123	0.916	0.648	0.531
0.35	1.235	1.070	0.945	0.776	0.557	0.459
0.40	0.981	0.853	0.758	0.632	0.475	0.401
0.45	0.755	0.658	0.589	0.503	0.406	0.355
0.50	0.567	0.496	0.448	0.397	0.350	0.316
0.60	0.394	0.370	0.351	0.323	0.280	0.259
0.70	0.322	0.304	0.290	0.269	0.237	0.218
0.80	0.270	0.253	0.243	0.227	0.204	0.189
0.90	0.222	0.212	0.205	0.194	0.178	0.167
1.00	0.187	0.180	0.175	0.168	0.157	0.149
1.10	0.159	0.155	0.151	0.147	0.140	0.135
1.20	0.138	0.134	0.132	0.130	0.127	0.123
1.30	0.119	0.118	0.117	0.117	0.116	0.113
1.40	0.104	0.105	0.105	0.106	0.106	0.104
1.50	0.0987	0.0962	0.0969	0.0982	0.0985	0.0971
1.60	0.0902	0.0903	0.0908	0.0915	0.0917	0.0908
1.70	0.0869	0.0860	0.0858	0.0859	0.0859	0.0852
1.80	0.0834	0.0818	0.0813	0.0810	0.0808	0.0802
1.90	0.0789	0.0777	0.0770	0.0765	0.0762	0.0758

* Denotes extremum occurring during free vibration.

TABLE A.2d SPECTRAL VALUES OF ABSOLUTE VELOCITY OF THE MASS, $\dot{x}$ (CONTINUED)

$t_1 f$	Value of $\frac{\dot{x}}{p y_o}$					
	$\beta = 0$	$\beta = 0.05$	$\beta = 0.10$	$\beta = 0.20$	$\beta = 0.50$	$\beta = 1.00$
2.00	0.0747	0.0736	0.0730	0.0725	0.0722	0.0718
2.10	0.0705	0.0696	0.0692	0.0688	0.0686	0.0683
2.20	0.0667	0.0659	0.0656	0.0654	0.0653	0.0651
2.30	0.0626	0.0624	0.0624	0.0624	0.0623	0.0621
2.40	0.0591	0.0593	0.0595	0.0596	0.0596	0.0595
2.50	0.0571	0.0566	0.0569	0.0571	0.0571	0.0570
3.00	0.0480	0.0476	0.0474	0.0474	0.0473	0.0473
3.50	0.0404	0.0403	0.0404	0.0404	0.0404	0.0404
4.00	0.0356	0.0354	0.0353	0.0353	0.0353	0.0353
4.50	0.0313	0.0313	0.0313	0.0313	0.0313	0.0313

TABLE A.3a VALUES OF ABSOLUTE MAXIMUM DEFORMATIONS, u_o, AND THE ASSOCIATED TIMES, t_o

Elastic Systems, Eureka Earthquake

y_o = 10 in., $\dot{y}_o$ = 12.5 in./sec., $\ddot{y}_o$ = 0.178 g, Duration of Quake = 20 sec.

T sec.	f cps	β = 0 t_o sec.	β = 0 u_o in.	β = 0.02 t_o sec.	β = 0.02 u_o in.	β = 0.05 t_o sec.	β = 0.05 u_o in.	β = 0.10 t_o sec.	β = 0.10 u_o in.	β = 0.20 t_o sec.	β = 0.20 u_o in.	β = 0.40 t_o sec.	β = 0.40 u_o in.
25	0.04	4.80	9.666	4.80	9.563	4.80	9.410	4.80	9.162	4.80	8.687	4.80	7.817
15	0.067	16.62	9.031	4.80	8.861	4.80	8.620	4.80	8.233	4.70	7.539	3.98	6.541
10	0.1	18.00	-11.33	18.06	-9.616	11.82	8.562	6.72	-7.932	6.62	-7.197	6.22	-6.327
7	0.14	6.58	-12.32	6.32	-11.83	6.22	-11.29	6.22	-10.49	6.20	-9.055	6.18	-6.803
5	0.2	15.92	-15.60	5.88	-12.81	5.84	-12.04	5.84	-10.90	5.76	-9.027	5.76	-6.572
4	0.25	7.56	14.61	7.56	13.24	7.52	11.48	5.64	-9.800	5.60	-7.971	5.56	-5.675
3	0.33	19.00	14.22	8.28	-10.75	6.78	9.195	6.78	7.417	5.34	-5.243	5.34	-3.809
2.5	0.4	18.84	10.67	6.42	7.112	6.44	6.266	6.44	5.195	6.44	3.781	5.22	-2.753
2	0.5	4.44	-5.310	4.38	-4.986	4.38	-4.550	4.38	-3.933	4.38	-2.995	3.54	1.986
1.5	0.67	11.49	-6.490	6.27	4.433	6.22	3.509	4.22	-3.055	4.20	-2.389	4.20	-1.562
1.25	0.8	5.94	6.416	5.91	5.438	5.91	4.315	5.22	-3.293	5.19	-2.270	5.13	-1.474
1	1	12.57	3.549	5.07	-3.306	5.07	-3.019	5.07	-2.584	5.08	-1.948	5.08	-1.281
0.7	1.43	5.97	1.833	5.97	1.444	5.96	1.083	4.97	-0.877	5.01	-0.790	5.03	-0.691
0.5	2	6.84	1.640	6.59	-1.317	6.58	-1.036	6.57	-0.749	6.33	0.471	5.03	-0.371
0.4	2.5	15.94	2.275	7.74	-1.024	6.73	0.695	6.72	0.507	5.01	-0.348	5.00	-0.275
0.3	3.33	6.24	0.665	6.23	0.481	4.94	-0.380	4.94	-0.329	4.95	-0.245	4.96	-0.173
0.25	4	8.63	0.431	6.64	0.2907	6.00	-0.248	6.01	-0.209	4.91	-0.156	4.94	-0.119
0.2	5	6.12	0.262	6.01	-0.1509	5.99	-0.114	5.99	-0.101	5.99	-0.0866	4.92	-0.0718
0.15	6.67	5.85	0.1734	3.83	-0.0837	3.84	-0.0658	3.84	-0.0477	5.97	-0.0418	4.91	-0.0393
0.125	8	5.86	0.0689	6.46	-0.0466	6.47	-0.0404	6.47	-0.0347	6.47	-0.0296	4.91	-0.0274
0.1	10	3.64	-0.0302	5.85	0.0225	6.46	-0.0213	6.46	-0.0203	4.90	-0.0184	4.90	-0.0176
0.05	20	4.88	-0.0087	4.89	-0.0049	4.89	-0.0050	4.89	-0.0048	4.89	-0.0047	4.89	-0.0046

TABLE A.3b VALUES OF ABSOLUTE MAXIMUM RELATIVE VELOCITIES, $\dot{u}_o$, AND THE ASSOCIATED TIMES, t_o

Elastic Systems, Eureka Earthquake

y_o = 10 in., $\dot{y}_o$ = 12.5 in./sec., $\ddot{y}_o$ = 0.178 g, Duration of Quake = 20 sec.

T	f	β = 0		β = 0.02		β = 0.05		β = 0.10		β = 0.20		β = 0.40	
sec.	cps	t_o sec.	$\dot{u}_o$ in./sec	t_o sec.	$\dot{u}_o$ in./sec	t_o sec.	$\dot{u}_o$ in./sec	t_o sec.	$\dot{u}_o$ in./sec	t_o sec.	$\dot{u}_o$ in./sec	t_o sec.	$\dot{u}_o$ in./sec
25	0.04	3.42	12.03	3.42	12.00	3.42	11.95	3.42	11.87	3.42	11.71	3.42	11.39
15	0.067	3.42	12.02	3.42	11.96	3.42	11.88	3.42	11.74	5.00	-11.80	5.00	-12.20
10	0.1	5.40	-13.51	5.40	-13.41	5.00	-13.48	5.00	-13.58	5.00	-13.64	5.00	-13.41
7	0.14	5.00	-16.73	5.00	-16.63	5.00	-16.46	5.00	-16.14	5.00	-15.45	5.00	-14.04
5	0.2	5.04	-20.16	5.04	-19.65	5.04	-18.94	5.04	-17.83	5.04	-15.93	5.04	-13.15
4	0.25	8.52	-23.78	6.38	22.64	6.38	21.07	6.38	18.72	6.32	15.04	5.04	-11.276
3	0.33	19.72	-29.88	9.06	23.45	5.88	19.86	5.88	17.46	5.88	14.03	5.88	10.24
2.5	0.4	19.44	-27.58	13.14	17.49	5.88	15.96	5.88	14.16	4.02	-11.92	4.02	-9.453
2	0.5	4.02	-17.39	4.02	-16.63	4.02	-15.58	4.02	-14.06	4.02	-11.66	4.02	-8.478
1.5	0.67	11.85	26.49	5.88	21.33	4.50	17.86	4.50	15.52	4.50	11.92	4.98	-8.261
1.25	0.8	6.18	-30.39	5.01	-25.71	4.98	-22.38	4.98	-18.17	4.98	-12.90	4.96	-8.011
1	1	6.30	23.20	5.28	20.78	5.28	18.06	5.28	14.41	5.28	9.637	4.92	-5.879
0.7	1.43	6.12	-16.41	6.11	-12.89	6.09	-9.700	5.15	7.994	5.15	6.455	5.15	4.648
0.5	2	6.71	20.99	6.71	17.28	6.69	13.45	6.47	-9.683	6.47	-6.608	6.47	-4.023
0.4	2.5	16.04	-35.57	7.83	15.83	6.63	10.85	6.62	8.153	6.62	5.384	5.10	3.379
0.3	3.33	6.32	-13.11	6.31	-8.782	6.14	6.475	6.13	5.195	5.97	-3.973	5.96	-2.615
0.25	4	10.45	-10.54	6.70	-6.648	6.06	5.42	5.94	-4.369	5.95	-3.261	5.95	-2.183
0.2	5	6.06	7.920	6.06	4.246	6.05	2.873	6.04	2.257	5.94	-1.943	5.94	-1.513
0.15	6.67	5.89	-6.841	3.79	-3.019	3.79	-2.330	3.80	-1.607	6.52	1.053	5.92	-0.838
0.125	8	5.45	3.192	5.46	2.026	3.69	1.539	3.69	1.169	6.50	0.758	5.91	-0.579
0.1	10	3.67	1.611	3.67	1.035	3.67	0.896	3.67	0.725	5.89	-0.491	3.56	0.376
0.05	20	6.42	-0.548	5.86	-0.191	5.87	-0.174	3.55	0.163	3.55	0.146	3.55	0.110

TABLE A.3c VALUES OF ABSOLUTE MAXIMUM RELATIVE ACCELERATIONS, $\ddot{u}_o$, AND THE ASSOCIATED TIMES, t_o

Elastic Systems, Eureka Earthquake

y_o = 10 in., $\dot{y}_o$ = 12.5 in./sec., $\ddot{y}_o$ = 0.178 g, Duration of Quake = 20 sec.

T	f	β = 0		β = 0.02		β = 0.05		β = 0.10		β = 0.20		β = 0.40	
sec.	cps	t_o sec.	$\ddot{u}_o$ in./sec.^{2}	t_o sec.	$\ddot{u}_o$ in./sec.^{2}	t_o sec.	$\ddot{u}_o$ in./sec.^{2}	t_o sec.	$\ddot{u}_o$ in./sec.^{2}	t_o sec.	$\ddot{u}_o$ in./sec.^{2}	t_o sec.	$\ddot{u}_o$ in./sec.^{2}
25	0.04	4.88	-69.47	4.88	-69.43	4.88	-69.37	4.88	-69.25	4.88	-69.00	4.88	-68.42
15	0.067	4.88	-70.42	4.88	-70.31	4.88	-70.15	4.88	-69.86	4.88	-69.24	4.88	-67.91
10	0.10	4.88	-71.78	4.88	-71.52	4.88	-71.13	4.88	-70.47	4.88	-69.17	4.88	-66.71
7	0.14	4.88	-72.66	4.88	-72.11	4.88	-71.31	4.88	-70.05	4.88	-67.79	4.88	-64.21
5	0.2	5.84	79.01	5.84	78.24	5.84	77.04	5.84	74.98	5.84	70.87	5.84	63.68
4	0.25	5.84	87.53	5.84	85.37	5.84	82.34	5.84	77.81	5.84	70.45	6.44	-68.55
3	0.33	6.44	-98.04	6.44	-95.95	6.44	-93.07	6.44	-88.90	6.44	-82.72	6.44	-75.56
2.5	0.4	6.44	-107.8	6.44	-103.5	6.44	-98.18	6.44	-91.80	6.44	-84.06	6.44	-75.93
2	0.5	4.44	99.47	4.44	95.53	4.44	89.95	4.44	81.69	6.44	-78.14	6.44	-73.01
1.5	0.67	4.88	-134.1	4.88	-126.3	4.88	-115.6	4.88	-100.3	5.94	-87.79	5.94	-77.14
1.25	0.8	5.94	-212.3	5.94	-186.6	5.94	-157.2	5.94	-124.9	5.08	95.73	5.08	80.51
1	1	6.44	-172.6	5.08	161.0	5.08	149.6	5.08	133.0	5.08	108.8	5.08	82.06
0.7	1.43	5.94	-191.6	5.94	-164.4	5.94	-137.9	5.94	-112.4	5.94	-87.95	5.94	-69.85
0.5	2	6.59	283.4	6.58	242.7	6.58	197.4	6.58	150.6	6.58	103.2	6.57	62.64
0.4	2.5	16.14	559.1	7.74	250.9	6.72	-174.3	6.72	-127.5	6.54	81.77	5.94	-57.27
0.3	3.33	6.38	263.8	6.38	177.0	6.06	126.5	6.05	106.4	6.03	77.28	5.90	-50.09
0.25	4.	9.38	-265.2	6.64	-160.5	6.00	131.0	6.01	108.3	6.01	74.08	3.56	49.78
0.2	5	6.02	238.1	6.01	125.2	6.00	83.94	3.56	69.82	3.56	61.21	3.56	45.16
0.15	6.67	15.54	284.4	3.75	-117.9	3.75	-92.66	3.75	-65.39	3.55	44.90	3.55	33.20
0.125	8	5.42	149.0	5.42	93.40	3.66	70.84	3.66	55.25	3.60	-37.61	3.54	25.98
0.1	10	3.64	92.05	3.64	57.37	3.64	50.48	3.64	42.49	3.59	-30.32	3.59	-19.91
0.05	20	4.91	-62.64	4.07	-20.53	3.57	-14.08	3.57	-13.85	3.53	12.46	3.53	10.23

TABLE A.3d VALUES OF ABSOLUTE MAXIMUM DISPLACEMENTS, x_o, AND THE ASSOCIATED TIMES, t_o

Elastic Systems, Eureka Earthquake

y_o = 10 in., $\dot{y}_o$ = 12.5 in./sec., $\ddot{y}_o$ = 0.178 g, Duration of Quake = 20 sec.

T	f	β = 0		β = 0.02		β = 0.05		β = 0.10		β = 0.20		β = 0.40	
sec.	cps	t_o sec.	x_o in.	t_o sec.	x_o in.	t_o sec.	x_o in.	t_o sec.	x_o in.	t_o sec.	x_o in.	t_o sec.	x_o in.
25	0.04	11.46	-5.259			10.80	-4.910	10.20	-4.659	9.16	-4.361	7.88	-4.291
15	0.067	8.80	-8.201	8.64	-7.955	8.48	-7.635	8.20	-7.201	7.68	-6.599	6.22	-6.217
10	0.1	17.78	-14.78	17.66	-13.10	17.60	-11.05	6.92	-9.653	6.44	-8.823	5.80	-8.349
7	0.14	6.44	-14.07	6.38	-13.62	6.32	-13.03	6.18	-12.24	5.94	-11.16	5.52	-10.26
5	0.2	15.92	-17.50	5.76	-15.86	5.76	-15.19	5.64	-14.27	5.52	-12.97	5.28	-11.61
4	0.25	5.52	-16.88	5.52	-16.35	5.40	-15.73	5.40	-14.81	5.28	-13.50	5.16	-12.01
3	0.33	17.50	-17.29	5.08	-15.64	5.08	-15.09	5.04	-14.31	5.04	-13.15	4.98	-11.83
2.5	0.4	4.80	-15.01	4.80	-14.63	4.80	-14.12	4.80	-13.39	4.86	-12.34	4.92	-11.28
2	0.5	4.50	-13.88	4.50	-13.50	4.50	-12.99	4.50	-12.29	4.44	-11.28	4.92	-10.41
1.5	0.67	4.23	-11.89	4.23	-11.69	4.23	-11.41	4.22	-10.98	4.22	-10.31	4.98	-9.996
1.25	0.8	5.22	-12.89	5.20	-12.36	5.16	-11.73	5.13	-11.06	5.07	-10.45	4.98	-10.24
1	1	4.98	-12.65	4.98	-12.39	4.98	-12.04	4.98	-11.58	4.98	-10.97	4.95	-10.47
0.7	1.43	4.88	-11.28	4.88	-11.14	4.86	-10.99	4.86	-10.83	4.86	-10.63	4.88	-10.40
0.5	2	4.73	-10.67	4.73	-10.56	4.74	-10.44	4.76	-10.32	4.77	-10.22	4.85	-10.18
0.4	2.5	4.86	-10.53	4.86	-10.28	4.82	-10.12	4.76	-10.05	4.87	-10.04	4.86	-10.10
0.3	3.33	4.90	-10.41	4.90	-10.33	4.90	-10.26	4.90	-10.20	4.89	-10.13	4.87	-10.10
0.25	4	4.86	-10.29	4.87	-10.29	4.87	-10.25	4.87	-10.20	4.86	-10.15	4.86	-10.10
0.2	5	4.83	-10.24	4.83	-10.18	4.83	-10.15	4.84	-10.13	4.85	-10.11	4.85	-10.09
0.15	6.67	4.87	-10.13	4.86	-10.08	4.85	-10.07	4.84	-10.08	4.84	-10.08	4.85	-10.07
0.125	8	4.81	-10.04	4.83	-10.06	4.84	-10.06	4.84	-10.07	4.84	-10.07	4.84	-10.07
0.1	10	4.84	-10.07	4.84	-10.06	4.84	-10.06	4.84	-10.06	4.84	-10.06	4.84	-10.06
0.05	20	4.84	-10.05	4.84	-10.05	4.84	-10.05	4.84	-10.05	4.84	-10.05	4.84	-10.05

TABLE A.3e VALUES OF ABSOLUTE MAXIMUM VELOCITIES, $\dot{x}_o$, AND THE ASSOCIATED TIMES, t_o

Elastic Systems, Eureka Earthquake

y_o = 10 in., $\dot{y}_o$ = 12.5 in./sec., $\ddot{y}_o$ = 0.178 g, Duration of Quake = 20 sec.

T sec.	f cps	β = 0 t_o sec.	β = 0 $\dot{x}_o$ in./sec	β = 0.02 t_o sec.	β = 0.02 $\dot{x}_o$ in./sec	β = 0.05 t_o sec.	β = 0.05 $\dot{x}_o$ in./sec	β = 0.10 t_o sec.	β = 0.10 $\dot{x}_o$ in./sec	β = 0.20 t_o sec.	β = 0.20 $\dot{x}_o$ in./sec	β = 0.40 t_o sec.	β = 0.40 $\dot{x}_o$ in./sec
25	0.04	6.44	-1.057	6.00	-1.045	5.52	-1.047	5.00	-1.151	4.92	-1.511	4.80	-2.146
15	0.067	19.60	-4.194	19.48	-3.792	19.20	-3.288	18.80	-2.636	4.92	-2.926	4.80	-3.560
10	0.1	15.10	-7.846	15.00	-7.003	14.96	-5.958	9.38	5.046	4.88	-4.815	4.80	-5.058
7	0.14	11.82	-11.59	8.20	10.50	8.08	9.514	7.88	8.175	4.80	-6.884	4.70	-6.353
5	0.2	14.64	-19.20	7.06	17.13	6.96	15.60	6.84	13.55	6.62	10.84	6.22	8.602
4	0.25	6.58	22.94	6.48	21.46	6.48	19.60	6.38	17.10	6.24	13.81	6.12	10.28
3	0.33	19.74	-29.77	6.00	23.20	5.94	21.26	5.94	18.66	5.84	15.30	5.76	11.97
2.5	0.4	19.44	-27.17	5.70	20.28	5.70	18.65	5.70	16.62	5.70	14.15	3.84	-12.13
2	0.5	3.98	-18.71	3.96	-18.07	3.96	-17.20	3.90	-16.14	3.84	-14.61	3.78	-12.89
1.5	0.67	11.88	27.85	5.84	22.91	5.79	20.02	5.73	16.90	3.72	-15.16	3.63	-13.75
1.25	0.8	5.58	36.20	5.58	32.22	5.56	27.58	5.52	22.23	5.49	16.49	3.60	-14.00
1	1	5.28	30.12	5.28	28.08	5.28	25.36	5.28	21.71	5.28	16.93	3.56	-13.89
0.7	1.43	5.13	20.49	5.13	18.71	5.13	17.07	5.15	15.61	5.15	14.07	3.54	-13.30
0.5	2	6.96	-20.71	6.71	16.39	3.59	-13.99	3.59	-13.71	3.56	-13.46	3.53	-13.42
0.4	2.5	16.44	-36.66	3.57	-17.21	3.57	-16.09	3.55	-15.20	3.54	-14.40	3.51	-13.81
0.3	3.33	5.01	17.09	5.02	15.61	3.51	-15.58	3.51	-15.45	3.50	-14.78	3.50	-14.01
0.25	4	3.47	-16.72	3.48	-15.99	3.48	-15.37	3.49	-14.85	3.49	-14.35	3.49	-13.90
0.2	5	5.47	-16.09	3.51	-13.23	3.50	-13.44	3.50	-13.65	3.49	-13.79	3.48	-13.69
0.15	6.67	3.47	-15.15	3.48	-14.32	3.48	-14.11	3.48	-13.94	3.48	-13.72	3.48	-13.52
0.125	8	3.46	-13.81	3.48	-13.41	3.47	-13.50	3.47	-13.56	3.47	-13.53	3.47	-13.41
0.1	10	3.45	-13.04	3.47	-13.46	3.47	-13.47	3.47	-13.44	3.47	-13.38	3.47	-13.29
0.05	20	3.47	-13.14	3.46	-13.03	3.46	-13.04	3.46	-13.04	3.46	-13.05	3.47	-13.06

TABLE A.3f VALUES OF ABSOLUTE MAXIMUM ACCELERATIONS, $\ddot{x}_o$, AND THE ASSOCIATED TIMES, t_o

Elastic Systems, Eureka Earthquake

y_o = 10 in., $\dot{y}_o$ = 12.5 in./sec., $\ddot{y}_o$ = 0.178 g, Duration of Quake = 20 sec.

T	f	β = 0		β = 0.02		β = 0.05		β = 0.10		β = 0.20		β = 0.40	
sec.	cps	t_o sec.	$\ddot{x}_o$ in./sec.^{2}	t_o sec.	$\ddot{x}_o$ in./sec.^{2}	t_o sec.	$\ddot{x}_o$ in./sec.^{2}	t_o sec.	$\ddot{x}_o$ in./sec.^{2}	t_o sec.	$\ddot{x}_o$ in./sec.^{2}	t_o sec.	$\ddot{x}_o$ in./sec.^{2}
25	0.04	4.80	-0.611	4.56	-0.617	4.56	-0.680	3.52	-0.841	3.52	-1.388	3.42	-2.480
15	0.067	16.62	-1.584	4.56	-1.564	4.56	-1.622	3.80	-1.758	3.52	-2.553	3.42	-4.192
10	0.1	18.00	4.471	11.82	-3.806	6.58	3.424	6.58	3.439	3.52	-4.254	3.42	-6.370
7	0.14	6.58	9.922	6.22	9.585	6.18	9.354	6.18	8.882	5.52	8.672	5.52	10.34
5	0.2	15.92	24.63	5.84	20.29	5.76	19.27	5.76	17.94	5.52	16.67	5.04	16.78
4	0.25	7.56	-36.05	7.52	-32.70	7.44	-28.40	5.52	25.10	5.40	21.53	5.04	21.63
3	0.33	19.00	-62.37	8.28	47.20	6.74	-40.57	6.72	-33.21	5.04	25.94	5.04	26.15
2.5	0.4	18.84	-67.42	6.42	-44.97	6.42	-39.84	6.38	-33.90	6.38	-27.31	5.04	27.03
2	0.5	4.44	52.41	4.38	49.46	4.38	45.21	4.32	40.00	4.24	33.44	5.04	28.58
1.5	0.67	11.49	113.9	6.24	-77.76	6.20	-61.66	4.17	54.67	4.11	45.32	5.04	41.47
1.25	0.8	5.94	-162.11	5.91	-137.8	5.91	-109.7	5.19	85.00	5.08	63.36	5.01	55.11
1	1	12.57	-140.1	5.04	130.72	5.04	119.7	5.04	104.3	5.04	83.37	5.01	66.49
0.7	1.43	5.97	-147.7	5.96	-116.6	5.96	-87.73	4.95	70.98	4.98	66.03	4.98	65.55
0.5	2	6.84	-259.0	6.58	208.3	6.57	164.7	6.56	121.2	6.54	81.33	4.98	68.18
0.4	2.5	15.94	-561.3	7.73	252.9	6.72	-172.3	6.71	-127.5	4.99	90.56	4.97	77.32
0.3	3.33	6.24	-291.8	6.23	-210.8	4.93	167.4	4.93	146.3	4.93	112.0	4.92	84.16
0.25	4	8.63	-272.0	6.64	-183.25	6.00	157.4	6.00	134.4	4.90	101.3	4.90	82.11
0.2	5	6.12	-258.8	6.00	148.5	5.99	113.1	5.98	101.3	5.98	89.98	4.90	74.82
0.15	6.67	5.85	-304.2	3.83	147.0	3.83	116.0	3.83	84.61	6.47	76.34	4.90	71.83
0.125	8	5.86	-174.2	6.46	117.8	6.47	102.1	6.47	88.64	6.46	77.68	4.90	71.79
0.1	10	3.64	119.4	5.85	-88.98	6.46	84.20	6.46	80.94	6.45	74.57	4.89	71.47
0.05	20	4.88	130.0	4.88	76.89	4.89	74.69	4.89	71.70	4.89	70.67	4.89	70.09

TABLE A.4a VALUES OF ABSOLUTE MAXIMUM DEFORMATIONS, u_o, AND THE ASSOCIATED TIMES, t_o

Elastic Systems, El Centro Earthquake

y_o = 8.28 in., $\dot{y}_o$ = 13.68 in./sec., $\ddot{y}_o$ = 0.32 g, Duration of Quake = 29.5 sec.

T sec.	f cps	β = 0 t_o sec.	β = 0 u_o in.	β = 0.02 t_o sec.	β = 0.02 u_o in.	β = 0.05 t_o sec.	β = 0.05 u_o in.	β = 0.10 t_o sec.	β = 0.10 u_o in.	β = 0.20 t_o sec.	β = 0.20 u_o in.	β = 0.40 t_o sec.	β = 0.40 u_o in.
25	0.04	11.00	11.12	10.92	11.03	10.92	10.89	10.86	10.63	10.86	10.08	10.86	8.885
15	0.067	10.80	13.37	10.80	12.87	10.80	12.16	10.80	11.09	10.80	9.309	10.56	6.945
10	0.10	28.40	20.83	28.23	17.09	13.11	-14.30	13.11	-12.54	13.11	-9.763	13.00	-6.448
7	0.14	25.40	-20.41	18.69	-15.82	18.69	-12.41	11.82	-9.522	4.13	-7.224	4.01	-6.113
5	0.2	28.68	-15.93	28.56	-12.02	4.08	-10.46	4.01	-9.563	4.01	-8.058	3.96	-5.810
4	0.25	27.84	-16.74	5.40	13.91	5.34	12.70	5.28	11.00	5.22	8.541	5.16	5.563
3	0.33	27.20	-27.82	13.62	18.04	6.20	-13.70	4.86	11.05	4.72	7.818	3.24	-5.094
2.5	0.4	7.01	19.00	7.01	15.89	5.80	-13.37	5.80	-10.39	5.80	-6.918	3.18	-4.267
2	0.5	12.33	9.878	12.33	7.486	6.54	5.627	6.56	5.079	5.70	-4.408	5.71	-3.224
1.5	0.67	28.89	9.237	6.30	5.166	6.30	4.654	6.30	3.903	6.27	2.788	5.64	-2.162
1.25	0.8	29.16	-7.412	6.15	5.179	6.15	4.166	6.15	3.173	6.15	2.216	4.53	1.621
1	1	4.98	-7.424	4.95	-5.968	4.50	4.657	4.50	3.281	4.50	2.026	2.07	-1.400
0.7	1.43	10.08	5.932	5.87	3.320	2.33	2.464	2.31	2.078	2.03	-1.537	2.03	-1.054
0.5	2	9.12	2.986	2.43	-2.242	2.43	-1.919	2.45	-1.484	2.22	1.118	2.22	0.698
0.4	2.5	26.26	3.140	5.17	-1.470	2.78	-1.184	2.39	-0.948	2.40	-0.714	2.20	0.487
0.3	3.33	28.21	0.990	2.64	-0.752	2.65	-0.723	2.65	-0.601	2.51	0.392	2.17	0.282
0.25	4	22.84	-1.258	2.60	-0.746	2.61	-0.588	2.61	-0.458	2.62	-0.316	2.16	0.200
0.2	5	26.34	0.854	2.78	-0.423	2.78	-0.344	2.58	-0.264	2.59	-0.199	2.14	0.130
0.15	6.67	19.80	0.652	4.95	0.262	4.96	0.186	4.96	0.1401	4.97	0.0983	2.46	0.0718
0.125	8	5.24	-0.2029	4.94	0.114	4.94	0.0964	4.94	0.0801	4.95	0.0625	2.45	0.0487
0.1	10	4.04	0.0958	4.79	0.0575	4.79	0.0545	4.79	0.0463	4.79	0.0363	2.13	0.0315
0.05	20	24.62	-0.0225	9.55	0.0101	9.55	0.0091	2.12	0.0085	2.12	0.0085	2.12	0.0082

TABLE A.4b VALUES OF ABSOLUTE MAXIMUM RELATIVE VELOCITIES, $\dot{u}_o$, AND THE ASSOCIATED TIMES, t_o

Elastic Systems, El Centro Earthquake

y_o = 8.28 in., $\dot{y}_o$ = 13.68 in./sec., $\ddot{y}_o$ = 0.32 g, Duration of Quake = 29.5 sec.

T	f	β = 0		β = 0.02		β = 0.05		β = 0.10		β = 0.20		β = 0.40	
sec.	cps	t_o sec.	$\dot{u}_o$ in./sec	t_o sec.	$\dot{u}_o$ in./sec	t_o sec.	$\dot{u}_o$ in./sec	t_o sec.	$\dot{u}_o$ in./sec	t_o sec.	$\dot{u}_o$ in./sec	t_o sec.	$\dot{u}_o$ in./sec
25	0.04	1.63	12.42	1.63	12.43	1.63	12.44	1.63	12.45	1.63	12.48	1.63	12.48
15	0.067	11.60	-13.01	11.60	-13.05	11.60	-13.09	11.60	-13.08	11.60	-12.88	1.63	12.65
10	0.1	26.40	19.78	11.60	-17.70	11.60	-16.94	11.60	-15.82	11.60	-14.04	1.63	12.78
7	0.14	23.87	-20.30	26.40	17.32	26.40	14.74	5.00	14.35	3.00	-14.19	3.00	-13.50
5	0.2	5.04	22.43	5.04	21.73	5.04	20.04	5.04	18.84	4.44	18.63	4.44	17.01
4	0.25	4.52	29.94	4.44	29.00	4.44	27.92	4.44	26.08	4.44	22.67	4.44	17.60
3	0.33	29.46	-56.89	5.58	-45.59	5.58	-40.14	5.58	-33.09	5.58	-24.01	5.58	-15.29
2.5	0.4	7.62	-47.59	6.36	40.69	6.36	33.11	5.41	-24.85	5.41	-19.05	5.41	-13.37
2	0.5	11.94	36.00	11.94	29.84	11.94	23.32	1.98	-18.69	1.98	-16.63	1.98	-13.71
1.5	0.67	11.97	38.58	11.97	28.40	11.94	19.93	1.98	-17.81	1.98	-15.85	1.98	-12.73
1.25	0.8	28.2	37.02	5.88	24.76	5.85	20.33	3.42	16.36	1.98	-14.14	1.95	-11.62
1	1	4.74	-52.23	4.74	-43.82	4.74	-34.47	4.74	-24.77	4.74	-15.96	2.19	11.13
0.7	1.43	9.54	-53.40	6.05	-27.75	2.19	25.56	2.18	22.55	2.16	17.68	2.15	11.90
0.5	2	9.24	-39.46	2.34	-27.63	2.34	-25.08	2.34	-21.18	2.34	-15.48	2.34	-9.407
0.4	2.5	26.15	49.69	3.49	-21.99	2.48	19.27	2.48	16.40	2.47	11.78	2.33	-6.824
0.3	3.33	28.28	-20.89	2.58	-14.42	2.58	-14.83	2.58	-13.15	2.58	-9.398	2.44	5.867
0.25	4	22.65	31.23	2.66	17.83	2.66	13.26	2.55	-10.06	2.56	-7.434	2.56	-4.588
0.2	5	26.38	-26.40	3.25	12.27	2.64	9.197	2.64	7.150	2.54	-4.839	2.43	3.151
0.15	6.67	19.84	-26.98	4.99	-9.49	5.00	-6.379	5.00	-4.396	4.77	2.941	4.77	2.265
0.125	8	5.27	9.173	5.35	-4.523	4.98	-3.335	4.91	2.602	4.77	2.166	4.77	1.807
0.10	10	4.07	-5.376	4.82	-3.010	4.82	-2.706	4.76	2.213	4.76	1.783	4.76	1.360
0.05	20	16.37	2.506	9.57	-0.947	9.54	0.768	9.54	0.728	9.54	0.636	9.54	0.499

TABLE A.4c VALUES OF ABSOLUTE MAXIMUM RELATIVE ACCELERATIONS, $\ddot{u}_o$, AND THE ASSOCIATED TIMES, t_o

Elastic Systems, El Centro Earthquake

y_o = 8.28 in., $\dot{y}_o$ = 13.68 in./sec., $\ddot{y}_o$ = 0.32 g, Duration of Quake = 29.5 sec.

T	c	β = 0		β = 0.02		β = 0.05		β = 0.10		β = 0.20		β = 0.40	
sec.	cps	t_o sec.	$\ddot{u}_o$ in./sec²	t_o sec.	$\ddot{u}_o$ in./sec²	t_o sec.	$\ddot{u}_o$ in./sec²	t_o sec.	$\ddot{u}_o$ in./sec²	t_o sec.	$\ddot{u}_o$ in./sec²	t_o sec.	$\ddot{u}_o$ in./sec²
25	0.04	2.11	123.4	2.11	123.4	2.11	123.3	2.11	123.2	2.11	123.1	2.11	122.8
15	0.067	2.11	123.2	2.11	123.1	2.11	123.0	2.11	122.9	2.11	122.7	2.11	122.3
10	0.1	2.11	122.6	2.11	122.5	2.11	122.3	2.11	122.1	2.11	121.8	2.11	121.5
7	0.14	2.11	120.8	2.11	120.7	2.11	120.5	2.11	120.3	2.11	120.1	2.11	120.4
5	0.2	2.33	-123.5	2.33	-123.3	2.33	-123.0	2.33	-122.5	2.33	-121.1	2.11	119.6
4	0.25	2.33	-129.2	2.33	-128.4	2.33	-127.3	2.33	-125.5	2.33	-122.2	2.11	120.1
3	0.33	4.72	-168.4	4.72	-163.4	4.72	-156.5	4.72	-146.5	4.72	-131.4	2.11	124.3
2.5	0.4	5.80	190.7	5.80	177.5	5.80	161.9	5.80	143.5	4.72	-125.7	2.11	130.2
2	0.5	2.43	153.8	2.43	151.6	2.43	148.4	2.43	143.6	2.11	139.7	2.11	140.1
1.5	0.67	2.11	179.9	2.11	178.0	2.11	175.4	2.11	171.7	2.11	165.1	2.11	151.3
1.25	0.8	20.31	211.0	2.11	189.4	2.11	188.2	2.11	185.2	2.11	176.4	2.11	154.1
1	1	4.93	386.6	4.93	339.9	4.93	284.5	4.93	223.0	2.11	188.6	2.11	149.4
0.7	1.43	9.04	521.3	2.33	-331.9	2.33	-311.7	2.33	-274.5	2.33	-209.6	4.76	154.4
0.5	2	2.43	505.2	2.43	470.2	2.43	422.3	2.43	357.5	2.43	271.3	2.43	180.8
0.4	2.5	26.25	-748.9	2.55	-374.8	2.55	-354.5	2.55	-306.8	2.42	229.8	2.41	154.6
0.3	3.33	28.35	442.6	2.63	342.8	2.63	330.0	2.63	285.0	2.51	-203.7	2.50	-126.8
0.25	4	22.71	-788.1	3.08	453.7	2.61	352.0	2.61	275.4	2.61	192.5	4.76	125.9
0.2	5	27.44	823.9	3.30	-397.9	2.68	-302.4	2.68	-230.7	4.76	152.5	4.76	110.1
0.15	6.67	19.87	1125.	4.95	-379.7	4.96	-257.1	4.96	-182.0	4.96	-113.5	9.53	96.81
0.125	8	5.36	456.7	5.38	219.8	4.94	-154.8	4.94	-115.7	9.53	99.80	9.53	91.86
0.1	10	4.04	-327.4	4.79	-178.9	4.79	-166.4	4.79	-134.6	9.53	97.95	9.53	84.67
0.05	20	16.38	-307.5	9.55	-117.0	9.55	- 94.22	9.55	- 83.78	9.53	67.03	9.53	53.35

TABLE A.4d VALUES OF ABSOLUTE MAXIMUM DISPLACEMENTS, x_o, AND THE ASSOCIATED TIMES, t_o

Elastic Systems, El Centro Earthquake

y_o = 8.28 in., $\dot{y}_o$ = 13.68 in./sec., $\ddot{y}_o$ = 0.32 g, Duration of Quake = 29.5 sec.

T	f	β = 0		β = 0.02		β = 0.05		β = 0.10		β = 0.20		β = 0.40	
sec.	cps	t_o sec.	x_o in.	t_o sec.	x_o in.	t_o sec.	x_o in.	t_o sec.	x_o in.	t_o sec.	x_o in.	t_o sec.	x_o in.
25	0.04	25.00	-7.306	11.33	7.019	11.00	6.824	10.56	6.612	9.91	6.422	8.56	6.510
15	0.067	9.80	10.44	9.62	10.17	9.52	9.820	9.20	9.360	8.60	8.821	8.00	8.521
10	0.10	23.65	-18.27	23.60	-15.34	13.38	-12.62	8.23	11.06	7.95	10.31	7.60	9.469
7	0.14	25.60	-18.39	15.40	15.78	15.36	13.27	7.46	12.22	7.23	11.01	7.01	9.686
5	0.2	28.68	-18.93	6.38	15.52	6.32	14.41	6.24	12.92	6.12	10.83	6.53	8.891
4	0.25	27.84	-20.14	5.52	18.51	5.45	16.96	5.34	14.95	5.16	12.36	4.93	9.940
3	0.33	27.20	-30.13	7.70	21.97	4.86	19.52	4.80	17.02	4.74	13.78	4.62	10.79
2.5	0.4	7.01	26.40	7.01	23.29	6.96	19.76	6.96	15.86	4.50	12.90	4.50	10.63
2	0.5	6.42	13.70	6.47	13.75	6.53	13.64	6.54	13.06	6.54	11.64	6.48	9.829
1.5	0.67	6.27	13.77	6.27	13.38	6.30	12.86	6.30	12.11	6.27	11.01	6.27	9.768
1.25	0.8	6.15	14.43	6.15	13.44	6.15	12.43	6.15	11.44	6.15	10.48	6.15	9.573
1	1	4.47	14.25	4.47	13.17	4.44	11.95	4.44	10.70	6.09	9.836	6.06	9.304
0.7	1.43	5.87	12.76	5.87	11.35	5.87	10.31	5.88	9.632	5.93	9.148	5.96	8.819
0.5	2	6.08	10.59	6.05	9.675	6.00	9.200	5.99	8.785	5.96	8.528	5.94	8.458
0.4	2.5	6.18	10.45	6.18	9.153	6.22	8.586	6.26	8.428	5.90	8.370	5.92	8.330
0.3	3.33	4.29	8.491	6.19	8.465	6.21	8.387	6.22	8.325	6.22	8.302	6.22	8.287
0.25	4	6.21	9.252	6.20	8.426	4.35	8.311	4.34	8.318	6.22	8.287	6.22	8.287
0.2	5	6.26	8.777	6.24	8.482	6.24	8.368	6.23	8.317	6.22	8.296	6.21	8.288
0.15	6.67	6.13	8.661	4.33	8.356	6.14	8.293	6.16	8.283	6.19	8.287	6.20	8.285
0.125	8	6.17	8.434	6.20	8.302	6.20	8.283	6.20	8.282	6.20	8.283	6.20	8.283
0.1	10	6.14	8.293	6.20	8.292	6.20	8.286	6.19	8.284	6.19	8.283	6.19	8.281
0.05	20	6.20	8.285	6.19	8.278	6.19	8.278	6.19	8.278	6.19	8.278	6.19	8.278

TABLE A.4e VALUES OF ABSOLUTE MAXIMUM VELOCITIES, $\dot{x}_o$, AND THE ASSOCIATED TIMES, t_o

Elastic Systems, El Centro Earthquake

y_o = 8.28 in., $\dot{y}_o$ = 13.68 in./sec., $\ddot{y}_o$ = 0.32 g, Duration of Quake = 29.5 sec.

T	f	β = 0		β = 0.02		β = 0.05		β = 0.10		β = 0.20		β = 0.40	
sec.	cps	t_o sec.	$\dot{x}_o$ in./sec	t_o sec.	$\dot{x}_o$ in./sec	t_o sec.	$\dot{x}_o$ in./sec	t_o sec.	$\dot{x}_o$ in./sec	t_o sec.	$\dot{x}_o$ in./sec	t_o sec.	$\dot{x}_o$ in./sec
25	0.04	18.00	-1.677	17.70	-1.582	17.47	-1.469	7.18	1.384	6.53	1.455	11.20	-1.762
15	0.067	12.86	-4.872	12.70	-4.655	12.47	-4.379	12.14	-4.056	11.40	-3.821	10.86	-3.785
10	0.1	26.20	12.36	26.08	10.23	15.63	8.169	11.16	-7.180	10.86	-6.090	10.80	-4.864
7	0.14	23.91	-17.12	17.20	-13.97	17.20	-11.31	13.51	9.001	13.29	6.545	4.33	6.445
5	0.2	27.38	-19.75	27.36	-15.32	4.86	12.07	4.76	11.03	4.41	10.30	4.21	9.907
4	0.25	26.86	-27.86	4.57	21.44	4.52	19.81	4.44	17.72	4.32	15.03	4.08	12.39
3	0.33	26.46	-59.05	12.90	38.57	5.52	-29.14	4.08	23.77	4.01	18.79	3.96	13.76
2.5	0.4	7.62	-49.97	7.62	-40.97	6.36	32.42	3.70	23.86	3.60	18.59	3.42	13.91
2	0.5	11.88	33.29	11.88	26.82	3.42	23.88	3.42	21.33	3.37	18.20	3.32	15.11
1.5	0.67	29.27	-39.02	11.94	24.80	5.89	21.07	5.88	19.21	5.85	16.51	3.27	15.45
1.25	0.8	21.24	-38.67	5.82	27.47	5.82	23.33	3.36	20.41	3.30	18.16	3.23	16.01
1	1	4.74	-50.57	4.74	-42.16	4.72	-33.20	3.23	26.06	3.18	20.78	3.15	16.29
0.7	1.43	10.25	-56.99	5.66	36.80	5.64	28.55	5.63	22.75	5.63	17.77	1.77	-15.88
0.5	2	5.48	41.96	3.05	35.07	2.33	-28.76	2.31	-24.75	2.31	-19.32	1.74	-16.01
0.4	2.5	26.35	-54.91	2.89	30.36	2.88	26.18	2.88	20.85	2.27	-17.49	1.72	-15.19
0.3	3.33	26.47	-24.65	3.01	18.61	2.57	-17.30	2.57	-15.65	2.24	-14.86	1.72	-14.47
0.25	4	3.14	37.56	3.15	27.28	2.90	20.88	1.74	-16.13	1.73	-15.02	1.72	-14.51
0.2	5	26.38	-34.03	3.05	23.45	3.05	19.62	3.05	16.16	1.71	-14.98	1.70	-14.50
0.15	6.67	21.49	-30.78	4.99	-18.63	5.00	-15.73	1.70	-14.43	1.70	-14.40	1.70	-14.26
0.125	8	5.64	17.55	1.70	-14.48	1.70	-14.29	1.70	-14.28	1.70	-14.23	1.69	-14.12
0.1	10	3.02	15.91	1.67	-14.52	1.68	-14.33	1.68	-14.21	1.69	-14.09	1.69	-13.98
0.05	20	1.70	-13.79	1.70	-13.74	1.70	-13.73	1.70	-13.72	1.69	-13.73	1.69	-13.74

TABLE A.4f VALUES OF ABSOLUTE MAXIMUM ACCELERATIONS, $\ddot{x}_o$, AND THE ASSOCIATED TIMES, t_o

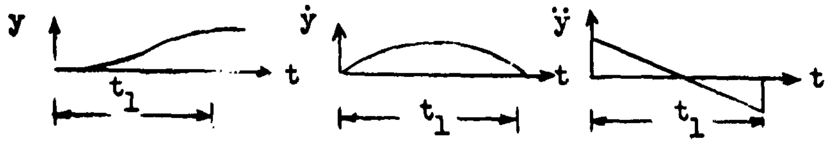
Elastic Systems, El Centro Earthquake

y_o = 8.28 in., $\dot{y}_o$ = 13.68 in./sec., $\ddot{y}_o$ = 0.32 g, Duration of Quake = 29.5 sec.

T	f	β = 0		β = 0.02		β = 0.05		β = 0.10		β = 0.20		β = 0.40	
sec.	cps	t_o sec.	$\ddot{x}_o$ in./sec^2	t_o sec.	$\ddot{x}_o$ in./sec^2	t_o sec.	$\ddot{x}_o$ in./sec^2	t_o sec.	$\ddot{x}_o$ in./sec^2	t_o sec.	$\ddot{x}_o$ in./sec^2	t_o sec.	$\ddot{x}_o$ in./sec^2
25	0.04	11.00	-.702	10.80	-.717	10.48	-.750	10.48	-.877	3.80	1.298	1.63	-2.475
15	0.067	10.80	-2.345	10.80	-2.262	10.48	-2.208	10.48	-2.164	3.80	2.406	1.63	-4.227
10	0.1	28.40	-8.222	28.20	-6.751	13.00	5.802	12.92	5.309	12.33	4.617	11.60	6.771
7	0.14	25.40	16.44	18.69	12.78	18.60	10.13	11.60	8.179	11.60	8.027	1.63	-10.08
5	0.2	28.68	25.15	28.56	19.06	4.01	16.83	3.96	16.02	3.84	14.52	1.68	-16.90
4	0.25	27.84	41.30	5.34	-34.43	5.30	-31.60	5.16	-28.75	5.04	-26.10	3.08	23.35
3	0.33	27.20	122.0	13.62	-79.31	6.18	60.47	4.68	-50.39	4.62	-40.29	4.50	-36.01
2.5	0.4	7.01	-120.0	6.96	-100.4	5.76	85.18	5.76	67.76	5.71	49.22	5.58	40.76
2	0.5	12.33	-97.50	12.30	-73.94	6.53	-55.67	6.53	-51.05	5.58	49.02	5.58	46.02
1.5	0.67	28.89	-162.1	6.27	-90.81	6.27	-82.15	6.24	-70.02	5.58	53.04	2.01	50.54
1.25	0.8	29.16	187.3	6.15	-130.9	6.12	-105.9	6.09	-81.98	2.04	61.85	2.01	64.35
1	1	4.98	293.1	4.95	236.4	4.47	-184.1	4.47	-132.3	2.04	86.93	1.98	82.16
0.7	1.43	10.08	-477.9	5.85	-267.8	2.31	-199.6	2.30	-173.2	2.27	-136.1	1.97	106.8
0.5	2	9.12	-471.5	2.43	355.5	2.43	306.8	2.43	241.9	2.19	-189.6	2.18	-143.6
0.4	2.5	26.26	-774.9	5.16	363.5	2.77	293.1	2.38	239.7	2.38	192.3	2.15	-147.5
0.3	3.33	28.21	-434.4	2.64	329.9	2.64	319.5	2.63	268.3	2.49	-188.1	2.14	-144.2
0.25	4	22.84	794.4	2.60	471.5	2.60	373.9	2.60	294.6	2.60	214.9	2.58	144.7
0.2	5	26.34	-842.6	2.78	418.4	2.78	340.0	2.58	265.3	2.58	208.6	2.58	152.4
0.15	6.67	19.80	-1144.	4.95	-460.2	4.96	-328.0	4.96	-249.1	4.96	-181.5	2.44	-146.9
0.125	8	5.24	512.7	4.94	-288.5	4.94	-244.5	4.94	-204.7	4.94	-163.8	2.44	-138.8
0.1	10	4.04	-378.4	4.79	-227.0	4.79	-216.5	4.79	-185.3	4.79	-151.0	2.12	-129.7
0.05	20	24.62	337.1	9.55	-159.8	9.55	-137.0	2.11	-127.7	2.11	-127.8	2.11	-126.5

TABLE A.5 VALUES OF MAXIMUM AND MINIMUM PSEUDO-VELOCITIES AND ASSOCIATED TIMES

Elasto-Plastic Systems Subjected to Ground Motion Shown:



$t_1 f$	$\frac{u_y}{u_o}$	β = 0 $\frac{t_{max}}{t_1}$	β = 0 $\frac{pu_{max}}{\dot{y}_o}$	β = 0 $\frac{t_{min}}{t_1}$	β = 0 $\frac{pu_{min}}{\dot{y}_o}$	β = 0.10 $\frac{t_{max}}{t_1}$	β = 0.10 $\frac{pu_{max}}{\dot{y}_o}$	β = 0.10 $\frac{t_{min}}{t_1}$	β = 0.10 $\frac{pu_{min}}{\dot{y}_o}$
0.05	1	10.40	0.209	20.40	-0.209	10.00	0.155	1.00	-0.201
	0.75	10.40	0.110	1.00	-0.206	9.6	0.070	1.00	-0.201
	0.5	10.40	0.047	1.00	-0.207	0	0	1.00	-0.201
	0.2	0	0	1.00	-0.208	0	0	1.00	-0.203
	0.15	0	0	1.00	-0.208	0	0	1.00	-0.203
	0.1	0	0	1.00	-0.209	0	0	1.00	-0.203
0.1	1	15.60	0.414	20.60	-0.414	5.20	0.308	1.00	-0.371
	0.75	5.40	0.253	1.00	-0.395	5.00	0.151	1.00	-0.372
	0.5	5.40	0.039	1.00	-0.398	0	0	1.00	-0.376
	0.2	0	0	1.00	-0.405	0	0	1.00	-0.385
	0.15	0	0	1.00	-0.410	0	0	1.00	-0.387
	0.1	-	-	-	-	0	0	1.00	-0.389
0.2	1	3.00	0.804	5.50	-0.804	2.90	0.598	0.90	-0.615
	0.75	3.00	0.663	0.90	-0.682	2.80	0.336	0.90	-0.622
	0.5	3.00	0.242	0.90	-0.697	2.80	0.527	0.90	-0.641
	0.25	0	0	0.90	-0.737	0	0	0.90	-0.676
	0.15	0	0	1.00	-0.770	0	0	0.90	-0.695
	0.1	0	0	1.00	-0.788	0	0	0.90	-0.708
	0.15	-	-	-	-	0	0	1.00	-0.722
0.3	1	5.49	1.147	7.17	-1.147	2.08	0.854	0.80	-0.740
	0.9	2.21	1.152	7.17	-0.913	2.08	0.858	0.80	-0.740
	0.75	2.28	1.195	0.80	-0.840	2.08	0.698	0.80	-0.745
	0.6	2.28	0.711	0.80	-0.846	2.08	0.469	0.80	-0.761
	0.5	2.28	0.664	0.80	-0.860	2.14	0.304	0.80	-0.779
	0.4	2.28	0.380	0.87	-0.891	2.14	0.131	0.80	-0.804
	0.3	2.35	0.061	0.87	-0.938	0	0	0.87	-0.844
	0.25	0	0	0.87	-0.967	0	0	0.87	-0.867
	0.15	0	0	0.94	-1.053	0	0	0.87	-0.924
	0.1	0	0	0.94	-1.106	0	0	0.94	-0.961
	0.05	0	0	1.00	-1.170	0	0	0.94	-1.005
0.4	1	1.75	1.424	3.00	-1.424	1.70	1.060	0.70	-0.790
	0.9	1.75	1.431	3.00	-1.131	1.70	1.066	0.70	-0.790
	0.75	1.80	1.483	0.70	-0.905	1.75	1.100	0.70	-0.790
	0.6	1.90	1.485	0.70	-0.905	1.75	0.849	0.70	-0.801
	0.5	1.90	1.199	0.75	-0.920	1.80	0.655	0.75	-0.822
	0.4	1.95	0.856	0.75	-0.959	1.85	0.429	0.75	-0.863

TABLE A.5 CONTINUED

$t_1 f$	$\frac{u_y}{u_o}$	β = 0 $\frac{t_{max}}{t_1}$	$\frac{pu_{max}}{\dot{y}_o}$	$\frac{t_{min}}{t_1}$	$\frac{pu_{min}}{\dot{y}_o}$	β = 0.10 $\frac{t_{max}}{t_1}$	$\frac{pu_{max}}{\dot{y}_o}$	$\frac{t_{min}}{t_1}$	$\frac{pu_{min}}{\dot{y}_o}$
	0.3	2.00	0.443	0.80	-1.032	1.90	0.176	0.80	-0.924
	0.25	2.05	0.203	0.85	-1.085	1.95	0.037	0.80	-0.967
	0.15	0	0	0.90	-1.244	0	0	0.85	-1.075
	0.1	0	0	0.90	-1.353	0	0	0.90	-1.144
	0.05	0	0	0.95	-1.494	0	0	0.90	-1.231
0.5	1	3.52	1.617	4.52	-1.617	1.44	1.204	2.48	-0.879
	0.9	1.52	1.283	2.52	-1.629				
	0.8	1.52	1.660	2.52	-0.927				
	0.75	1.56	1.689	0.64	-0.919	1.52	1.252	0.60	-0.794
	0.7	1.56	1.724	0.64	-0.919				
	0.6	1.64	1.837	0.64	-0.919				
	0.5	1.68	1.696	0.64	-0.924	1.60	0.981	0.64	-0.818
	0.4	1.72	1.342	0.68	-0.961				
	0.3	1.80	0.709	0.72	-1.050				
	0.25	1.84	0.596	0.76	-1.125	1.76	0.274	0.76	-1.002
	0.2	1.80	0.253	0.80	-1.227				
	0.15	0	0	0.84	-1.362	0	0	0.80	-1.168
	0.1	0	0	0.88	-1.540	0	0	0.84	-1.278
	0.05	0	0	0.96	-1.773				
0.7	1	4.08	1.736	3.36	-1.736	1.20	1.296	1.92	-0.946
	0.9	2.64	1.744	1.92	-1.383	1.20	1.303	0.50	-0.750
	0.75	1.24	1.809	0.52	-0.872	1.24	1.344	0.50	-0.750
	0.6	4.16	1.967	0.52	-0.872	1.28	1.446	0.50	-0.750
	0.5	1.36	2.165	0.52	-0.872	1.32	1.321	0.52	-0.759
	0.4	4.28	1.887	0.56	-0.894	1.36	1.106	0.56	-0.802
	0.3	1.52	1.514	0.60	-0.996	1.44	0.826	0.60	-0.905
	0.25	3.00	1.247	0.64	-1.098	1.48	0.644	0.64	-0.990
	0.15	3.16	0.365	0.76	-1.478	1.64	0.168	0.72	-1.254
	0.1	0	0	0.84	-1.805	0	0	0.80	-1.448
	0.05	0	0	0.92	-2.271	0	0	0.84	-1.699
1	1	1.00	1.274	1.50	-1.274	1.02	0.966	1.52	-0.705
	0.9	1.00	1.286	1.50	-1.007				
	0.8	1.02	1.333	0.40	-0.761				
	0.75	1.04	1.373	0.40	-0.761	1.04	1.030	0.40	-0.653
	0.6	1.08	1.591	0.40	-0.761				
	0.5	1.14	1.498	0.42	-0.777	1.14	0.962	0.44	-0.693

TABLE A.5 CONTINUED

$t_1 f$	$\frac{u_y}{u_o}$	β = 0 $\frac{t_{max}}{t_1}$	β = 0 $\frac{pu_{max}}{\dot{y}_o}$	β = 0 $\frac{t_{min}}{t_1}$	β = 0 $\frac{pu_{min}}{\dot{y}_o}$	β = 0.10 $\frac{t_{max}}{t_1}$	β = 0.10 $\frac{pu_{max}}{\dot{y}_o}$	β = 0.10 $\frac{t_{min}}{t_1}$	β = 0.10 $\frac{pu_{min}}{\dot{y}_o}$
	0.4	1.22	1.450	0.46	-0.850				
	0.3	1.34	1.358	0.54	-1.042				
	0.25	1.44	1.218	0.60	-1.220	1.36	0.607	0.60	-1.081
	0.15	1.64	0.367	0.74	-1.873				
	0.1	0	0	0.82	-2.414	0	0	0.74	-1.759
1.25	1	0.80	0.815	0.34	-0.676	0.83	0.576	0.34	-0.580
	0.9	0.82	0.823	0.34	-0.676	0.86	0.499	0.34	-0.585
	0.75	0.86	0.746	0.34	-0.680	1.02	0.498	0.37	-0.618
	0.6	1.04	0.756	0.34	-0.733	1.09	0.572	0.42	-0.707
	0.5	1.12	0.410	0.38	-0.828	1.15	0.605	0.46	-0.820
	0.4	1.23	1.066	0.43	-1.028	1.23	0.576	0.53	-1.000
	0.3	1.38	1.039	0.50	-1.414	1.33	-.440	0.59	-1.241
	0.25	1.47	0.843	0.59	-1.714	1.39	0.318	0.62	-1.446
	0.15	0	0	0.66	-2.635	0	0	0.70	-1.895
	0.1	0	0	0.78	-3.305	0	0	0.75	-2.176
	0.05	0	0	0.93	-4.155	0	0	0.78	-2.499
1.50	1	0.66	0.566	0.29	-0.603	0.69	0.366	0.29	-0.518
	0.9	0.68	0.457	0.30	-0.608	0.71	0.290	0.29	-0.523
	0.75	0.78	0.293	0.32	-0.647	0.76	0.176	0.32	-0.556
	0.6	1.01	0.191	0.37	-0.764	1.03	0.145	0.37	-0.648
	0.5	1.10	0.532	0.43	-0.934	1.09	0.298	0.42	-0.772
	0.4	1.24	0.831	0.52	-1.251	1.16	0.400	0.49	-0.973
	0.3	2.06	0.841	0.62	-1.803	1.26	0.374	0.56	-1.284
	0.25	1.49	0.556	0.67	-2.204	1.32	0.295	0.60	-1.489
	0.15	0	0	0.80	-3.351	0	0	0.68	-2.013
	0.1	0	0	0.86	-4.139	0	0	0.72	-2.341
	0.05	0	0	0.94	-5.103	0	0	0.77	-2.716
2	1	1.00	0.637	1.25	-0.637	1.01	0.404	0.22	-0.424
	0.9	1.00	0.643	1.25	-0.504	1.01	0.345	0.23	-0.428
	0.8	1.01	0.668	0.23	-0.493	1.02	0.277	0.24	-0.444
	0.75	1.02	0.651	0.23	-0.493	1.02	0.237	0.25	-0.459
	0.6	1.04	0.464	0.25	-0.525	1.04	0.061	0.31	-0.690
	0.5	1.06	0.266	0.29	-0.601	0	0	0.36	-0.690
	0.4	0	0	0.36	-0.799	0	0	0.43	-0.925
	0.3	1.12	0.086	0.48	-1.290	1.18	0.140	0.51	-1.297
	0.25	1.24	0.475	0.55	-1.740	1.23	0.164	0.56	-1.546
	0.15	0	0	0.72	-3.310	1.38	0.002	0.64	-2.190
	0.1	0	0	0.81	-4.559	0	0	0.68	-2.590

TABLE A.5 CONTINUED

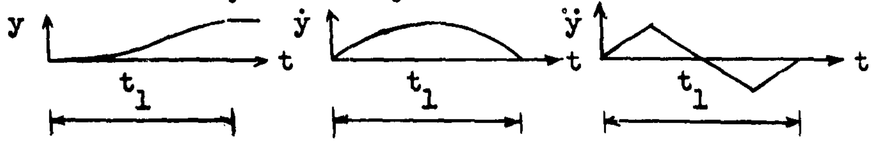
$t_1 f$	$\frac{u_y}{u_o}$	β = 0 $\frac{t_{max}}{t_1}$	$\frac{pu_{max}}{\dot{y}_o}$	$\frac{t_{min}}{t_1}$	$\frac{pu_{min}}{\dot{y}_o}$	β = 0.10 $\frac{t_{max}}{t_1}$	$\frac{pu_{max}}{\dot{y}_o}$	$\frac{t_{min}}{t_1}$	$\frac{pu_{min}}{\dot{y}_o}$
2.5	1	0.80	0.407	0.18	-0.416	0.82	0.227	0.18	-0.357
	0.9	0.81	0.327	0.19	-0.419	0.84	0.180	0.18	-0.361
	0.75	0.84	0.196	0.21	-0.452	0.89	0.100	0.21	-0.390
	0.6	1.01	0.096	0.26	-0.565	1.02	0.019	0.26	-0.483
	0.5	1.06	0.069	0 33	-0.764	0	0	0.32	-0.625
	0.4	0	0	0.42	-1.196	0	0	0.40	-0.883
	0.3	1.22	0.082	0.55	-2.076	0	0	0.48	-1.302
	0.25	1.34	0.277	0.62	-2.768	0	0	0.52	-1.586
	0.15	0	0	0.77	-4.858	0	0	0.61	-2.316
	0.1	0	0	0.84	-6.345	0	0	0.66	-2.769
	0.05	0	0	0.92	-8.193	0	0	0.70	-3.284
3	1	1.00	0.425	1.50	-0.425	1.00	0.241	0.16	-0.309
	0.9	1.00	0.429	0.16	-0.358	1.01	0.202	0.16	-0.312
	0.75	1.01	0.358	0.16	-0.363	1.01	0.119	0.18	-0.339
	0.6	1.02	0.187	0.19	-0.409	0	0	0.23	-0.429
	0.5	0	0	0.23	-0.511	0	0	0.29	-0.575
	0.4	0	0	0.32	-0.793	0	0	0.37	-0.849
	0.3	0	0	0.46	-1.550	0	0	0.46	-1.304
	0.25	0	0	0.53	-2.261	0	0	0.50	-1.614
	0.15	0	0	0.71	-4.747	0	0	0.59	-2.409
	0.1	0	0	0.80	-6.704	0	0	0.63	-2.901
	0.05	0	0	0.90	-9.266	0	0	0.67	-3.460
4	1	1.00	0.318	1.13	-0.318	1.01	0.170	0.12	-0.242
	0.9	1.00	0.321	0.12	-0.280	1.01	0.140	0.12	-0.244
	0.75	1.01	0.236	0.13	-0.288	1.01	0.077	0.14	-0.268
	0.6	1.02	0.080	0.16	-0.338	0	0	0.19	-0.351
	0.5	0	0	0.20	-0.452	0	0	0.25	-0.500
	0.4	0	0	0.30	-0.802	0	0	0.33	-0.797
	0.3	0	0	0.44	-1.813	0	0	0.42	-1.307
	0.25	0	0	0.53	-2.783	0	0	0.46	-1.652
	0.15	0	0	0.71	-6.185	0	0	0.55	-2.539
	0.1	0	0	0.81	-8.847	0	0	0.59	-3.083
	0.05	0	0	0.90	-12.32	0	0	0.64	-3.696
5	1	1.00	0.255	1.10	-0.255	1.00	0.131	0.10	-0.198
	0.75	1.00	0.172	0.10	-0.239	1.00	0.056	0.12	-0.221
	0.5	0	0	0.18	-0.411	0	0	0.22	-0.446
	0.4	0	0	0.28	-0.818	0	0	0.30	-0.759
	0.25	0	0	0.52	-3.302	0	0	0.44	-1.675
	0.15	0	0	0.71	-7.620	0	0	0.52	-2.618

TABLE A.5 CONTINUED

$t_1 f$	$\frac{u_y}{u_o}$	β = 0 $\frac{t_{max}}{t_1}$	$\frac{pu_{max}}{\dot{y}_o}$	$\frac{t_{min}}{t_1}$	$\frac{pu_{min}}{\dot{y}_o}$	β = 0.10 $\frac{t_{max}}{t_1}$	$\frac{pu_{max}}{\dot{y}_o}$	$\frac{t_{min}}{t_1}$	$\frac{pu_{min}}{\dot{y}_o}$
7	1	1.00	0.182	1.22	-0.182	1.00	0.091	1.07	-0.146
	0.9	1.29	0.172	0.07	-0.169	1.00	0.075	0.07	-0.148
	0.75	1.29	0.108	0.08	-0.179	1.00	0.037	0.08	-0.163
	0.6	0	0	0.10	-0.227	0	0	0.13	-0.230
	0.5	0	0	0.15	-0.355	0	0	0.19	-0.375
	0.4	0	0	0.26	-0.866	0	0	0.27	-0.709
	0.3	0	0	0.42	-2.606	0	0	0.36	-1.298
	0.25	0	0	0.52	-4.346	0	0	0.40	-1.696
	0.15	0	0	0.71	-10.49	0	0	0.49	-2.705
	0.1	0	0	0.80	-15.28	0	0	0.53	-3.315
	0.05	0	0	0.90	-21.48	0	0	0.57	-3.998
10	1	1.00	0.127	1.05	-0.127	1.00	0.063	0.05	-0.104
	0.75	1.00	0.067	0.06	-0.130	1.00	0.024	0.06	-0.118
	0.5	0	0	0.13	-0.303	0	0	0.16	-0.313
	0.4	0	0	0.25	-0.948	0	0	0.24	-0.663
	0.25	0	0	0.51	-5.920	0	0	0.37	-1.706
	0.15	0	0	0.70	-14.80	0	0	0.46	-2.757

TABLE A.6 VALUES OF MAXIMUM AND MINIMUM PSEUDO-VELOCITIES AND ASSOCIATED TIMES

Elasto-Plastic Systems Subjected to Ground Motion Shown:



		$\beta = 0$				$\beta = 0.10$			
$t_1 f$	$\frac{u_y}{u_o}$	$\frac{t_{max}}{t_1}$	$\frac{pu_{max}}{\dot{y}_o}$	$\frac{t_{min}}{t_1}$	$\frac{pu_{min}}{\dot{y}_o}$	$\frac{t_{max}}{t_1}$	$\frac{pu_{max}}{\dot{y}_o}$	$\frac{t_{min}}{t_1}$	$\frac{pu_{min}}{\dot{y}_o}$
0.1	1	15.60	0.311	20.60	-0.311	5.20	0.232	0.80	-0.281
	0.9	5.60	0.279	1.00	-0.297	5.20	0.185	0.80	-0.281
	0.75	5.40	0.185	1.00	-0.298	5.00	0.114	0.80	-0.282
	0.6	5.40	0.089	1.00	-0.299	5.00	0.042	0.80	-0.282
	0.5	5.40	0.024	1.00	-0.301	0	0	0.80	-0.283
	0.4	0	0	1.00	-0.302	0	0	1.00	-0.284
	0.3	0	0	1.00	-0.304	0	0	1.00	-0.286
	0.25	0	0	1.00	-0.306	0	0	1.00	-0.287
	0.15	0	0	1.00	-0.309	0	0	1.00	-0.290
	0.1	0	0	1.00	-0.310	0	0	1.00	-0.291
	0.05	0	0	1.00	-0.312	0	0	1.00	-0.293
0.2	1	3.00	0.612	5.50	-0.612	2.90	0.455	0.80	-0.502
	0.9	3.00	0.616	0.80	-0.543	2.80	0.372	0.80	-0.502
	0.75	3.00	0.452	0.80	-0.544	2.80	0.244	0.80	-0.504
	0.6	3.00	0.265	0.80	-0.547	2.70	0.112	0.80	-0.508
	0.5	3.00	0.138	0.80	-0.550	2.70	0.022	0.80	-0.512
	0.4	0	0	0.90	-0.559	0	0	0.80	-0.517
	0.3	0	0	0.90	-0.569	0	0	0.80	-0.524
	0.25	0	0	0.90	-0.577	0	0	0.80	-0.528
	0.15	0	0	0.90	-0.591	0	0	0.90	-0.539
	0.1	0	0	0.90	-0.601	0	0	0.90	-0.546
	0.05	0	0	0.90	-0.611	0	0	0.90	-0.555
0.4	1	1.75	1.136	3.00	-1.136	1.70	0.845	0.70	-0.730
	0.9	1.75	1.142	3.00	-0.903	1.70	0.850	0.70	-0.730
	0.75	1.80	1.183	0.70	-0.821	1.70	0.701	0.70	-0.732
	0.6	1.85	0.930	0.70	-0.826	1.70	0.480	0.70	-0.745
	0.5	1.85	0.681	0.75	-0.842	1.75	0.323	0.75	-0.759
	0.4	1.85	0.404	0.75	-0.871	1.75	0.149	0.75	-0.787
	0.3	1.90	0.104	0.75	-0.911	0	0	0.75	-0.822
	0.25	0	0	0.80	-0.942	0	0	0.75	-0.844
	0.15	0	0	0.80	-1.020	0	0	0.80	-0.900
	0.1	0	0	0.85	-1.078	0	0	0.80	-0.936
	0.05	0	0	0.90	-1.149	0	0	0.80	-0.978

TABLE A.6 CONTINUED

$t_1 f$	$\frac{u_y}{u_o}$	β = 0 $\frac{t_{max}}{t_1}$	$\frac{pu_{max}}{\dot{y}_o}$	$\frac{t_{min}}{t_1}$	$\frac{pu_{min}}{\dot{y}_o}$	β = 0.10 $\frac{t_{max}}{t_1}$	$\frac{pu_{max}}{\dot{y}_o}$	$\frac{t_{min}}{t_1}$	$\frac{pu_{min}}{\dot{y}_o}$
0.6	1	1.34	1.500	2.18	-1.500	1.31	1.117	2.14	-0.815
	0.9	1.34	1.509	2.18	-1.191	1.31	1.123	0.60	-0.779
	0.75	1.38	1.563	0.64	-0.896	1.34	1.160	0.60	-0.779
	0.6	1.44	1.703	0.64	-0.896	1.37	1.015	0.64	-0.784
	0.5	1.48	1.432	0.64	-0.906	1.37	0.811	0.64	-0.806
	0.4	1.51	1.098	0.67	-0.941	1.41	0.584	0.67	-0.845
	0.3	1.54	0.673	0.70	-1.019	1.47	0.318	0.67	-0.913
	0.25	1.57	0.414	0.70	-1.080	1.51	0.164	0.70	-0.962
	0.15	0	0	0.75	-1.264	0	0	0.74	-1.090
	0.1	0	0	0.80	-1.399	0	0	0.75	-1.178
	0.05	0	0	0.84	-1.587	0	0	0.77	-1.286
0.8	1	1.13	1.663	1.75	-1.663	1.10	1.239	1.73	-0.903
	0.9	1.13	1.672	1.75	-1.321	1.10	1.244	0.55	-0.753
	0.75	1.15	1.733	0.55	-0.871	1.13	1.286	0.55	-0.753
	0.6	1.20	1.887	0.55	-0.871	1.18	1.362	0.55	-0.753
	0.5	1.25	1.963	0.55	-0.872	1.20	1.173	0.55	-0.753
	0.4	1.30	1.649	0.58	-0.902	1.23	0.941	0.58	-0.807
	0.3	1.35	1.234	0.63	-0.991	1.30	0.651	0.63	-0.896
	0.25	1.40	0.954	0.65	-1.075	1.33	0.478	0.65	-0.965
	0.15	1.50	0.155	0.70	-1.366	1.45	0.051	0.70	-1.171
	0.1	0	0	0.75	-1.609	0	0	0.73	-1.322
	0.05	0	0	0.83	-1.946	0	0	0.75	-1.513
1	1	1.00	1.621	1.50	-1.621	1.00	1.209	1.50	-0.882
	0.9	1.00	1.629	1.50	-1.288	1.00	1.216	0.50	-0.698
	0.75	1.02	1.691	0.50	-0.810	1.02	1.260	0.50	-0.698
	0.6	1.06	1.852	0.50	-0.810	1.06	1.363	0.50	-0.698
	0.5	1.12	2.060	0.50	-0.810	1.08	1.260	0.50	-0.707
	0.4	1.16	1.817	0.52	-0.832	1.12	1.074	0.52	-0.750
	0.3	1.24	1.507	0.56	-0.932	1.18	0.826	0.56	-0.855
	0.25	1.28	1.275	0.60	-1.035	1.22	0.658	0.60	-0.943
	0.15	1.42	0.448	0.68	-1.430	1.34	0.204	0.66	-1.220
	0.1	0	0	0.72	-1.776	0	0	0.70	-1.427
	0.05	0	0	0.80	-2.285	0	0	0.74	-2.339
1.25	1	0.90	1.348	1.30	-1.325	0.90	1.015	1.31	-0.727
	0.9	0.90	1.358	2.11	-1.048	0.90	1.021	0.45	-0.622
	0.75	0.91	1.423	0.45	-0.720	0.93	1.071	0.45	-0.622
	0.6	0.96	1.606	0.45	-0.720	0.96	1.166	0.45	-0.622
	0.5	1.01	1.703	0.45	-0.722	1.00	1.076	0.46	-0.640
	0.4	1.06	1.607	0.48	-0.761	1.04	0.978	0.50	-0.701

TABLE A.6 CONTINUED

$t_1 f$	$\frac{u_y}{u_o}$	β = 0 $\frac{t_{max}}{t_1}$	$\frac{pu_{max}}{\dot{y}_o}$	$\frac{t_{min}}{t_1}$	$\frac{pu_{min}}{\dot{y}_o}$	B = 0.10 $\frac{t_{max}}{t_1}$	$\frac{pu_{max}}{\dot{y}_o}$	$\frac{t_{min}}{t_1}$	$\frac{pu_{min}}{\dot{y}_o}$
	0.3	1.15	1.479	0.53	-0.903	1.10	0.815	0.54	-0.840
	0.25	1.20	1.335	0.56	-1.044	1.15	0.685	0.58	-0.956
	0.15	1.36	0.541	0.66	-1.594	1.28	0.259	0.64	-1.319
	0.1	0	0	0.72	-2.078	0	0	0.67	-1.587
	0.05	0	0	0.78	-2.777	0	0	0.72	-1.933
1.5	1	0.81	0.997	1.83	-0.871	0.82	0.762	0.40	-0.549
	0.9	0.82	1.006	1.18	-0.677	0.83	0.769	0.41	-0.549
	0.75	0.84	1.075	0.41	-0.632	0.86	0.824	0.41	-0.549
	0.6	0.90	1.195	0.41	-0.633	0.92	0.804	0.42	-0.563
	0.5	0.96	1.211	0.43	-0.660	0.97	0.808	0.44	-0.611
	0.4	1.03	1.307	0.47	-0.755	1.02	0.801	0.49	-0.715
	0.3	1.14	1.340	0.53	-0.993	1.09	0.697	0.54	-0.919
	0.25	1.20	1.214	0.58	-1.211	1.14	0.586	0.58	-1.071
	0.15	1.37	0.311	0.67	-1.965	1.27	0.193	0.64	-1.512
	0.1	0	0	0.73	-2.577	0	0	0.67	-1.816
	0.05	0	0	0.79	-3.412	0	0	0.71	-2.190
2	1	0.65	0.480	0.35	-0.480	0.69	0.356	0.35	-0.424
	0.9	0.67	0.408	0.35	-0.485	0.71	0.307	0.36	-0.428
	0.75	0.79	0.362	0.38	-0.527	0.83	0.289	0.38	-0.465
	0.6	0.95	0.712	0.44	-0.663	0.93	0.441	0.43	-0.573
	0.5	1.04	1.050	0.49	-0.869	0.99	0.554	0.47	-0.715
	0.4	1.14	1.215	0.55	-1.232	1.05	0.582	0.52	-0.940
	0.3	1.26	0.939	0.62	-1.829	1.12	0.474	0.57	-1.268
	0.25	1.32	0.540	0.66	-2.242	1.16	0.363	0.59	-1.475
	0.15	0	0	0.73	-3.361	1.29	0.030	0.64	-1.994
	0.1	0	0	0.77	-4.102	0	0	0.67	-2.312
	0.05	0	0	0.83	-5.017	0	0	0.69	-2.674
2.5	1	0.88	0.358	0.31	-0.363	0.86	0.271	0.32	-0.327
	0.9	0.88	0.275	0.32	-0.368	0.85	0.221	0.33	-0.332
	0.75	0.62	0.095	0.35	-0.418	0.84	0.104	0.36	-0.376
	0.6	0.90	0.214	0.42	-0.607	0.90	0.145	0.42	-0.517
	0.5	1.02	0.819	0.49	-0.901	0.97	0.350	0.46	-0.703
	0.4	1.13	1.183	0.55	-1.414	1.02	0.467	0.51	-0.987
	0.3	1.26	0.921	0.62	-2.229	1.10	0.422	0.56	-1.384
	0.25	1.32	0.437	0.66	-2.780	1.14	0.334	0.58	-1.630
	0.15	0	0	0.74	-4.235	1.26	0.046	0.63	-2.224
	0.1	0	0	0.78	-5.174	0	0	0.66	-2.577
	0.05	0	0	0.83	-6.314	0	0	0.67	-2.975

TABLE A.6 CONTINUED

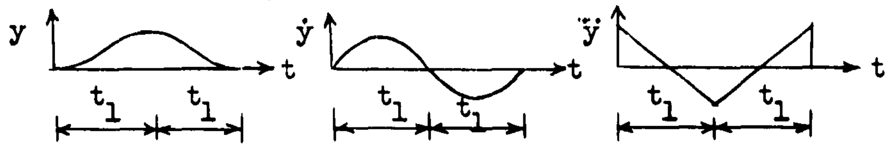
$t_1 f$	$\frac{u_y}{u_o}$	β = 0 $\frac{t_{max}}{t_1}$	β = 0 $\frac{pu_{max}}{\dot{y}_o}$	β = 0 $\frac{t_{min}}{t_1}$	β = 0 $\frac{pu_{min}}{\dot{y}_o}$	β = 0.10 $\frac{t_{max}}{t_1}$	β = 0.10 $\frac{pu_{max}}{\dot{y}_o}$	β = 0.10 $\frac{t_{min}}{t_1}$	β = 0.10 $\frac{pu_{min}}{\dot{y}_o}$
3	1	0.81	0.344	0.28	-0.273	0.82	0.266	0.29	-0.254
	0.9	0.81	0.348	0.28	-0.273	0.82	0.247	0.30	-0.256
	0.75	0.83	0.345	0.29	-0.275	0.85	0.165	0.34	-0.298
	0.6	0.88	0.204	0.35	-0.352	0	0	0.41	-0.467
	0.5	0	0	0.42	-0.573	0.95	0.178	0.46	-0.692
	0.4	1.03	0.822	0.50	-1.079	1.01	0.366	0.50	-1.026
	0.3	1.51	1.196	0.59	-2.011	1.07	0.375	0.55	-1.480
	0.25	1.59	0.888	0.63	-2.687	1.11	0.309	0.58	-1.754
	0.15	0	0	0.72	-4.558	1.22	0.063	0.62	-2.406
	0.1	0	0	0.76	-5.808	0	0	0.64	-2.787
	0.05	0	0	0.82	-7.338	0	0	0.66	-3.209
4	1	0.75	0.159	0.25	-0.159	0.79	0.165	0.29	-0.164
	0.9	0.87	0.290	0.33	-0.207	0.82	0.164	0.31	-0.174
	0.75	0.95	0.256	0.43	-0.538	0.89	0.164	0.38	-0.305
	0.6	1.04	0.737	0.51	-1.316	0	0	0.49	-0.630
	0.5	1.14	1.284	0.57	-2.157	0.98	0.133	0.48	-0.960
	0.4	1.25	1.044	0.63	-3.311	1.03	0.278	0.52	-1.382
	0.3	0	0	0.78	-4.824	1.08	0.271	0.56	-1.897
	0.25	0	0	0.71	-5.734	1.11	0.216	0.57	-2.190
	0.2	0	0	0.77	-7.895	1.20	0.034	0.61	-2.849
	0.15	0	0	0.81	-9.184	0	0	0.62	-3.218
	0.1	0	0	0.86	-10.68	0	0	0.64	-3.614
5	1	0.80	0.166	0.30	-0.134	0.79	0.139	0.29	-0.132
	0.9	0.80	0.169	0.30	-0.134	0.80	0.134	0.30	-0.133
	0.75	0.83	0.167	0.31	-0.136	0.85	0.107	0.35	-0.188
	0.6	0.91	0.415	0.38	-0.262	0.92	0.151	0.42	-0.488
	0.5	0.97	0.162	0.47	-0.896	0	0	0.46	-0.848
	0.4	1.09	1.189	0.55	-2.108	1.00	0.134	0.50	-1.319
	0.3	1.24	1.175	0.63	-4.007	1.04	0.226	0.53	-1.897
	0.25	1.31	0.317	0.66	-5.252	1.07	0.207	0.55	-2.226
	0.15	0	0	0.74	-8.417	1.14	0.074	0.58	-2.962
	0.1	0	0	0.78	-10.38	0	0	0.60	-3.371
	0.05	0	0	0.84	-12.71	0	0	0.62	-3.810
7	1	0.78	0.115	0.27	-0.102	0.78	0.097	0.27	-0.097
	0.9	0.78	0.118	0.26	-0.102	0.79	0.082	0.29	-0.105
	0.75	0.81	0.057	0.31	-0.134	0.85	0.079	0.35	-0.214
	0.6	0.91	0.090	0.41	-0.489	0.91	0.049	0.41	-0.529
	0.5	0.99	0.670	0.47	-1.176	0.95	0.125	0.45	-0.900
	0.4	1.23	1.286	0.55	-2.721	0.98	0.110	0.48	-1.423

TABLE A.6 CONTINUED

$t_1 f$	$\frac{u_y}{u_o}$	β = 0: $\frac{t_{max}}{t_1}$	β = 0: $\frac{pu_{max}}{\dot{y}_o}$	β = 0: $\frac{t_{min}}{t_1}$	β = 0: $\frac{pu_{min}}{\dot{y}_o}$	β = 0.10: $\frac{t_{max}}{t_1}$	β = 0.10: $\frac{pu_{max}}{\dot{y}_o}$	β = 0.10: $\frac{t_{min}}{t_1}$	β = 0.10: $\frac{pu_{min}}{\dot{y}_o}$
	0.3	1.24	1.419	0.63	-5.473	1.02	0.120	0.51	-2.058
	0.25	1.30	2.320	0.67	-7.272	1.04	0.142	0.53	-2.413
	0.15	0	0	0.74	-11.80	1.09	0.082	0.56	-3.196
	0.10	0	0	0.78	-14.58	1.14	0.017	0.58	-3.625
	0.05	0	0	0.84	-17.84	0	0	0.59	-4.079
10	1	0.73	0.070	0.27	-0.070	0.77	0.065	0.27	-0.066
	0.9	1.56	0.075	0.23	-0.075	0.79	0.060	0.29	-0.076
	0.75	1.37	0.232	0.36	-0.221	0.85	0.059	0.35	-0.219
	0.6	1.46	0.069	0.46	-1.357	0.90	0.041	0.41	-0.627
	0.5	1.02	0.093	0.52	-2.966	0.94	0.027	0.44	-1.032
	0.4	1.15	1.932	0.59	-5.419	0.97	0.073	0.47	-1.545
	0.3	1.20	0.534	0.65	-9.312	0.99	0.004	0.49	-2.197
	0.25	0	0	0.69	-11.87	1.01	0.056	0.51	-2.565
	0.15	0	0	0.76	-18.10	1.05	0.069	0.54	-3.376
	0.1	0	0	0.79	-21.81	1.08	0.033	0.55	-3.814
	0.05	0	0	0.85	-26.09	0	0	0.56	-4.274

TABLE A.7 VALUES OF MAXIMUM AND MINIMUM PSEUDO-VELOCITIES AND ASSOCIATED TIMES

Elasto-Plastic Systems Subjected to Ground Motion Shown:



		$\beta = 0$				$\beta = 0.10$			
$t_1 f$	$\frac{u_y}{u_o}$	$\frac{t_{max}}{t_1}$	$\frac{pu_{max}}{\dot{y}_o}$	$\frac{t_{min}}{t_1}$	$\frac{pu_{min}}{\dot{y}_o}$	$\frac{t_{max}}{t_1}$	$\frac{pu_{max}}{\dot{y}_o}$	$\frac{t_{min}}{t_1}$	$\frac{pu_{min}}{\dot{y}_o}$
0.05	1	6.00	0.067	1.00	-0.206	5.20	0.058	1.00	-0.201
	0.75	24.00	0.029	1.00	-0.206	3.40	0.034	1.00	-0.201
	0.5	2.80	0.015	1.00	-0.207	2.40	0.023	1.00	-0.201
	0.25	2.40	0.006	1.00	-0.208	2.00	0.016	1.00	-0.202
	0.15	22.40	0.003	1.00	-0.208	2.00	0.014	1.00	-0.203
	0.1	22.40	0.002	1.00	-0.209	2.00	0.013	1.00	-0.203
0.1	1	-	0.255	1.00	-0.394	3.00	0.225	1.00	-0.371
	0.75	2.80	0.160	1.00	-0.395	2.60	0.163	1.00	-0.372
	0.5	2.60	0.097	1.00	-0.399	2.20	0.119	1.00	-0.376
	0.25	2.40	0.044	1.00	-0.407	2.00	0.079	1.00	-0.383
	0.15	2.20	0.026	1.00	-0.410	2.00	0.066	1.00	-0.387
	0.1	2.20	0.018	1.00	-0.413	2.00	0.059	1.00	-0.389
0.2	1	7.30	0.943	9.80	-0.943	2.00	0.831	0.90	-0.615
	0.75	2.40	0.972	0.90	-0.681	2.00	0.842	0.90	-0.615
	0.5	2.30	0.725	0.90	-0.691	2.00	0.651	0.90	-0.626
	0.25	2.30	0.388	0.90	-0.728	2.00	0.418	0.90	-0.662
	0.15	2.20	0.236	1.00	-0.760	2.00	0.317	0.90	-0.685
	0.1	2.20	0.159	1.00	-0.781	2.00	0.267	0.90	-0.699
0.3	1	8.51	1.855	6.83	-1.855	1.88	1.497	3.35	-1.190
	0.9	1.94	1.809	3.48	-1.577	1.88	1.502	3.35	-0.924
	0.75	1.94	1.839	3.55	-0.971	1.88	1.525	0.80	-0.740
	0.6	2.00	1.899	0.80	-0.840	1.94	1.571	0.80	-0.740
	0.5	2.00	1.971	0.80	-0.840	1.94	1.629	0.80	-0.740
	0.4	2.08	1.882	0.80	-0.842	1.94	1.456	0.80	-0.749
	0.3	2.08	1.560	0.80	-0.863	1.94	1.229	0.80	-0.774
	0.25	2.08	1.362	0.87	-0.889	1.94	1.108	0.80	-0.794
	0.15	2.14	0.905	0.87	-0.972	1.94	0.824	0.87	-0.861
	0.1	2.14	0.632	0.94	-1.041	1.94	0.665	0.87	-0.909
	0.05	5.43	0.331	0.94	-1.130	2.00	0.499	0.94	-0.971
0.4	1	6.65	2.703	5.40	-2.703	1.70	1.850	2.80	-1.740
	0.9	1.75	2.323	2.90	-2.722	1.70	1.859	2.80	-1.407
	0.75	1.75	2.343	2.90	-2.111	1.75	1.903	2.80	-0.856
	0.6	1.80	2.440	2.95	-1.131	1.75	2.007	0.70	-0.790
	0.5	1.85	2.575	0.70	-0.905	1.80	2.124	0.70	-0.790
	0.4	1.90	2.796	0.70	-0.905	1.85	2.179	0.70	-0.791

TABLE A.7 CONTINUED

		β = 0				β = 0.10			
$t_1 f$	$\frac{u_y}{u_o}$	$\frac{t_{max}}{t_1}$	$\frac{pu_{max}}{\dot{y}_o}$	$\frac{t_{min}}{t_1}$	$\frac{pu_{min}}{\dot{y}_o}$	$\frac{t_{max}}{t_1}$	$\frac{pu_{max}}{\dot{y}_o}$	$\frac{t_{min}}{t_1}$	$\frac{pu_{min}}{\dot{y}_o}$
	0.3	1.95	2.864	0.70	-0.907	1.90	1.920	0.70	-0.815
	0.25	1.95	2.608	0.75	-0.928	1.90	1.757	0.75	-0.847
	0.15	2.00	1.918	0.80	-1.048	1.90	1.360	0.80	-0.957
	0.1	2.00	1.424	0.85	-1.172	1.95	1.118	0.85	-1.045
	0.05	2.05	0.798	0.95	-1.365	1.95	0.837	0.90	-1.165
0.5	1	5.52	3.235	4.52	-3.235	1.52	1.930	2.48	-2.083
	0.9	3.52	2.566	2.52	-3.257				
	0.8	1.56	2.473	2.52	-3.321				
	0.75	1.56	2.474	2.56	-3.265	1.56	1.974	2.48	-1.399
	0.7	1.56	2.483	2.56	-2.934				
	0.6	1.60	2.546	2.56	-2.192				
	0.5	1.64	2.693	2.60	-1.326	1.68	2.255	0.60	-0.794
	0.4	1.72	2.988	0.64	-0.919				
	0.3	1.80	3.500	0.64	-0.919				
	0.25	1.34	3.490	0.64	-0.924	1.80	2.238	0.68	-0.844
	0.2	1.88	3.234	0.68	-0.961				
	0.15	1.92	2.862	0.72	-1.050	1.88	1.846	0.76	-0.989
	0.1	1.96	2.282	0.80	-1.226	1.92	1.552	0.80	-1.120
	0.05	2.00	1.372	0.88	-1.540	1.92	1.202	0.84	-1.312
0.7	1	5.64	2.814	2.08	-2.814	1.28	1.664	2.08	-1.844
	0.9	2.80	2.233	2.08	-2.832	1.28	1.664	2.12	-1.842
	0.75	1.28	2.153	4.96	-2.848	1.32	1.706	2.12	-1.391
	0.6	1.32	2.241	2.16	-2.066	1.40	1.862	2.20	-0.864
	0.5	1.40	2.444	3.64	-1.455	1.44	2.085	0.50	-0.750
	0.4	1.48	2.874	0.52	-0.872	1.52	2.421	0.50	-0.751
	0.3	1.64	3.628	0.52	-0.872	1.64	2.435	0.56	-0.784
	0.25	1.68	3.681	0.56	-0.891	1.68	2.436	0.56	-0.837
	0.15	1.84	3.685	0.64	-1.114	1.80	2.298	0.68	-1.069
	0.1	1.92	3.295	0.76	-1.410	1.84	2.085	0.78	-1.278
	0.05	2.00	2.167	0.88	-1.962	1.88	1.700	0.80	-1.586
1	1	1.00	1.274	1.60	-0.764	1.04	0.976	0.40	-0.653
	0.90	1.02	1.289	0.40	-0.761	1.06	0.991	0.40	-0.653
	0.75	1.08	1.412	0.40	-0.761	1.16	1.116	0.40	-0.653
	0.6	1.24	1.854	0.40	-0.761	1.30	1.354	0.40	-0.659
	0.5	1.38	2.116	0.42	-0.777	1.40	1.596	0.42	-0.690

$t_1 f$	$\frac{u_y}{u_o}$	β = 0: $\frac{t_{max}}{t_1}$	β = 0: $\frac{pu_{max}}{\dot{y}_o}$	β = 0: $\frac{t_{min}}{t_1}$	β = 0: $\frac{pu_{min}}{\dot{y}_o}$	β = 0.10: $\frac{t_{max}}{t_1}$	β = 0.10: $\frac{pu_{max}}{\dot{y}_o}$	β = 0.10: $\frac{t_{min}}{t_1}$	β = 0.10: $\frac{pu_{min}}{\dot{y}_o}$
	0.4	1.54	2.766	0.46	-0.850	1.52	1.953	0.48	-0.774
	0.3	1.70	3.698	0.54	-1.042	1.62	2.324	0.54	-0.941
	0.25	1.78	4.121	0.60	-1.220	1.68	2.458	0.60	-1.073
	0.15	1.92	4.191	0.74	-1.873	1.78	2.510	0.68	-1.470
	0.1	1.96	3.509	0.82	-2.414	1.82	2.378	0.74	-1.753
1.25	1	0.80	0.815	2.00	-0.760	0.83	0.576	0.34	-0.580
	0.9	0.82	0.823	0.34	-0.676	0.86	0.499	0.34	-0.584
	0.75	0.86	0.746	0.34	-0.680	1.23	0.631	0.37	-0.618
	0.6	1.20	0.896	0.38	-0.731	1.41	1.206	0.42	-0.707
	0.5	1.42	1.746	0.43	-0.828	1.50	1.696	0.46	-0.820
	0.4	1.60	3.080	0.50	-0.028	1.58	2.177	0.53	-1.000
	0.3	1.76	4.436	0.59	-1.414	1.66	2.548	0.59	-1.270
	0.25	1.82	4.875	0.66	-1.714	1.71	2.662	0.62	-1.446
	0.15	1.94	4.636	0.78	-2.635	1.78	2.695	0.70	-1.895
	0.1	1.97	3.730	0.85	-3.305	1.81	2.606	0.75	-2.176
	0.05	2.00	2.200	0.93	-4.155	1.84	2.436	0.78	-2.499
1.5	1	1.26	0.702	0.29	-0.603	1.21	0.472	0.29	-0.518
	0.9	1.26	0.709	0.29	-0.603	1.20	0.397	0.29	-0.523
	0.75	1.27	0.546	0.30	-0.612	1.20	0.251	0.32	-0.556
	0.6	1.25	0.247	0.34	-0.675	1.30	0.413	0.37	-0.648
	0.5	1.25	0.439	0.38	-0.788	1.43	1.090	0.42	-0.772
	0.4	1.52	2.252	0.47	-1.025	1.52	1.821	0.49	-0.973
	0.3	1.72	4.440	0.56	-1.496	1.62	2.447	0.56	-1.284
	0.25	1.80	5.265	0.64	-1.870	1.66	2.676	0.60	-1.489
	0.15	1.92	5.446	0.77	-3.024	1.74	2.903	0.68	-2.013
	0.1	1.97	4.495	0.84	-3.865	1.78	2.885	0.72	-2.341
	0.05	1.99	2.695	0.92	-4.932	1.81	2.777	0.77	-2.716
2	1	1.00	0.637	1.78	-0.495	1.03	0.411	0.22	-0.424
	0.75	1.05	0.671	0.23	-0.493	1.13	0.316	0.25	-0.459
	0.5	1.24	0.639	1.29	-0.601	1.34	0.399	0.36	-0.690
	0.4	1.35	0.612	0.36	-0.799	1.45	1.026	0.43	-0.925
	0.25	1.68	4.443	0.55	-1.740	1.80	3.218	0.76	-3.491
	0.15	1.88	7.070	0.72	-3.310	1.68	3.127	0.64	-2.190
2.5	1	1.17	0.438	0.18	-0.416	1.12	0.265	0.18	-0.357
	0.9	1.17	-.394	0.18	-0.417	1.11	0.219	0.18	-0.361
	0.75	0.82	0.233	0.20	-0.438	1.10	0.125	0.21	-0.390
	0.6	0.92	0.089	0.25	-0.529	1.18	0.139	0.26	-0.484
	0.5	1.27	0.573	0.30	-0.694	1.31	0.472	0.32	-0.625
	0.4	1.44	1.205	0.40	-1.075	1.41	0.778	0.40	-0.883

TABLE A.7 CONTINUED

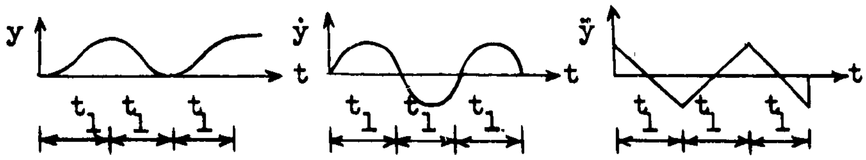
t_1f	$\frac{u_y}{u_o}$	β = 0 $\frac{t_{max}}{t_1}$	$\frac{pu_{max}}{\dot{y}_o}$	$\frac{t_{min}}{t_1}$	$\frac{pu_{min}}{\dot{y}_o}$	β = 0.10 $\frac{t_{max}}{t_1}$	$\frac{pu_{max}}{\dot{y}_o}$	$\frac{t_{min}}{t_1}$	$\frac{pu_{min}}{\dot{y}_o}$
	0.3	1.63	3.891	0.53	-1.892	1.50	1.704	0.48	-1.302
	0.25	1.74	6.397	0.60	-2.560	1.55	2.314	0.52	-1.586
	0.15	1.90	8.491	0.75	-4.646	1.64	3.207	0.61	-2.316
	0.1	1.96	7.381	0.83	-6.168	1.69	3.466	0.66	-2.769
	0.05	1.99	4.570	0.91	-8.083	1.73	3.596	0.70	-3.284
3	1	1.00	0.425	1.85	-0.359	1.03	0.247	0.16	-0.308
	0.9	1.01	0.430	0.16	-0.358	1.04	0.210	0.16	-0.312
	0.75	1.03	0.369	0.16	-0.363	1.06	0.136	0.18	-0.339
	0.6	1.09	0.270	0.19	-0.409	1.16	0.076	0.23	-0.429
	0.5	1.18	0.196	0.23	-0.511	1.27	0.202	0.29	-0.575
	0.4	1.28	0.290	0.32	-0.793	1.38	0.833	0.37	-0.849
	0.3	1.51	1.800	0.46	-1.550	1.48	1.344	0.46	-1.304
	0.25	1.64	4.287	0.53	-2.261	1.52	2.078	0.50	-1.614
	0.15	1.87	9.831	0.71	-4.747	1.61	3.217	0.59	-2.409
	0.1	1.94	9.375	0.80	-6.704	1.65	3.584	0.63	-2.901
	0.05	1.99	6.147	0.90	-0.266	1.69	3.812	0.67	-3.460
4	1	1.00	0.318	1.89	-0.281	1.03	0.175	0.12	-0.242
	0.9	1.01	0.322	0.12	-0.280	1.03	0.146	0.12	-0.244
	0.75	1.02	0.244	0.13	-0.288	1.04	0.086	0.14	-0.267
	0.6	1.07	0.141	0.16	-0.338	0	0	0.19	-0.351
	0.5	0	0	0.20	-0.452	1.23	0.168	0.25	-0.500
	0.4	1.30	0.569	0.30	-0.802	1.34	0.578	0.33	-0.797
	0.3	1.52	2.953	0.44	-1.813	1.43	1.455	0.42	-1.307
	0.25	1.61	3.973	0.53	-4.411	1.48	1.820	0.46	-1.652
	0.15	1.86	12.54	0.71	-6.185	1.57	3.144	0.55	-2.539
	0.1	1.94	12.27	0.81	-8.847	1.61	3.676	0.59	-3.083
	0.05	1.99	8.137	0.90	-12.32	1.65	4.059	0.64	-3.696
5	1	1.00	0.255	1.90	-0.231	1.02	0.135	0.10	-0.198
	0.75	1.02	0.178	0.10	-0.239	1.03	0.062	0.12	-0.221
	0.5	0	0	0.18	-0.411	1.20	0.118	0.22	-0.446
	0.4	1.26	0.167	0.28	-0.818	1.31	0.578	0.30	-0.759
	0.25	1.61	0.515	0.52	-3.302	1.45	1.863	0.44	-1.675
	0.15	1.86	1.525	0.71	-7.620	1.53	3.033	0.52	-2.618
7	1	1.00	0.182	1.93	-0.170	1.02	0.094	0.07	-0.146
	0.9	1.00	0.172	0.07	-0.169	1.02	0.077	0.07	-0.148
	0.75	1.01	0.113	0.08	-0.179	1.03	0.040	0.08	-0.163
	0.6	1.04	0.022	0.10	-0.227	0	0	0.13	-0.230
	0.5	0	0	0.15	-0.355	1.17	0.065	0.19	-0.375
	0.4	1.26	0.390	0.26	-0.866	1.28	0.518	0.27	-0.709

TABLE A.7 CONTINUED

$t_1 f$	$\frac{u_y}{u_o}$	β = 0 $\frac{t_{max}}{t_1}$	$\frac{pu_{max}}{\dot{y}_o}$	$\frac{t_{min}}{t_1}$	$\frac{pu_{min}}{\dot{y}_o}$	β = 0.10 $\frac{t_{max}}{t_1}$	$\frac{pu_{max}}{\dot{y}_o}$	$\frac{t_{min}}{t_1}$	$\frac{pu_{min}}{\dot{y}_o}$
	0.3	1.49	3.750	0.42	-2.606	1.37	1.269	0.36	-1.298
	0.25	1.61	7.398	0.52	-4.346	1.41	1.762	0.40	-1.696
	0.15	1.85	20.63	0.71	-10.49	1.49	2.883	0.49	-2.705
	0.1	1.94	20.95	0.80	-15.28	1.53	3.610	0.53	-3.315
	0.05	1.98	14.11	0.90	-21.48	1.57	4.253	0.57	-3.998
10	1	1.00	0.127	1.95	-0.121	1.02	0.065	0.05	-0.104
	0.75	1.01	0.070	0.06	-0.130	1.02	0.025	0.06	-0.118
	0.5	0	0	0.13	-0.303	1.15	0.053	0.16	-0.313
	0.4	1.25	0.424	0.25	-0.948	1.24	0.504	0.24	-0.663
	0.25	1.61	1.002	0.51	-5.920	1.37	1.718	0.37	-1.706
	0.15	1.85	2.868	0.70	-14.80	1.46	2.844	0.46	-2.757

TABLE A.8 VALUES OF MAXIMUM AND MINIMUM PSEUDO-VELOCITIES AND ASSOCIATED TIMES

Elasto-Plastic Systems Subjected to Ground Motion Shown:



		$\beta = 0$				$\beta = 0.10$			
$t_1 f$	$\frac{u_y}{u_o}$	$\frac{t_{max}}{t_1}$	$\frac{pu_{max}}{\dot{y}_o}$	$\frac{t_{min}}{t_1}$	$\frac{pu_{min}}{\dot{y}_o}$	$\frac{t_{max}}{t_1}$	$\frac{pu_{max}}{\dot{y}_o}$	$\frac{t_{min}}{t_1}$	$\frac{pu_{min}}{\dot{y}_o}$
0.1	1	16.6	0.256	1.00	-0.394	6.40	0.192	1.00	-0.371
	0.9	6.60	0.181	1.00	-0.394	2.00	0.166	1.00	-0.371
	0.75	2.00	0.122	1.00	-0.395	2.00	0.149	1.00	-0.372
	0.6	2.00	0.100	1.00	-0.397	2.00	0.130	1.00	-0.374
	0.5	7.40	0.094	1.00	-0.399	2.00	0.116	1.00	-0.376
	0.4	2.00	0.068	1.00	-0.401	2.00	0.102	1.00	-0.378
	0.3	2.00	0.050	1.00	-0.405	2.00	0.087	1.00	-0.381
	0.25	2.00	0.041	1.00	-0.407	2.00	0.079	1.00	-0.383
	0.15	2.00	0.025	1.00	-0.410	2.00	0.066	1.00	-0.387
	0.1	2.00	0.017	1.00	-0.413	2.00	0.059	1.00	-0.389
	0.05	2.00	0.008	1.00	-0.416	2.00	0.051	1.00	-0.392
0.2	1	2.10	0.904	0.90	-0.681	2.00	0.831	0.90	-0.616
	0.9	2.10	0.907	0.90	-0.681	2.00	0.832	0.90	-0.616
	0.75	2.10	0.912	0.90	-0.681	2.00	0.836	0.90	-0.616
	0.6	2.10	0.769	0.90	-0.685	2.00	0.734	0.90	-0.619
	0.5	2.00	0.660	0.90	-0.693	2.00	0.651	0.90	-0.627
	0.4	2.00	0.547	0.90	-0.704	2.00	0.560	0.90	-0.638
	0.3	2.00	0.426	0.90	-0.720	2.00	0.466	0.90	-0.652
	0.25	2.00	0.359	0.90	-0.731	2.00	0.418	0.90	-0.662
	0.15	2.00	0.221	1.00	-0.763	2.00	0.305	0.90	-0.685
	0.1	2.00	0.150	1.00	-0.783	2.00	0.268	0.90	-0.699
	0.05	2.00	0.073	1.00	-0.809	2.00	0.215	0.90	-0.716
0.3	1	8.17	1.853	6.50	-1.853	1.88	1.497	3.00	-1.435
	0.9	1.94	1.809	3.15	-1.600	1.88	1.502	2.95	-1.208
	0.75	1.94	1.839	3.08	-1.021	1.88	1.525	2.95	-0.828
	0.6	2.00	1.899	0.80	-0.840	1.94	1.571	0.80	-0.740
	0.5	2.00	1.971	0.80	-0.840	1.94	1.629	0.80	-0.740
	0.4	2.01	1.877	0.80	-0.843	1.94	1.456	0.80	-0.749
	0.3	2.01	1.553	0.80	-0.863	1.94	1.229	0.80	-0.774
	0.25	2.01	1.355	0.87	-0.890	1.94	1.108	0.80	-0.794
	0.15	2.01	0.898	0.87	-0.973	1.94	0.824	0.87	-0.861
	0.1	2.01	0.627	0.94	-1.042	1.94	0.665	0.87	-0.909
	0.05	2.00	0.328	0.94	-1.130	2.00	0.499	0.94	-0.971

TABLE A.8 CONTINUED

$t_1 f$	$\frac{u_y}{u_o}$	β = 0 $\frac{t_{max}}{t_1}$	$\frac{pu_{max}}{\dot{y}_o}$	$\frac{t_{min}}{t_1}$	$\frac{pu_{min}}{\dot{y}_o}$	β = 0.10 $\frac{t_{max}}{t_1}$	$\frac{pu_{max}}{\dot{y}_o}$	$\frac{t_{min}}{t_1}$	$\frac{pu_{min}}{\dot{y}_o}$
0.4	1	4.00	3.724	5.25	-3.724	3.90	2.088	2.75	-2.501
	0.9	4.00	3.378	2.80	-3.528	1.70	1.850	2.75	-2.523
	0.75	1.75	2.323	2.85	-3.604	1.70	1.850	2.80	-2.571
	0.6	1.75	2.326	2.90	-3.585	1.70	1.880	2.80	-1.966
	0.5	1.75	2.370	2.90	-2.778	1.75	1.949	2.80	-1.443
	0.4	1.80	2.499	2.90	-1.785	1.80	2.074	2.85	-0.829
	0.3	1.90	2.759	0.70	-0.905	1.85	2.191	0.70	-0.791
	0.25	1.95	2.958	0.70	-0.905	1.85	2.024	0.70	-0.803
	0.15	1.95	2.348	0.75	-0.963	1.90	1.589	0.75	-0.888
	0.1	2.00	1.812	0.80	-1.071	1.95	1.292	0.80	-0.979
	0.05	2.00	1 052	0.90	-1.284	1.95	0.938	0.85	-1.119
0.5	1	5.52	4.853	6.52	-4.853	3.48	2.725	2.52	-2.797
	0.9	3.52	4.886	2.52	-4.071	3.48	2.218	2.52	-2.814
	0.75	3.52	4.027	2.56	-4.101	1.52	1.930	2.56	-2.908
	0.6	1.56	2.473	2.60	-4.312	1.56	1.949	2.60	-2.573
	0.5	1.56	2.474	2.64	-4.511	1.60	2.024	2.64	-2.067
	0.4	1.60	2.546	2.68	-3.542	1.64	2.192	2.68	-1.441
	0.3	1.68	2.819	2.76	-2.277	1.72	2.488	0.60	-0.794
	0.25	1.72	3.087	2.80	-1.407	1.76	2.463	0.64	-0.800
	0.15	1.88	3.380	0.68	-0.935	1.84	2.077	0.68	-0.897
	0.1	1.92	2.862	0.72	-1.050	1.88	1.762	0.76	-1.025
	0.05	2.00	1.876	0.84	-1.362	1.92	1.323	0.84	-1.238
0.7	1	2.92	2.878	2.12	-2.878	3.04	1.742	2.16	-1.951
	0.9	2.92	2.356	2.12	-2.899	1.28	1.664	2.16	-1.971
	0.75	1.28	2.153	2.16	-3.066	1.32	1.683	2.24	-1.730
	0.6	1.32	2.227	2.24	-2.441	1.36	1.808	2.32	-1.405
	0.5	1.40	2.408	2.36	-2.120	1.44	2.004	2.44	-1.177
	0.4	1.48	2.822	2.50	-1.721	1.52	2.352	2.52	-0.835
	0.3	1.60	3.624	0.52	-0.872	1.60	2.427	0.52	-0.774
	0.25	1.68	3.677	0.52	-0.886	1.68	2.432	0.56	-0.821
	0.15	1.84	3.696	0.64	-1.101	1.80	2.322	0.68	-1.040
	0.1	1.92	3.328	0.72	-1.392	1.84	2.112	0.72	-1.252
	0.05	2.00	2.201	0.84	-1.946	1.88	1.727	0.80	-1.565
1	1	1.00	1.274	3.50	-1.274	1.04	0.976	0.40	-0.653
	0.9	1.02	1.289	3.50	-1.001	1.06	0.991	0.40	-0.653
	0.75	1.08	1.412	0.40	-0.761	1.16	1.116	0.40	-0.653
	0.6	1.24	1.855	0.40	-0.761	1.30	1.354	0.40	-0.659
	0.5	1.38	2.116	0.42	-0.777	1.40	1.597	0.42	-0.690
	0.4	1.54	2.766	2.56	-1.171	1.52	1.953	2.52	-0.855

TABLE A.8 CONTINUED

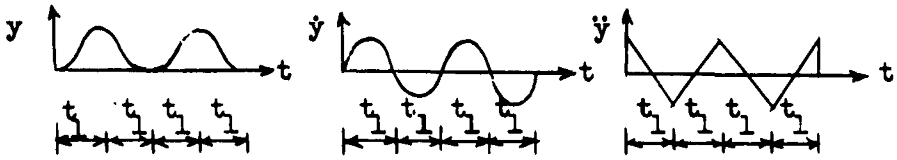
		β = 0				β = 0.10			
$t_1 f$	$\frac{u_y}{u_o}$	$\frac{t_{max}}{t_1}$	$\frac{pu_{max}}{y_o}$	$\frac{t_{min}}{t_1}$	$\frac{pu_{min}}{y_o}$	$\frac{t_{max}}{t_1}$	$\frac{pu_{max}}{y_o}$	$\frac{t_{min}}{t_1}$	$\frac{pu_{min}}{y_o}$
	0.3	1.70	3.698	0.54	-1.042	1.62	2.324	0.54	-0.941
	0.25	1.78	4.121	0.60	-1.220	1.68	2.458	0.60	-1.073
	0.15	1.92	4.191	0.74	-1.873	1.78	2.510	0.68	-1.470
	0.1	1.96	3.509	0.82	-2.414	1.82	2.378	0.74	-1.752
	0.05	2.00	2.144	0.90	-3.160	1.86	2.128	0.80	-2.101
1.25	1	0.80	0.815	2.00	-0.760	0.83	0.576	0.34	-0.580
	0.9	0.82	0.814	0.34	-0.676	0.86	0.499	0.34	-0.585
	0.75	0.86	0.746	0.34	-0.680	1.23	0.631	0.37	-0.618
	0.6	1.20	0.896	0.38	-0.732	1.41	1.206	0.42	-0.707
	0.5	1.42	1.746	0.43	-0.828	1.50	1.696	2.50	-0.917
	0.4	1.60	3.080	2.64	-1.511	1.58	2.177	2.59	-1.067
	0.30	1.76	4.436	0.59	-1.414	1.66	2.548	0.59	-1.270
	0.25	1.82	4.875	0.66	-1.714	1.71	2.662	0.62	-1.446
	0.15	1.94	4.636	0.78	-2.635	1.78	2.695	0.70	-1.895
1.5	1	1.26	0.702	2.24	-0.837	1.21	0.472	0.29	-0.518
	0.9	1.26	0.702	2.24	-0.844	1.20	0.397	2.17	-0.566
	0.75	1.26	0.710	2.26	-0.698	1.20	0.251	2.21	-0.633
	0.6	1.27	0.484	2.26	-0.676	1.30	0.412	0.37	-0.648
	0.5	1.25	0.240	2.27	-0.825	1.43	1.089	2.42	-0.775
	0.4	1.33	0.767	0.40	-0.827	1.52	1.821	2.53	-1.088
	0.3	1.61	3.197	2.65	-1.815	1.62	2.447	2.63	-1.293
	0.25	1.73	4.475	0.58	-1.508	1.66	2.676	0.60	-1.488
	0.15	1.90	5.657	0.73	-2.625	1.74	2.903	0.68	-2.013
2	1	1.00	0.637	3.25	-0.637	1.03	0.411	0.22	-0.424
	0.9	1.01	0.645	3.25	-0.500	1.05	0.359	0.23	-0.428
	0.75	1.05	0.671	0.23	-0.493	1.13	0.316	0.25	-0.459
	0.6	1.13	0.591	0.25	-0.524	1.26	0.360	0.31	-0.554
	0.5	1.24	0.639	0.29	-0.601	1.34	0.399	2.35	-0.722
	0.4	1.35	0.612	2.36	-0.848	1.45	1.025	2.45	-0.992
	0.3	1.54	2.169	2.57	-2.133	1.55	2.097	2.55	-1.394
	0.25	1.68	4.443	2.70	-2.241	1.60	2.539	2.60	-1.580
	0.15	1.88	7.070	0.72	-3.311	1.68	3.127	0.64	-2.190
	0.1	1.95	6.463	0.81	-4.559	1.72	3.256	0.68	-2.590
	0.05	1.99	4.149	0.90	-6.214	1.76	3.272	0.73	-3.046

TABLE A.8 CONTINUED

$t_1 f$	$\frac{u_y}{u_o}$	β = 0: $\frac{t_{max}}{t_1}$	β = 0: $\frac{pu_{max}}{\dot{y}_o}$	β = 0: $\frac{t_{min}}{t_1}$	β = 0: $\frac{pu_{min}}{\dot{y}_o}$	β = 0.10: $\frac{t_{max}}{t_1}$	β = 0.10: $\frac{pu_{max}}{\dot{y}_o}$	β = 0.10: $\frac{t_{min}}{t_1}$	β = 0.10: $\frac{pu_{min}}{\dot{y}_o}$
2.5	1	1.17	0.438	2.16	-0.472	1.12	0.265	0.18	-0.357
	0.9	1.17	0.438	2.16	-0.449	1.11	0.219	0.18	-0.361
	0.75	1.18	0.300	2.16	-0.445	1.10	0.125	2.08	-0.397
	0.6	0.87	0.132	0.23	-0.488	1.18	0.139	0.26	-0.484
	0.5	1.18	0.272	0.28	-0.613	1.31	0.472	0.32	-0.625
	0.4	1.40	1.164	0.37	-0.921	1.41	0.778	0.40	-0.883
	0.3	1.57	2.568	2.60	-2.840	1.50	1.704	2.50	-1.410
	0.25	1.70	5.496	2.72	-2.676	1.55	2.314	2.55	-1.661
	0.15	1.89	8.517	0.74	-4.341	1.64	3.207	0.61	-2.316

TABLE A.9 VALUES OF MAXIMUM AND MINIMUM PSEUDO-VELOCITIES AND ASSOCIATED TIMES

Elasto-Plastic Systems Subjected to Ground Motion Shown:



		$\beta = 0$				$\beta = 0.10$			
$t_1 f$	$\frac{u_y}{u_o}$	$\frac{t_{max}}{t_1}$	$\frac{pu_{max}}{\dot{y}_o}$	$\frac{t_{min}}{t_1}$	$\frac{pu_{min}}{\dot{y}_o}$	$\frac{t_{max}}{t_1}$	$\frac{pu_{max}}{\dot{y}_o}$	$\frac{t_{min}}{t_1}$	$\frac{pu_{min}}{\dot{y}_o}$
0.1	1	14.60	0.413	19.60	-0.413	4.20	0.364	1.00	-0.371
	0.9	4.40	0.379	1.00	-0.394	4.00	0.316	1.00	-0.371
	0.75	4.20	0.276	1.00	-0.395	4.00	0.243	1.00	-0.372
	0.6	4.00	0.181	1.00	-0.397	4.00	0.207	1.00	-0.374
	0.5	4.40	0.172	1.00	-0.398	4.00	0.200	1.00	-0.376
	0.4	4.40	0.145	1.00	-0.401	4.00	0.176	1.00	-0.378
	0.3	4.40	0.108	1.00	-0.404	4.00	0.151	1.00	-0.381
	0.25	4.40	0.088	1.00	-0.406	4.00	0.138	1.00	-0.383
	0.15	4.20	0.054	1.00	-0.410	4.00	0.117	1.00	-0.387
	0.1	4.20	0.038	1.00	-0.412	4.00	0.104	1.00	-0.389
	0.05	4.00	0.016	1.00	-0.416	4.00	0.091	1.00	-0.392
0.2	1	2.10	0.904	0.90	-0.682	2.00	0.831	0.90	-0.616
	0.9	2.10	0.907	0.90	-0.681	2.00	0.832	0.90	-0.616
	0.75	2.10	0.912	0.90	-0.681	2.00	0.836	0.90	-0.616
	0.6	4.00	0.832	0.90	-0.685	4.00	0.798	0.90	-0.619
	0.5	4.00	0.859	0.90	-0.693	4.00	0.839	0.90	-0.627
	0.4	4.10	0.861	0.90	-0.704	4.00	0.762	0.90	-0.638
	0.3	4.10	0.725	0.90	-0.720	4.00	0.661	0.90	-0.652
	0.25	4.10	0.632	0.90	-0.731	4.00	0.607	0.90	-0.662
	0.15	4.10	0.414	1.00	-0.763	4.00	0.480	0.90	-0.685
	0.1	4.20	0.291	1.00	-0.783	4.00	0.415	0.90	-0.699
	0.05	4.10	0.144	1.00	-0.809	4.00	0.341	0.90	-0.716
0.3	1	1.94	1.805	3.08	-1.805	1.88	1.497	3.00	-1.435
	0.9	1.94	1.811	3.08	-1.483	1.88	1.502	2.95	-1.208
	0.75	1.94	1.845	3.02	-0.927	1.88	1.525	2.95	-0.828
	0.6	2.00	1.908	0.80	-0.840	1.94	1.571	0.80	-0.740
	0.5	4.00	2.066	0.80	-0.840	3.95	1.701	0.80	-0.740
	0.4	3.95	2.149	0.80	-0.844	3.95	1.601	0.80	-0.749
	0.3	3.95	1.969	0.80	-0.866	3.95	1.431	0.80	-0.774
	0.25	4.00	1.818	0.87	-0.894	3.95	1.335	0.80	-0.794
	0.15	4.00	1.381	0.87	-0.977	3.95	1.071	0.87	-0.861
	0.1	4.00	1.040	0.94	-1.046	3.95	0.901	0.87	-0.909
	0.05	4.02	0.591	0.94	-1.132	3.95	0.702	0.94	-0.971

TABLE A.9 CONTINUED

		β = 0				β = 0.10			
$t_1 f$	$\frac{u_y}{u_o}$	$\frac{t_{max}}{t_1}$	$\frac{pu_{max}}{\dot{y}_o}$	$\frac{t_{min}}{t_1}$	$\frac{pu_{min}}{\dot{y}_o}$	$\frac{t_{max}}{t_1}$	$\frac{pu_{max}}{\dot{y}_o}$	$\frac{t_{min}}{t_1}$	$\frac{pu_{min}}{\dot{y}_o}$
0.4	1	8.90	4.369	7.65	-4.369	3.80	2.755	2.75	-2.501
	0.9	3.90	4.333	5.15	-3.579	3.80	2.728	2.75	-2.501
	0.75	3.95	3.884	2.80	-3.532	3.80	1.982	2.75	-2.529
	0.6	3.95	2.351	2.85	-3.647	1.70	1.860	2.80	-2.248
	0.5	1.75	2.328	2.90	-3.486	1.75	1.906	2.80	-1.722
	0.4	1.75	2.397	2.90	-2.499	1.75	2.010	2.80	-1.098
	0.3	1.85	2.605	2.90	-1.220	1.85	2.202	0.70	-0.790
	0.25	1.90	2.785	0.70	-0.905	3.85	2.161	0.70	-0.795
	0.15	3.90	2.941	0.75	-0.933	3.90	1.833	0.75	-0.867
	0.1	3.95	2.618	0.80	-1.025	3.90	1.578	0.80	-0.958
	0.05	4.00	1.871	0.90	-1.237	3.95	1.223	0.85	-1.103
0.5	1	7.52	6.470	6.52	-6.470	3.50	3.436	4.48	-3.194
	0.9	3.52	5.688	4.52	-6.514	3.52	3.454	2.52	-2.797
	0.8	3.52	5.710	4.52	-5.489				
	0.75	3.56	5.759	4.52	-4.773	3.56	3.128	2.52	-2.808
	0.7	3.56	5.838	2.52	-4.071				
	0.6	3.60	5.652	2.52	-4.073	3.60	2.161	2.56	-2.923
	0.5	3.64	4.269	2.56	-4.185	1.56	1.943	2.60	-2.640
	0.4	3.68	2.629	2.64	-4.509	1.60	2.035	2.64	-2.018
	0.3	1.60	2.546	2.68	-3.543	1.68	2.265	2.72	-1.215
	0.25	1.64	2.693	2.72	-2.749	1.72	2.463	0.60	-0.794
	0.2	1.72	2.988	2.80	-1.717				
	0.15	1.80	3.500	0.64	-0.919	3.80	2.274	0.68	-0.846
	0.1	3.88	3.516	0.68	-0.961	3.84	2.056	0.72	-0.958
	0.05	3.92	3.036	0.80	-1.226	3.88	1.657	0.80	-1.181
	0.03	3.96	2.418	0.88	-1.462				
	0.01	4.00	1.111	0.96	-1.832				
0.7	1	2.92	2.869	2.12	-2.869	3.04	1.755	2.16	-1.951
	0.9	2.92	2.356	2.12	-2.899	1.28	1.664	2.16	-1.971
	0.75	1.28	2.153	2.16	-3.066	1.32	1.683	2.24	-1.730
	0.6	1.32	2.227	2.24	-2.441	1.36	1.808	2.32	-1.405
	0.5	1.40	2.408	2.36	-2.120	1.44	2.004	2.44	-1.177
	0.4	3.52	2.885	2.50	-1.721	3.52	2.355	2.52	-0.835
	0.3	1.60	3.624	0.52	-0.872	1.60	2.427	0.52	-0.774
	0.25	1.68	3.677	0.52	-0.887	1.68	2.432	0.56	-0.821
	0.15	3.84	3.981	0.64	-1.101	3.76	2.360	0.68	-1.040
	0.1	3.88	4.133	0.72	-1.392	3.84	2.225	0.72	-1.252
	0.05	3.96	3.330	0.84	-1.946	3.88	1.917	0.80	-1.565

TABLE A.9 CONTINUED

		β = 0				β = 0.10			
$t_1 f$	$\frac{u_y}{u_o}$	$\frac{t_{max}}{t_1}$	$\frac{pu_{max}}{\dot{y}_o}$	$\frac{t_{min}}{t_1}$	$\frac{pu_{min}}{\dot{y}_o}$	$\frac{t_{max}}{t_1}$	$\frac{pu_{max}}{\dot{y}_o}$	$\frac{t_{min}}{t_1}$	$\frac{pu_{min}}{\dot{y}_o}$
1	1	1.00	1.274	3.60	-0.767	1.04	0.976	0.40	-0.653
	0.9	3.02	1.289	0.40	-0.761	1.06	0.991	0.40	-0.653
	0.75	1.08	1.412	0.40	-0.761	1.16	1.116	0.40	-0.653
	0.6	1.24	1.855	0.40	-0.761	1.30	1.354	0.40	-0.659
	0.5	1.38	2.116	0.42	-0.777	1.40	1.597	0.42	-0.690
	0.4	3.58	2.862	2.56	-1.171	3.54	1.966	2.52	-0.855
	0.3	1.70	3.698	0.54	-1.042	1.62	2.324	0.54	-0.941
	0.25	3.78	4.190	0.60	-1.220	1.68	2.458	0.60	-1.073
	0.15	3.88	5.170	0.74	-1.873	3.76	2.552	0.68	-1.470
	0.1	3.94	4.963	0.82	-2.414	3.80	2.465	0.74	-1.752
	0.05	3.98	3.551	0.90	-3.160	3.86	2.254	0.80	-2.101
1.5	1	3.22	0.990	2.24	-0.837	3.15	0.534	0.29	-0.518
	0.9	3.23	0.999	2.23	-0.838	3.16	0.461	2.16	-0.548
	0.75	3.24	0.809	2.24	-0.847	1.20	0.277	2.16	-0.644
	0.6	3.25	0.712	2.26	-0.642	1.27	0.306	0.36	-0.632
	0.5	3.25	0.484	2.26	-0.683	1.40	0.976	0.42	-0.748
	0.4	0.78	0.198	2.27	-0.841	3.52	1.743	2.52	-1.058
	0.3	3.50	1.958	2.50	-1.263	1.61	2.396	2.62	-1.276
	0.25	3.66	3.358	2.65	-1.798	1.66	2.646	0.60	-1.454
	0.15	3.83	6.309	0.68	-2.241	3.73	2.917	0.67	-1.984
2	1	1.00	0.637	3.78	-0.496	1.03	0.411	0.22	-0.424
	0.9	3.01	0.645	0.23	-0.493	1.05	0.359	0.23	-0.428
	0.75	1.05	0.671	0.23	-0.493	1.13	0.315	0.25	-0.459
	0.6	1.13	0.591	0.25	-0.524	1.26	0.360	0.31	-0.554
	0.5	1.24	0.639	0.29	-0.601	1.34	0.399	2.35	-0.722
	0.4	1.35	0.612	2.36	-0.848	3.45	1.032	2.45	-0.992
	0.3	3.58	2.509	2.57	-2.133	3.55	2.102	2.55	-1.394
	0.25	1.68	4.443	2.70	-2.241	1.60	2.539	2.60	-1.588
	0.15	3.85	8.363	0.72	-3.311	3.68	3.129	0.64	-2.189
	0.1	3.92	8.951	0.81	-4.559	3.72	3.264	0.68	-2.590
	0.05	3.97	6.849	0.90	-6.214	3.76	3.290	0.73	-3.046
2.5	1	3.15	0.515	2.16	-0.472	3.08	0.281	0.18	-0.357
	0.9	3.15	0.500	2.16	-0.473	3.08	0.242	0.18	-0.361
	0.75	3.15	0.387	2.16	-0.432	3.09	0.152	2.08	-0.397
	0.6	3.15	0.239	0.21	-0.455	3.18	0.142	0.26	-0.484
	0.5	3.15	0.125	0.26	-0.543	1.31	0.472	0.32	-0.625

TABLE A.9 CONTINUED

$t_1 f$	$\frac{u_y}{u_o}$	β = 0 $\frac{t_{max}}{t_1}$	$\frac{pu_{max}}{\dot{y}_o}$	$\frac{t_{min}}{t_1}$	$\frac{pu_{min}}{\dot{y}_o}$	β = 0.10 $\frac{t_{max}}{t_1}$	$\frac{pu_{max}}{\dot{y}_o}$	$\frac{t_{min}}{t_1}$	$\frac{pu_{min}}{\dot{y}_o}$
	0.4	1.33	0.837	0.34	-0.777	3.41	0.778	0.40	-0.883
	0.3	3.50	1.184	2.49	-2.893	3.50	1.709	2.50	-1.410
	0.25	3.68	4.290	2.67	-3.018	3.55	2.317	2.55	-1.661
	0.15	3.84	9.830	0.71	-3.986	3.64	3.207	0.61	-2.316

TABLE A.10 VALUES OF MAXIMUM AND MINIMUM DEFORMATIONS AND DISPLACEMENTS WITH THE ASSOCIATE TIMES

Elasto-Plastic System, Damping Factor, β = 0.02, Eureka Earthquake

y_o = 10.00 in., $\dot{y}_o$ = 12.50 in./sec., $\ddot{y}_o$ = 0.178 g, Duration of Quake = 20 sec.

$\frac{u_y}{u_o}$	Deformations				Absolute Displacements			
	t_{max} sec.	u_{max} in.	t_{min} sec.	u_{min} in.	t_{max} sec.	x_{max} in.	t_{min} sec.	x_{min} in.
				f = 1/25 = .04 cps				
1.00	4.80	9.563	12.60	-5.783				
0.80	4.80	9.571	10.22	-3.632	19.84	3.341		
0.70	4.80	9.597	10.22	-2.647	19.40	4.963		
0.60	17.78	9.868	9.00	-1.872	19.00	6.622		
0.50	4.80	9.674	8.64	-1.175	18.40	6.182		
0.40	4.80	9.724	8.64	-0.743				
0.30	4.80	9.778	8.64	-0.468			7.20	-0.746
0.25	4.80	9.812	8.64	-0.344			7.06	-0.609
0.20	4.80	9.846	2.80	-0.320			6.96	-0.479
0.15	4.80	9.879	2.80	-0.320			6.88	-0.363
0.10	4.80	9.915	12.60	-0.355			6.72	-0.255
0.05	4.80	9.957	12.60	-0.671			6.44	-0.146
				f = 1/15 = 0.067 cps				
1.00	4.80	8.861	9.00	-7.897	15.60	6.218	8.64	-7.955
0.818	14.96	9.724	8.64	-5.920	15.34	7.519		
0.716	14.78	9.885	8.64	-4.569	15.10	7.573		
0.614	14.60	9.911	8.60	-3.289	14.96	7.518		
0.511	14.56	9.237	8.60	-2.382	14.88	6.811		
0.409	4.80	9.329	8.60	-1.634	14.88	5.840		
0.307	4.80	9.479	8.64	-1.047	14.80	4.618		
0.256	4.80	9.567	8.64	-0.760	14.78	3.987		
0.205	4.80	9.656	8.64	-0.509	14.78	3.308		
0.153	4.80	9.745	8.64	-0.307	14.78	2.557	6.74	-0.781
0.102	4.80	9.843	2.80	-0.270			6.60	-0.500
0.051	4.80	9.953	2.80	-0.270			6.20	-0.223
				f = 1/10 = 0.1 cps				
1.00	12.00	9.603	18.06	-9.616	12.36	9.562	17.66	-13.10
0.80	4.60	7.688	7.44	-9.117	12.50	7.019	17.84	-12.00
0.70	4.60	7.766	7.44	-7.849	12.50	6.468	17.84	-10.34
0.60	4.70	7.911	7.44	-6.389	12.50	6.133	17.96	-8.382
0.50	4.70	8.130	7.00	-4.964	12.60	5.738	7.10	-6.727
0.40	4.80	8.427	6.96	-3.530	12.66	5.346	7.06	-5.270
0.30	4.80	8.773	8.58	-2.337	12.88	4.567	6.96	-3.874
0.25	4.80	8.982	8.58	-1.735	13.00	4.140	6.88	-3.123
0.20	4.80	9.193	8.60	-1.217	13.16	3.613	6.80	-2.420

TABLE A.10 CONTINUED

$\frac{u_y}{u_o}$	Deformations				Absolute Displacements			
	t_{max} sec.	u_{max} in.	t_{min} sec.	u_{min} in.	t_{max} sec.	x_{max} in.	t_{min} sec.	x_{min} in.
0.15	4.80	9.403	8.60	-0.779	13.38	3.007	6.72	-1.762
0.10	4.80	9.641	8.64	-0.370	13.78	2.417	6.62	-1.102
0.05	4.80	9.902	1.00	-0.213			6.32	-0.450
				f = 1/7 = 0.143 cps				
1.00	9.90	11.009	6.32	-11.83	10.00	11.139	6.38	-13.62
0.80	3.98	7.067	6.60	-12.06	10.12	6.380	6.44	-13.76
0.70	3.98	7.067	6.62	-12.37	10.32	3.855	6.58	-14.00
0.60	3.98	7.067	6.62	-12.78	10.40	1.374	6.72	-14.40
0.50	3.98	7.087	6.62	-11.03	10.54	0.876	6.72	-12.66
0.40	3.98	7.151	6.62	-9.090	10.60	0.614	6.72	-10.72
0.30	3.98	7.269	6.62	-6.970	10.72	0.585	6.78	-8.613
0.25	4.60	7.485	6.62	-5.742	10.86	0.745	6.78	-7.383
0.20	4.70	7.887	6.62	-4.282	10.98	1.111	6.78	-5.910
0.15	4.80	8.354	6.62	-2.830	11.20	1.525	6.72	-4.448
0.10	4.80	8.903	8.58	-1.484	11.46	1.816	6.72	-2.955
0.05	4.80	9.550	8.64	-0.531	13.48	1.324	6.58	-1.351
				f = 1/5 = 0.2 cps				
1.00	8.22	12.272	5.88	-12.81	8.32	11.56	5.76	-15.86
0.80	8.22	6.697	6.12	-13.14	8.40	6.043	5.84	-15.92
0.70	3.90	6.273	6.18	-13.57	8.40	2.903	5.88	-16.07
0.60	3.90	6.273	6.18	-14.17	3.12	0.429	5.94	-16.32
0.50	3.90	6.273	6.22	-14.95	3.12	0.429	16.20	-16.97
0.40	3.90	6.307	6.22	-13.08	3.12	0.429	6.12	-14.85
0.30	3.90	6.448	6.24	-10.59	3.12	0.429	6.18	-12.32
0.25	3.96	6.569	6.24	-9.250	3.12	0.429	6.22	-10.96
0.20	3.96	6.744	12.46	-7.987	3.12	0.429	6.32	-9.495
0.15	3.96	6.962	12.60	-7.363	3.12	0.429	16.20	-8.853
0.10	4.68	7.395	15.84	-6.916	3.12	0.429	19.80	-9.343
0.05	4.80	8.594	8.58	-2.061	3.12	0.430	19.08	-4.252
				f = 1/4 = 0.25 cps				
1.00	7.56	13.24	5.70	-11.82	7.52	11.521	5.52	-16.35
0.80	7.58	10.87	5.70	-11.89	7.56	9.134	5.52	-16.36
0.70	7.60	7.990	5.76	-12.08	7.58	6.248	5.52	-16.39
0.60	3.84	5.476	5.76	-12.49	7.60	3.022	5.52	-16.51
0.50	3.84	5.476	5.84	-13.17	3.00	0.413	5.60	-16.76
0.40	3.84	5.477	5.84	-13.65	3.00	0.413	5.70	-16.86
0.30	3.84	5.585	6.12	-11.46	3.00	0.413	5.76	-14.26
0.25	3.90	5.708	6.18	-10.33	3.00	0.413	5.76	-12.81
0.20	3.90	5.907	6.20	-9.033	3.00	0.413	19.14	-11.42

TABLE A.10 CONTINUED

$\frac{u_y}{u_o}$	Deformations				Absolute Displacements			
	t_{max} sec.	u_{max} in.	t_{min} sec.	u_{min} in.	t_{max} sec.	x_{max} in.	t_{min} sec.	x_{min} in.
0.15	3.90	6.191	15.76	-7.956	3.00	0.413	19.20	-10.49
0.10	3.96	6.588	12.60	-6.813	3.00	0.413	18.96	-8.887
0.05	4.68	7.461	12.60	-5.046	3.00	0.413	18.84	-7.532
				f = 1/3 = 0.33 cps				
1.00	9.72	10.61	8.28	-10.75	9.78	10.51	5.08	-15.64
0.80	6.84	10.78	5.28	-7.974	9.78	10.29	5.08	-15.64
0.70	6.88	10.14	5.34	-7.986	9.84	9.607	5.08	-15.64
0.60	6.90	7.908	5.34	-8.161	13.02	7.393	5.08	-15.64
0.50	13.26	5.250	5.52	-8.681	13.08	4.863	5.10	-15.68
0.40	3.72	4.352	5.56	-9.624	13.16	1.835	5.16	-15.77
0.30	3.78	4.491	5.64	-8.001	13.26	1.627	5.22	-13.49
0.25	3.84	4.643	5.76	-7.210	13.38	1.450	5.22	-12.21
0.20	3.84	4.903	5.82	-6.321	13.44	1.324	5.28	-10.77
0.15	3.84	5.248	5.82	-5.467	2.16	0.341	5.40	-9.264
0.10	3.90	5.793	12.36	-5.088	2.16	0.341	5.56	-7.573
0.05	3.96	6.554	12.54	-6.441	2.16	0.341	18.48	-8.010
				f = 1/2.5 = 0.4 cps				
1.00	6.42	7.112	10.22	-6.694	9.00	6.591	4.80	-14.63
0.80	6.44	7.268	5.10	-5.113	9.02	7.034	4.80	-14.63
0.70	9.12	7.243	5.10	-5.115	9.06	7.214	4.80	-14.63
0.60	9.18	6.112	5.10	-5.428	9.16	6.020	4.86	-14.70
0.50	9.30	4.417	5.22	-5.660	9.24	4.239	4.88	-14.34
0.40	9.38	4.171	5.46	-4.924	9.38	3.925	4.96	-12.96
0.30	3.78	4.180	5.56	-4.587	9.48	2.402	5.04	-11.36
0.25	3.78	4.422	5.60	-4.161	9.54	1.931	5.04	-10.30
0.20	3.84	4.792	5.76	-3.608	13.26	2.305	5.10	-9.014
0.15	3.90	5.244	5.82	-3.202	13.38	2.313	5.20	-7.681
0.10	3.90	5.861	6.18	-2.922	13.38	0.345	5.40	-6.252
0.05	4.62	6.731	12.50	-4.858	1.80	0.326	18.34	-6.188
				f = 1/2 = 0.5 cps				
1.00	19.02	3.228	4.38	-4.986	12.96	3.325	4.50	-13.50
0.80	3.56	3.221	4.44	-5.066	12.88	2.055	4.56	-13.74
0.70	3.56	3.221	4.44	-5.190	12.84	1.320	4.62	-14.04
0.60	3.56	3.225	4.44	-4.865	12.78	1.151	4.68	-13.91
0.50	3.60	3.283	4.44	-3.819	12.84	3.554	4.70	-13.03
0.40	13.20	4.753	5.20	-3.069	13.02	4.683	4.80	-12.03

TABLE A 10 CONTINUED

$\frac{u_y}{u_o}$	Deformations				Absolute Displacements			
	t_{max} sec.	u_{max} in.	t_{min} sec.	u_{min} in.	t_{max} sec.	x_{max} in.	t_{min} sec.	x_{min} in.
0.30	13.32	4.488	5.52	-2.858	13.08	3.918	4.88	-10.81
0.25	3.78	4.030	5.56	-3.001	13.14	2.738	4.96	-10.06
0.20	3.84	4.518	5.64	-2.769	13.16	2.309	5.04	-8.871
0.15	3.84	5.132	5.76	-2.581	13.20	1.904	5.16	-7.516
0.10	3.90	5.902	6.18	-2.619	13.26	1.026	5.40	-6.194
0.05	4.68	7.030	8.52	-2.950	3.12	0.419	5.76	-4.640
				$f = 1/1.5 = 0.67$ cps				
1.00	6.27	4.433	7.00	-4.162	10.68	3.795	4.23	-11.69
0.80	6.30	3.668	5.52	-4.024	12.27	3.311	4.23	-11.70
0.70	6.30	2.723	5.49	-4.153	10.68	2.510	4.26	-11.77
0.60	3.51	1.942	5.49	-4.287	10.68	1.760	4.28	-11.95
0.50	3.51	1.942	5.46	-4.471	10.65	1.042	4.35	-12.28
0.40	3.51	1.947	5.43	-4.495	10.65	0.554	4.41	-12.39
0.30	3.54	2.073	5.43	-4.105	2.79	0.512	5.04	-11.72
0.25	3.56	2.266	5.49	-3.975	2.79	0.512	5.01	-11.57
0.20	3.60	2.584	5.52	-3.942	2.79	0.511	5.01	-11.20
0.15	3.63	3.052	5.58	-4.078	2.79	0.512	5.01	-10.58
0.10	3.81	3.853	5.79	-4.397	2.79	0.512	5.16	-9.365
0.05	3.90	5.218	6.18	-5.138	2.79	0.512	5.58	-7.744
				$f = 1/1.25 = 0.80$				
1.00	5.91	5.438	5.25	-4.949	8.43	3.383	5.20	-12.36
0.80	5.94	4.356	5.25	-4.979	8.46	2.553	5.20	-12.36
0.70	4.68	3.414	5.28	-5.119	8.49	1.598	5.20	-12.39
0.60	4.68	3.417	5.31	-5.003	8.52	0.946	5.22	-12.16
0.50	4.70	2.983	5.31	-4.430	8.55	0.722	5.22	-11.55
0.40	4.71	1.698	5.34	-4.740	2.43	0.499	5.20	-11.81
0.30	3.48	1.599	5.40	-5.564	2.43	0.499	5.20	-12.53
0.25	3.48	1.622	5.46	-5.607	2.43	0.499	5.19	-12.45
0.20	3.52	1.802	5.49	-5.422	2.43	0.499	5.19	-12.09
0.15	3.56	2.176	5.52	-4.753	2.43	0.499	5.13	-11.42
0.10	3.63	2.850	5.58	-4.216	2.43	0.499	5.10	-10.53
0.05	3.84	4.212	5.82	-4.519	2.43	0.499	5.28	-8.810
				$f = 1$ cps				
1.00	5.55	2.994	5.07	-3.306	12.60	2.501	4.98	-12.39
0.80	4.53	1.872	5.07	-3.401	12.63	1.517	4.98	-12.39
0.70	4.53	1.872	5.08	-3.533	12.63	0.958	4.98	-12.41
0.60	4.53	1.872	5.10	-3.768	3.03	0.501	5.01	-12.48
0.50	4.53	1.884	5.13	-3.530	3.03	0.501	5.01	-12.07
0.40	4.56	1.729	5.19	-3.238	3.03	0.501	5.04	-11.52

TABLE A.10 CONTINUED

u_y/u_o	Deformations t_{max} sec.	u_{max} in.	t_{min} sec.	u_{min} in.	Absolute Displacements t_{max} sec.	x_{max} in.	t_{min} sec.	x_{min} in.
0.30	4.59	1.148	5.25	-3.464	3.03	0.501	5.08	-11.27
0.25	3.27	1.187	5.34	-3.973	3.03	0.501	5.10	-11.48
0.20	3.51	1.550	5.46	-4.231	3.03	0.501	5.13	-11.37
0.15	3.54	2.250	5.52	-4.289	3.03	0.502	5.13	-10.86
0.10	3.63	3.338	5.61	-4.302	3.03	0.505	5.16	-10.09
0.05	3.84	4.894	5.82	-3.560	3.06	0.377	5.28	-7.818
$f = 1/0.7 = 1.43$ cps								
1.00	5.97	1.444	4.92	-1.168	12.54	1.216	4.88	-11.14
0.80	5.97	1.447	4.92	-1.168	12.56	1.588	4.88	-11.14
0.70	5.97	1.121	4.96	-1.215	12.56	1.449	4.88	-11.14
0.60	4.53	0.770	5.01	-1.407	12.56	1.088	4.88	-11.17
0.50	4.53	0.771	5.06	-1.629	12.57	0.625	4.91	-11.13
0.40	4.53	0.804	5.09	-1.615	2.54	0.443	4.91	-10.85
0.30	4.53	0.777	5.15	-1.819	2.54	0.443	4.94	-10.70
0.25	3.27	0.848	6.17	-2.249	2.54	0.443	4.96	-10.62
0.20	3.51	1.333	5.30	-2.129	12.62	0.599	5.00	-10.17
0.15	3.56	2.037	5.51	-2.407	12.66	0.448	5.04	-9.674
0.10	3.63	2.931	5.60	-3.140	2.54	0.430	5.08	-9.411
0.05	3.84	4.338	5.81	-3.659	2.57	0.373	5.22	-8.177
$f = 1/0.5 = 2$ cps								
1.00	6.83	1.317	6.59	-1.317	12.76	1.060	4.73	-10.56
0.80	6.35	1.108	6.59	-1.231	12.75	0.850	4.73	-10.56
0.70	6.35	1.128	6.59	-0.946	12.74	0.991	4.73	-10.56
0.60	6.36	1.175	4.71	-0.749	12.71	1.149	4.73	-10.56
0.50	6.36	0.974	6.09	-0.821	12.68	1.088	4.74	-10.58
0.40	6.38	0.635	6.09	-0.941	12.65	0.905	4.76	-10.67
0.30	4.46	0.310	6.11	-1.046	12.63	0.776	4.79	-10.77
0.25	3.51	0.262	6.12	-1.093	12.63	0.727	4.82	-10.86
0.20	3.51	0.262	6.14	-1.508	2.87	0.452	4.83	-11.01
0.15	3.51	0.266	6.15	-1.888	2.87	0.452	4.88	-10.99
0.10	3.50	0.460	6.15	-2.150	2.87	0.452	4.94	-10.56
0.05	3.63	2.512	5.58	-2.454	12.65	0.617	5.03	-9.009
$f = 1/0.4 = 2.5$ cps								
1.00	7.54	0.983	7.74	-1.024	12.74	1.278	4.86	-10.28
0.80	6.74	0.936	6.54	-0.818	12.74	1.262	4.86	-10.28
0.70	6.74	0.735	6.54	-0.824	12.74	1.132	4.86	-10.28
0.60	6.33	0.604	6.56	-0.845	12.74	0.993	4.86	-10.28
0.50	6.35	0.526	6.56	-0.735	12.74	0.973	4.86	-10.32
0.40	6.33	0.446	4.07	-0.665	12.74	0.981	4.88	-10.36

TABLE A.10 CONTINUED

$\frac{u_y}{u_o}$	Deformations t_{max} sec.	u_{max} in.	t_{min} sec.	u_{min} in.	Absolute Displacements t_{max} sec.	x_{max} in.	t_{min} sec.	x_{min} in.
0.30	3.50	0.366	5.46	-0.773	12.74	0.857	4.77	-10.30
0.25	3.50	0.379	5.48	-0.855	12.74	0.811	4.76	-10.24
0.20	3.51	0.400	5.49	-0.927	12.74	0.705	4.77	-10.26
0.15	3.51	0.419	6.12	-1.091	12.74	0.474	4.83	-10.46
0.10	3.50	0.472	6.14	-1.848	2.73	0.408	4.89	-10.62
0.05	3.57	1.506	5.52	-2.482	2.78	0.429	4.98	-9.844
				f = 1/0.3 = 3.33 cps				
1.00	6.23	0.481	4.92	-0.416	12.62	1.024	4.90	-10.33
0.80	6.23	0.446	4.93	-0.417	12.62	1.088	4.90	-10.33
0.70	6.24	0.362	4.93	-0.431	12.63	1.054	4.90	-10.33
0.60	4.40	0.218	4.95	-0.501	12.63	0.947	4.90	-10.35
0.50	3.78	0.182	4.98	-0.710	12.63	0.695	4.91	-10.44
0.40	3.00	0.168	6.06	-1.071	2.70	0.460	4.92	-10.53
0.30	3.01	0.171	5.06	-1.144	2.70	0.460	4.91	-10.59
0.25	3.45	0.235	5.06	-1.194	2.70	0.460	4.90	-10.60
0.20	3.48	0.347	5.08	-1.177	12.65	0.459	4.89	-10.55
0.15	3.50	0.481	5.46	-1.193	12.63	0.472	4.86	-10.46
0.10	3.50	0.539	5.50	-1.721	2.74	0.383	4.90	-10.40
0.05	3.60	1.884	5.54	-2.220	12.61	0.610	4.98	-9.382
				f = 1/0.25 = 4 cps				
1.00	6.64	0.291	4.88	-0.278	12.63	0.992	4.87	-10.29
0.80	6.64	0.170	6.00	-0.347	12.62	0.926	4.87	-10.29
0.70	4.74	0.166	6.01	-0.372	12.62	0.892	4.87	-10.30
0.60	3.40	0.149	4.92	-0.356	12.62	0.898	4.88	-10.31
0.50	3.40	0.149	4.97	-0.497	12.62	0.830	4.89	-10.36
0.40	3.41	0.158	5.02	-0.717	12.61	0.630	4.89	-10.42
0.30	3.45	0.205	5.06	-1.013	12.61	0.438	4.90	-10.49
0.25	3.47	0.241	5.08	-1.241	2.87	0.374	4.90	-10.58
				f = 1/0.2 = 5 cps				
1.00	5.54	0.139	6.01	-0.151	12.65	0.982	4.83	-10.18
0.80	5.54	0.118	6.01	-0.144	12.65	0.959	4.83	-10.18
0.70	5.54	0.0850	6.01	-0.149	12.65	0.939	4.84	-10.19
0.60	3.34	0.0751	6.00	-0.175	12.65	0.899	4.85	-10.22
0.50	3.34	0.0751	6.02	-0.313	12.65	0.748	4.86	-10.26
0.40	3.34	0.0784	5.02	-0.562	12.65	0.555	4.87	-10.34
0.30	3.38	0.105	5.07	-1.021	2.74	0.355	4.89	-10.46
0.25	3.46	0.180	5.10	-1.341	2.74	0.355	4.90	-10.62
0.20	3.49	0.279	5.34	-1.512	2.74	0.356	4.89	-10.65

TABLE A.10 CONTINUED

$\frac{u_y}{u_o}$	Deformations				Absolute Displacements			
	t_{max} sec.	u_{max} in.	t_{min} sec.	u_{min} in.	t_{max} sec.	x_{max} in.	t_{min} sec.	x_{min} in.
				f = 1/0.15 = 6.67 cps				
1.00	5.85	0.0826	3.83	-0.0837	12.62	0.974	4.86	-10.08
0.80	3.75	0.0691	3.99	-0.0823	12.62	0.973	4.86	-10.08
0.70	3.75	0.0677	4.91	-0.0788	12.62	0.970	4.86	-10.08
0.60	3.76	0.0512	4.92	-0.0942	12.62	0.947	4.86	-10.09
0.50	3.43	0.0364	4.93	-0.0992	12.62	0.917	4.85	-10.11
0.40	3.43	0.0367	6.51	-0.1734	12.62	0.862	4.84	-10.13
0.30	3.45	0.0451	6.53	-0.6352	12.62	0.426	4.85	-10.27
0.25	3.46	0.0523	6.07	-0.9678	2.81	0.353	4.88	-10.37
				f = 1/0.125 = 8 cps				
1.00	5.49	0.0415	6.46	-0.0466	12.61	0.970	4.83	-10.06
0.719	3.73	0.0291	6.48	-0.0883	12.61	0.915	4.84	-10.06
0.63	3.73	0.0210	6.48	-0.1127	12.61	0.886	4.84	-10.07
0.54	3.38	0.0206	6.49	-0.1370	12.61	0.858	4.84	-10.08
0.45	3.38	0.0206	6.50	-0.1752	12.61	0.820	4.84	-10.08
				f = 1/0.1 = 10 cps				
1.00	5.85	0.0225	3.64	-0.0217	12.61	0.968	4.84	-10.06
0.90	5.85	0.0202	4.90	-0.0227	12.61	0.968	4.84	-10.06
0.80	3.10	0.0124	6.46	-0.0293	12.61	0.957	4.84	-10.06
0.75	3.10	0.0124	6.47	-0.0406	12.61	0.945	4.84	-10.06
0.70	3.10	0.0124	6.47	-0.0632	12.61	0.921	4.84	-10.07
0.65	3.10	0.0124	6.48	-0.1088	12.61	0.874	4.84	-10.07
0.60	3.10	0.0124	6.49	-0.1501	12.61	0.832	4.84	-10.08
0.50	3.43	0.0137	6.50	-0.2953	12.61	0.686	4.84	-10.09
0.40	3.45	0.0244	6.54	-0.6375	12.61	0.384	4.85	-10.15
0.30	3.48	0.1246	5.46	-1.186	2.82	0.360	4.88	-10.42
0.25	3.49	0.2195	5.47	-1.462	2.82	0.360	4.89	-10.52
				f = 1/0.05 = 20 cps				
1.00	5.84	0.00423	4.89	-0.00487	12.61	0.967	4.84	-10.05
0.90	4.44	0.00359	4.92	-0.00909	12.61	0.962	4.84	-10.05
0.70	4.44	0.00296	6.47	-0.1022	12.61	0.868	4.84	-10.05

TABLE A.11 VALUES OF MAXIMUM AND MINIMUM DEFORMATIONS AND DISPLACEMENTS WITH THE ASSOCIATED TIMES

Elasto-Plastic Systems, Damping Factor, β = 0.02, El Centro Earthquake

y_o = 8.28 in., $\dot{y}_o$ = 13.68 in./sec., $\ddot{y}_o$ = 0.32 g, Duration of Quake = 29.5 sec.

$\frac{u_y}{u_o}$	Deformations				Absolute Displacements			
	t_{max} sec.	u_{max} in.	t_{min} sec.	u_{min} in.	t_{max} sec.	x_{max} in.	t_{min} sec.	x_{min} in.
				f = 1/25 = 0.04 cps				
1.00	10.92	11.03	25.39	-8.797	11.33	7.019	24.85	-6.695
0.80	10.92	11.03	4.33	-7.670	11.52	7.070	25.39	-2.662
0.70	11.00	11.05	4.33	-7.670	11.82	7.141	25.80	-0.739
0.60	10.92	9.909	4.33	-7.671	11.40	5.886	0	0
0.50	10.92	8.680	4.33	-7.677	10.92	4.616	0	0
0.40	10.86	7.502	4.33	-7.686	10.48	3.463	0	0
0.30	22.00	7.028	4.33	-7.619	22.20	4.253	0	0
0.25	10.86	6.169	4.33	-7.686	21.60	2.951	0	0
0.20	10.86	5.589	4.33	-7.773	20.78	1.753	29.48	-0.038
0.15	10.86	5.287	4.33	-7.826	9.72	1.327	29.48	-0.591
0.10	10.86	4.866	4.33	-7.904	9.40	0.919	29.48	-0.382
0.05	10.86	4.585	4.33	-7.992	9.26	0.606	29.48	-0.659
				f = 1/15 = 0.067 cps				
1.00	10.80	12.87	15.58	-9.608	9.62	10.17	16.54	-9.224
0.80	10.80	12.94	4.33	-7.202	9.62	10.17	16.12	-4.540
0.70	10.80	13.10	4.33	-7.202	9.72	10.17	15.90	-1.925
0.60	10.86	13.47	4.33	-7.202	9.80	10.24	0	0
0.50	10.86	12.45	4.33	-7.205	22.00	9.414	0	0
0.40	10.86	10.32	4.33	-7.222	21.59	7.465	0	0
0.30	10.86	8.795	4.33	-7.040	9.40	5.551	0	0
0.25	10.86	8.161	4.33	-6.893	9.18	4.949	29.48	-1.251
0.20	10.86	6.832	4.33	-7.105	8.82	3.664	29.48	-3.254
0.15	10.86	6.936	4.33	-7.276	9.40	3.311	29.48	-2.489
0.10	10.86	5.928	4.33	-7.478	9.04	2.275	29.48	-1.213
0.05	10.86	5.106	4.33	-7.748	8.69	1.334	29.48	-1.658
				f = 1/10 = 0.10 cps				
1.00	28.23	17.09	13.29	-15.57	18.34	14.17	23.60	-15.34
0.90	28.23	16.81	13.29	-15.57	18.34	13.80	23.65	-15.37
0.80	28.23	13.35	13.78	-15.78	8.49	11.98	23.75	-15.76
0.70	8.80	10.52	13.78	-16.18	8.49	11.98	23.87	-16.36
0.60	8.80	10.52	24.80	-17.19	8.49	11.98	24.00	-16.68
0.50	8.80	10.59	24.85	-14.73	8.56	12.00	24.17	-13.69
0.40	8.82	10.18	24.89	-12.81	8.60	11.52	24.40	-11.26

TABLE A-11 CONTINUED

$\frac{u_y}{u_o}$	Deformations t_{max} sec.	u_{max} in.	t_{min} sec.	u_{min} in.	Absolute Displacements t_{max} sec.	x_{max} in.	t_{min} sec.	x_{min} in.
0.30	10.56	7.588	24.93	-12.51	8.49	8.474	24.40	-10.764
0.25	10.56	6.768	24.93	-12.01	8.33	7.581	24.48	-10.170
0.20	10.56	6.080	24.93	-11.51	8.17	6.782	24.62	-9.568
0.15	10.80	6.092	24.93	-9.548	8.17	5.672	29.48	-7.791
0.10	10.86	7.687	4.33	-6.505	8.69	5.194	29.48	-2.428
0.07	10.86	6.562	4.33	-6.884	8.56	3.767	29.48	-1.648
0.05	10.86	5.932	4.33	-7.176	8.44	2.871	29.48	-2.339
				f = 1/7 = 0.143 cps				
1.00	22.06	15.78	18.69	-15.82	15.40	15.78	18.85	-15.20
0.90	15.41	14.53	25.40	-15.38	15.40	15.78	18.95	-14.71
0.80	15.74	14.04	11.82	-13,03	15.41	15.13	11.80	-13.83
0.70	15.74	11.16	11.82	-13.04	7.68	13.68	11.80	-13.84
0.60	8.33	9.706	11.82	-12.68	7.68	13.68	11.82	-13.48
0.50	8.33	8.907	11.82	-10.41	7.68	12.75	11.82	-11.20
0.40	2.60	6.211	12.60	-10.95	7.70	9.544	12.08	-11.02
0.30	2.60	6.240	12.92	-10.41	7.60	8.042	12.20	-9.753
0.25	2.63	6.281	13.00	-10.12	7.40	7.474	12.33	-9.026
0.20	2.67	6.399	13.00	-9.279	7.33	7.423	29.48	-8.561
0.15	2.67	6.268	24.93	-7.833	7.70	8.212	29.48	-6.637
0.10	10.80	7.183	24.93	-6.342	8.17	7.321	24.24	-4.625
0.07	10.80	6.409	24.93	-6.411	8.17	5.533	24.07	-4.732
0.05	10.80	5.983	4.33	-6.519	8.17	4.322	24.00	-4.639
0.03	10.86	5.724	4.33	-7.042	8.17	3.106	24.00	-4.243
				f = 1/5 = 0.2 cps				
1.00	26.40	10.44	28.56	-12.02	6.38	15.52	28.68	-15.00
0.80	26.40	8.172	28.68	-12.89	6.38	12.71	28.68	-15.89
0.70	2.55	6.475	28.68	-12.59	6.38	10.20	28.68	-15.60
0.60	2.55	6.475	28.68	-12.31	6.48	7.541	28.68	-15.32
0.50	2.55	6.481	28.68	-11.27	6.53	5.488	28.68	-14.28
0.40	2.55	6.583	4.21	-9.219	6.36	5.014	22.80	-11.19
0.30	26.53	8.602	4.21	-6.462	26.21	6.997	11.16	-6.963
0.25	26.64	10.03	4.20	-4.970	7.23	8.598	23.19	-4.629
0.22	26.64	9.257	4.20	-4.050	7.33	9.893	23.39	-4.003
0.20	27.44	7.887	4.20	-3.700	7.40	10.44	23.52	-4.296
0.15	8.44	7.631	24.78	-7.063	7.70	10.48	23.75	-6.998
0.10	2.64	6.271	24.85	-9.770	7.68	8.287	23.75	-9.136
0.05	10.80	5.434	24.89	-8.839	7.79	5.297	23.65	-7.629

TABLE A.11 CONTINUED

$\frac{u_y}{u_o}$	Deformations t_{max} sec.	u_{max} in.	t_{min} sec.	u_{min} in.	Absolute Displacements t_{max} sec.	x_{max} in.	t_{min} sec.	x_{min} in.
				f = 1/4 = 0.25 cps				
1.00	5.40	13.91	3.94	-11.85	5.52	18.51	27.96	-13.58
0.80	5.40	12.73	3.96	-11.87	5.61	17.49	28.08	-11.18
0.70	5.34	9.845	3.96	-12.06	5.61	14.51	28.08	-12.11
0.60	5.34	6.302	4.01	-12.57	5.61	10.86	28.11	-13.71
0.50	2.33	6.050	4.01	-13.42	5.61	6.590	28.20	-15.99
0.40	2.33	6.067	4.08	-13.46	1.95	4.269	28.23	-15.87
0.30	2.55	6.581	4.08	-10.06	5.52	4.389	28.32	-9.833
0.25	2.55	7.122	4.08	-7.874	7.32	6.233	28.35	-6.216
0.20	2.55	7.744	4.08	-5.537	7.40	9.137	11.76	-3.931
0.15	26.53	10.82	4.08	-3.470	7.40	11.87	0	0
0.10	26.64	8.427	4.17	-3.706	7.40	10.48	22.88	-2.526
0.05	2.64	5.945	24.85	-9.220	7.46	7.211	23.64	-8.724
				f = 1/3 = 0.33 cps				
1.00	13.62	18.04	9.26	-17.72	7.70	21.97	12.12	-18.39
0.90	4.98	15.66	9.30	-17.82	4.86	21.49	12.12	-18.64
0.80	5.04	15.82	9.30	-15.40	4.86	21.53	12.12	-16.44
0.70	5.04	16.49	3.36	-11.56	4.92	21.78	12.08	-12.24
0.60	5.07	15.75	3.36	-11.58	7.70	21.30	3.24	-9.702
0.50	5.10	12.62	3.37	-11.76	7.70	18.07	3.24	-9.733
0.40	5.16	9.435	3.42	-12.21	7.68	14.31	3.32	-9.953
0.30	5.22	6.375	3.84	-11.82	7.56	11.14	27.30	-8.813
0.25	1.90	6.223	3.90	-10.77	7.46	10.35	27.36	-9.486
0.20	1.90	6.336	4.01	-9.915	7.38	9.434	27.42	-10.75
0.15	2.33	6.751	4.02	-8.609	7.26	9.141	27.58	-10.65
0.10	2.55	8.222	12.48	-5.208	7.14	11.05	27.84	-8.104
0.05	2.58	7.223	12.92	-5.119	7.09	9.503	22.94	-5.772
				f = 1/2.5 = 0.4 cps				
1.00	7.01	15.89	5.80	-15.88	7.01	23.29	10.92	-15.76
0.80	4.57	12.09	5.82	-16.13	4.50	18.96	10.98	-16.96
0.70	4.57	12.12	5.82	-14.36	4.50	18.96	11.00	-15.79
0.60	4.62	11.82	5.88	-11.55	4.50	18.51	11.05	-13.72
0.50	4.62	8.891	5.88	-11.33	4.56	15.31	11.10	-14.42
0.40	4.68	5.936	11.22	-11.26	4.62	11.99	11.22	-15.19
0.30	1.86	5.823	11.70	-10.47	4.76	10.83	11.40	-13.55
0.25	5.10	5.986	3.36	-7.918	4.86	11.03	27.30	-9.981
0.20	5.16	7.383	3.37	-6.623	7.09	13.12	27.42	-7.298
0.15	10.20	6.764	3.90	-6.292	7.02	13.03	27.54	-7.774
0.10	2.33	7.247	27.36	-6.262	6.96	9.443	27.66	-9.332
0.05	2.58	7.313	12.60	-6.848	6.91	9.279	22.80	-7.554

TABLE A-11 CONTINUED

$\frac{u_y}{u_o}$	Deformations t_{max} sec.	u_{max} in.	t_{min} sec.	u_{min} in.	Absolute Displacements t_{max} sec.	x_{max} in.	t_{min} sec.	x_{min} in.
				f = 1/2 = 0.50 cps				
1.00	12.33	7.486	11.34	-6.710	6.47	13.75	11.33	10.53
0.90	12.33	7.531	11.34	-6.710	6.47	13.75	11.33	-10.53
0.80	12.36	6.049	11.40	-6.779	6.47	13.75	11.33	-10.57
0.70	6.53	5.952	11.40	-5.819	6.48	14.00	11.34	-9.547
0.60	6.60	6.190	2.94	-5.127	6.56	14.12	2.70	-8.272
0.50	6.60	4.554	5.70	-5.142	6.60	12.41	11.34	-7.743
0.40	1.80	4.636	11.64	-7.045	4.26	10.62	11.34	-10.45
0.30	1.80	5.014	11.70	-6.299	4.50	10.50	11.33	-9.489
0.25	1.80	5.357	11.70	-5.610	4.68	10.83	11.28	-8.705
0.20	5.16	7.622	3.37	-2.674	6.78	12.49	22.20	-3.481
0.15	1.90	6.753	4.01	-4.359	6.72	11.05	27.36	-6.425
0.10	2.55	7.362	11.82	-6.575	6.72	9.196	27.77	-8.896
0.05	2.58	6.591	12.96	-8.253	6.84	8.411	22.80	-9.770
				f = 1/1.50 = 0.67 cps				
1.00	6.30	5.166	8.61	-4.998	6.27	13.38	11.55	-7.136
0.80	12.27	5.390	5.61	-3.489	6.30	13.50	2.33	-5.277
0.70	9.48	5.531	5.61	-3.489	6.33	13.69	2.33	-5.277
0.60	6.36	4.556	5.61	-3.735	6.36	12.73	2.34	-5.333
0.50	27.58	2.359	5.67	-4.741	6.39	10.44	28.29	-6.076
0.40	1.74	1.868	5.76	-6.030	6.42	7.670	28.26	-8.671
0.30	1.74	2.223	28.32	-5.451	4.50	8.044	28.29	-8.466
0.25	1.77	2.893	28.32	-5.316	4.50	8.273	28.29	-8.327
0.20	1.80	3.827	27.03	-5.436	4.53	8.544	28.29	-8.407
0.15	5.15	5.759	3.36	-1.815	4.83	10.55	29.48	-4.597
0.10	2.55	5.813	4.01	-4.238	6.54	8.767	28.14	-6.469
0.05	2.58	6.654	12.90	-6.649	6.66	8.927	28.14	-8.088
				f = 1/1.25 = 0.80 cps				
1.00	6.15	5.179	6.78	-4.621	6.15	13.44	27.99	-5.527
0.80	6.18	5.309	5.52	-3.678	6.18	13.58	27.99	-4.424
0.70	15.84	5.516	5.55	-3.679	6.20	13.67	29.31	-4.030
0.60	15.81	5.546	5.55	-3.273	6.21	13.11	2.22	-3.921
0.50	15.81	5.096	5.58	-3.442	6.24	12.02	2.22	-3.924
0.40	15.81	4.877	3.16	-3.458	6.27	11.08	2.94	-4.196
0.30	15.84	4.263	3.16	-3.832	6.32	9.637	11.16	-5.343
0.25	15.84	3.308	5.73	-4.122	6.36	8.353	11.16	-6.566
0.20	15.84	1.806	5.79	-4.801	4.56	6.797	11.22	-7.995
0.15	1.77	2.183	8.67	-5.997	4.59	6.599	11.19	-9.291
0.10	1.83	3.590	3.37	-4.154	4.86	8.165	11.16	-7.220
0.05	2.55	6.479	11.79	-4.302	6.51	8.948	29.48	-7.291

TABLE A.11 CONTINUED

$\frac{u_y}{u_o}$	Deformations				Absolute Displacements			
	t_{max} sec.	u_{max} in.	t_{min} sec.	u_{min} in.	t_{max} sec.	x_{max} in.	t_{min} sec.	x_{min} in.
				f = 1 cps				
1.00	4.50	5.967	4.95	-5.968	4.47	13.168	10.89	-6.797
0.80	4.52	6.114	3.99	-4.640	4.47	13.21	2.94	-5.364
0.70	4.53	5.934	3.99	-3.994	4.47	12.98	2.94	-5.364
0.60	4.53	5.795	3.03	-3.735	4.47	12.78	2.94	-5.364
0.50	4.56	4.638	3.06	-3.835	4.50	11.57	2.94	-5.366
0.40	12.14	3.991	3.06	-4.095	6.15	10.94	2.94	-5.504
0.30	8.13	2.584	3.08	-4.328	6.12	10.78	2.91	-5.749
0.25	8.17	2.913	3.08	-4.451	6.15	11.01	2.91	-5.897
0.20	8.19	3.007	3.12	-4.498	6.20	11.03	2.88	-5.942
0.15	27.33	2.173	3.16	-4.499	6.24	9.488	2.87	-5.730
0.10	1.80	1.505	5.79	-5.089	4.57	6.104	11.04	-8.668
0.05	1.83	2.746	11.76	-5.937	4.86	6.244	20.19	-9.091
				f = 1/0.7 = 1.43 cps				
1.00	5.87	3.320	5.48	-3.144	5.87	11.35	2.72	-6.754
0.80	2.34	2.703	5.48	-3.167	5.88	10.48	2.72	-6.670
0.70	2.34	2.727	5.49	-2.670	5.89	10.41	2.72	-6.036
0.60	2.34	2.181	5.49	-2.768	5.91	9.755	2.72	-5.944
0.50	2.34	1.403	5.49	-3.138	5.93	8.869	11.12	-6.122
0.40	1.74	1.013	5.51	-3.761	5.96	7.778	11.12	-6.585
0.30	1.74	1.013	5.58	-4.219	6.00	7.025	2.75	-6.886
0.25	1.74	1.024	5.61	-4.584	4.25	6.541	11.10	-7.273
0.20	1.76	1.116	5.64	-4.971	4.31	6.929	11.09	-7.754
0.15	1.76	1.316	5.69	-4.052	4.38	7.199	11.04	-6.765
0.10	1.77	1.766	3.18	-3.492	4.47	7.267	11.01	-6.301
0.05	1.82	2.620	11.76	-4.913	4.67	5.904	11.01	-7.750
				f = 1/0.5 = 2 cps				
1.00	3.17	1.853	2.43	-2.242	6.05	9.675	2.45	-5.297
0.80	2.21	1.833	5.30	-2.540	6.03	8.686	2.46	-5.309
0.70	2.21	1.865	5.31	-2.142	6.03	8.673	27.72	-5.080
0.60	2.22	1.942	26.70	-1.412	6.03	9.028	27.72	-4.542
0.50	5.04	1.811	1.91	-1.332	6.03	8.949	27.74	-4.519
0.40	2.27	1.450	1.94	-1.471	4.16	8.528	27.75	-4.526
0.30	5.07	1.359	2.01	-1.580	4.44	8.549	27.75	-4.154
0.25	5.07	1.163	2.04	-1.712	4.46	8.249	11.00	-4.497
0.20	1.68	1.233	2.07	-1.596	6.36	7.948	11.03	-4.923
0.15	1.74	1.696	5.58	-1.372	5.99	8.208	11.05	-4.677
0.10	1.77	1.882	5.66	-2.339	4.33	7.982	10.95	-5.702
0.05	1.80	2.570	3.90	-3.517	4.53	6.521	10.94	-6.243

TABLE A.11 CONTINUED

$\frac{u_y}{u_o}$	Deformations				Absolute Displacements			
	t_{max} sec.	u_{max} in.	t_{min} sec.	u_{min} in.	t_{max} sec.	x_{max} in.	t_{min} sec.	x_{min} in.
				f = 1/0.4 = 2.5 cps				
1.00	4.95	1.323	5.17	-1.470	6.18	9.153	2.76	-5.028
0.80	2.57	1.104	5.18	-1.607	6.20	8.559	2.76	-5.025
0.70	2.57	0.907	5.18	-1.619	6.21	8.313	2.76	-5.049
0.60	2.19	0.708	5.18	-1.641	6.21	8.069	10.92	-5.220
0.50	2.19	0.708	26.09	-1.831	6.23	7.853	10.92	-5.378
0.40	2.21	0.769	5.21	-1.482	6.24	7.855	10.94	-4.996
0.30	2.24	1.219	5.27	-0.881	6.27	8.298	10.95	-4.397
0.25	2.28	1.282	1.98	-0.647	6.30	8.549	10.97	-4.154
0.20	1.65	0.962	26.66	-1.223	4.37	7.911	10.98	-4.842
0.15	1.71	1.282	26.72	-1.962	4.38	7.266	11.00	-5.581
0.10	1.76	1.572	5.64	-2.443	6.05	6.906	10.98	-5.822
0.05	1.80	2.385	3.89	-3.184	4.49	6.481	10.97	-6.250
				f = 1/0.3 = 3.33 cps				
1.00	2.50	0.520	2.64	-0.752	6.19	8.465	2.64	-4.687
0.80	2.50	0.520	2.65	-0.769	6.20	8.327	2.65	-4.711
0.70	2.50	0.515	2.65	-0.818	6.20	8.225	2.66	-4.764
0.60	2.50	0.579	10.14	-0.686	6.21	8.296	2.66	-4.550
0.50	2.52	0.700	9.53	-0.419	4.37	8.499	2.67	-4.245
0.40	9.38	0.692	1.33	-0.268	6.22	8.638	2.68	-4.126
0.30	25.78	0.936	1.34	-0.273	6.21	8.868	2.69	-4.040
0.25	2.25	1.147	1.34	-0.274	6.23	8.948	2.70	-3.803
0.22	2.27	1.061	2.00	-0.357	6.22	8.735	10.90	-3.822
0.20	2.28	0.751	2.03	-0.659	6.16	8.314	10.91	-4.245
0.15	1.70	0.919	3.57	-1.433	6.22	7.134	10.94	-5.274
0.10	1.76	1.453	5.63	-2.443	6.28	6.534	10.96	-5.970
0.05	1.80	2.373	5.78	-3.048	4.46	6.347	11.00	-6.388
				f = 1/0.25 = 4 cps				
1.00	2.96	0.667	2.60	-0.746	6.20	8.426	2.60	-4.650
0.80	2.46	0.582	2.61	-0.761	6.19	8.241	2.61	-4.672
0.70	2.47	0.588	2.61	-0.656	6.18	8.259	2.61	-4.567
0.60	2.47	0.507	2.62	-0.615	6.18	8.211	2.62	-4.531
0.50	2.49	0.457	2.62	-0.539	6.18	8.199	2.63	-4.462
0.40	2.50	0.308	4.77	-0.567	4.36	8.111	2.64	-4.487
0.30	22.47	0.404	2.35	-0.568	6.21	8.211	2.66	-4.505
0.25	24.55	0.564	3.54	-0.537	6.22	8.242	2.66	-4.437
0.20	24.57	0.549	1.96	-0.422	6.22	8.291	2.67	-4.300
0.15	26.19	0.837	2.00	-0.361	6.14	8.655	2.69	-3.924
0.10	1.70	0.716	3.56	-1.400	6.22	7.338	10.91	-5.017
0.05	1.78	1.586	5.70	-2.788	6.14	6.578	10.98	-5.870

TABLE A-11 CONTINUED

$\frac{u_y}{u_o}$	Deformations t_{max} sec.	u_{max} in.	t_{min} sec.	u_{min} in.	Absolute Displacements t_{max} sec.	x_{max} in.	t_{min} sec.	x_{min} in.
				f = 1/0.20 = 5 cps				
1.00	3.30	0.415	2.78	-0.423	6.24	8.482	2.59	-4.243
0.80	2.69	0.279	2.79	-0.435	6.24	8.412	2.59	-4.244
0.70	2.48	0.227	5.13	-0.448	6.24	8.311	10.90	-4.325
0.60	2.48	0.227	3.01	-0.472	6.24	8.235	10.90	-4.381
0.50	2.48	0.228	3.01	-0.479	6.24	8.215	10.90	-4.380
0.40	2.48	0.246	26.06	-0.423	6.25	8.166	10.90	-4.406
0.30	2.49	0.350	5.14	-0.396	4.31	8.133	10.90	-4.232
0.25	2.51	0.476	5.14	-0.271	4.30	8.283	10.92	-4.080
0.20	2.22	0.278	5.15	-0.464	4.32	8.089	10.94	-4.214
0.15	2.26	0.303	2.02	-0.701	6.23	8.099	2.68	-4.296
0.10	1.70	0.541	2.92	-1.490	6.22	7.469	10.89	-4.814
				f = 1/0.15 = 6.67 cps				
1.00	4.95	0.262	5.03	-0.213	4.33	8.356	10.89	-4.157
0.80	4.96	0.286	3.65	-0.212	6.13	8.406	10.89	-4.081
0.70	4.96	0.275	3.65	-0.214	6.13	8.407	10.89	-4.065
0.60	4.97	0.280	3.65	-0.202	6.13	8.425	10.89	-4.035
0.50	4.98	0.314	3.66	-0.181	6.24	8.480	2.70	-4.012
0.40	5.81	0.388	3.67	-0.136	6.14	8.588	2.71	-3.987
0.30	5.00	0.309	3.68	-0.241	6.15	8.489	10.90	-4.003
0.25	5.01	0.315	3.68	-0.248	6.16	8.485	10.90	-3.979
0.20	2.50	0.374	3.69	-0.133	6.17	8.523	10.90	-3.902
0.10	26.37	0.236	3.73	-0.911	6.20	8.002	2.70	-4.287
				f = 1/0.125 = 8 cps				
1.000	4.95	0.114	5.38	-0.106	6.20	8.302	10.86	-4.097
0.716	4.95	0.121	2.73	-0.089	6.20	8.330	10.86	-4.059
0.626	4.96	0.141	2.73	-0.079	6.21	8.357	10.86	-4.029
0.537	4.97	0.172	2.35	-0.074	6.21	8.396	10.86	-3.986
0.447	13.83	0.207	2.36	-0.066	6.22	8.425	2.70	-3.958
0.358	2.49	0.169	2.78	-0.152	6.22	8.297	10.86	-4.043
0.268	5.02	0.416	3.19	-0.092	6.23	8.546	2.65	-3.912
0.224	26.30	0.380	1.99	-0.233	6.15	8.516	2.66	-4.043
0.179	26.34	0.634	2.03	-0.535	6.16	8.447	2.68	-4.119
0.134	1.70	0.315	3.73	-1.350	6.20	7.610	10.90	-4.616
				f = 1/0.10 = 10 cps				
1.00	4.79	0.0575	2.69	-0.0495	6.20	8.292	10.90	-4.085
0.80	4.79	0.0561	2.69	-0.0498	6.20	8.301	10.90	-4.075
0.70	9.58	0.0578	2.70	-0.0504	6.20	8.304	10.90	-4.069
0.60	26.30	0.0899	2.70	-0.0421	6.20	8.328	10.90	-4.036
0.50	26.30	0.1417	4.74	-0.0425	6.20	8.370	10.90	-3.987
0.40	26.30	0.2096	4.74	-0.0714	6.20	8.414	2.70	-3.926

TABLE A.11 CONTINUED

$\frac{u_y}{u_o}$	Deformations				Absolute Displacements			
	t_{max} sec.	u_{max} in.	t_{min} sec.	u_{min} in.	t_{max} sec.	x_{max} in.	t_{min} sec.	x_{min} in.
0.35	26.30	0.1797	2.81	-0.153	6.20	8.364	2.70	-3.961
0.30	5.02	0.1183	3.02	-0.2709	6.19	8.206	10.89	-4.089
0.25	26.29	0.3359	2.01	-0.3496	6.19	8.344	2.66	-4.043
				f = 1/0.05 = 20 cps				
1.00	9.55	0.01009	2.56	-0.00836	6.19	8.278	10.89	-4.069
0.90	9.56	0.00935	2.56	-0.00923	6.19	8.278	10.88	-4.069
0.80	9.56	0.0115	2.34	-0.0073	6.19	8.280	10.88	-4.066
0.50	14.27	0.1366	1.94	-0.0328	6.19	8.396	10.88	-3.939

TABLE A.12a

MAXIMUM DEFORMATIONS OF SINGLE DEGREE-OF-FREEDOM BILINEAR SYSTEMS

2 Percent Critical Damping; Eureka Earthquake

k_2/k_1	$u_y/u_o = 0.75$ t_{max}	u_{max}	$u_y/u_o = 0.50$ t_{max}	u_{max}	$u_y/u_o = 0.25$ t_{max}	u_{max}	$u_y/u_o = 0.10$ t_{max}	u_{max}
				f = 0.10 cps				
1.0	18.06	- 9.62	18.06	- 9.62	18.06	- 9.62	18.06	- 9.62
0.9	18.16	- 9.31	18.26	- 9.23	18.34	- 9.55	18.34	- 9.98
0.8	18.20	- 8.99	13.58	8.82	13.66	9.38	13.66	10.00
0.7	7.44	- 8.87	13.60	8.53	13.78	8.99	13.92	9.79
0.6	7.44	- 8.83	13.66	8.05	13.98	8.18	14.38	9.03
0.5	7.44	- 8.79	4.60	7.89	4.70	8.24	4.80	8.54
0.4	7.44	- 8.74	4.70	7.93	4.80	8.38	4.80	8.76
0.3	7.44	- 8.70	4.70	7.98	4.80	8.53	4.80	8.97
0.2	7.44	- 8.65	4.70	8.03	4.80	8.68	4.80	9.19
0.1	7.44	- 8.59	4.70	8.08	4.80	8.83	4.80	9.42
0	7.44	- 8.53	4.70	8.13	4.80	8.98	4.80	9.64
				f = 1/3 cps				
1.0	8.28	-10.75	8.28	-10.75	8.28	-10.75	8.28	-10.75
0.9	6.78	10.58	6.84	10.34	6.84	10.42	14.88	-10.92
0.8	6.78	10.61	6.84	10.12	6.92	10.16	7.02	10.99
0.7	6.78	10.64	6.88	9.82	7.06	9.77	7.20	11.06
0.6	6.78	10.67	6.90	9.47	7.14	9.21	7.30	10.70
0.5	6.84	10.72	6.92	9.05	5.56	- 8.59	5.64	-10.05
0.4	6.84	10.76	6.96	8.54	5.58	- 8.52	5.76	-10.10
0.3	6.84	10.81	5.46	- 8.36	5.60	- 8.35	5.82	- 9.74
0.2	6.84	10.86	5.46	- 8.46	5.64	- 8.08	6.12	- 8.63
0.1	6.84	10.91	5.46	- 8.57	5.70	- 7.71	6.18	- 6.88
0	6.84	10.96	5.52	- 8.68	5.76	- 7.21	3.90	5.79
				f = 1 cps				
1.0	5.07	-3.306	5.07	-3.306	5.07	-3.306	5.07	-3.306
0.9	5.07	-3.321	5.07	-3.337	5.08	-3.435	5.08	-3.709
0.8	5.07	-3.523	5.07	-3.362	5.10	-3.491	5.13	-3.981
0.7	5.07	-3.351	5.08	-3.382	5.13	-3.481	5.16	-4.069
0.6	5.07	-3.366	5.08	-3.405	5.16	-3.400	5.94	4.011
0.5	5.07	-3.381	5.08	-3.423	5.19	-3.278	6.00	3.737
0.4	5.07	-3.397	5.10	-3.447	5.20	-3.130	4.18	-2.876
0.3	5.07	-3.413	5.10	-3.469	5.22	-3.029	4.26	-3.060
0.2	5.07	-3.428	5.10	-3.482	5.25	-3.039	4.32	-2.930
0.1	5.08	-3.446	5.13	-3.507	5.25	-3.287	5.31	-4.123
0	5.08	-3.465	5.13	-3.530	5.34	-3.973	5.61	-4.302
				f = 5 cps				
1.0	6.01	-0.151	6.01	-0.151	6.01	-0.151	6.01	-0.151
0.9	6.01	-0.145	4.83	-0.132	4.84	-0.139	5.56	0.150
0.8	6.01	-0.140	4.84	-0.129	4.86	-0.143	4.89	-0.165
0.7	6.01	-0.137	4.84	-0.129	4.87	-0.149	5.99	-0.199
0.6	4.82	-0.135	4.85	-0.129	4.89	-0.160	4.90	-0.245
0.5	4.83	-0.136	4.85	-0.131	4.90	-0.179	6.18	0.329
0.4	4.83	-0.136	4.86	-0.136	4.93	-0.212	4.97	-0.359
0.3	4.83	-0.137	4.86	-0.146	4.94	-0.249	6.27	0.480
0.2	4.83	-0.138	4.86	-0.163	4.97	-0.300	6.74	0.486
0.1	6.01	-0.140	4.87	-0.194	5.00	-0.430	5.02	-0.544
0.075	-	-	4.89	-0.206	5.01	-0.502	5.03	-0.633
0.05	-	-	6.00	-0.222	5.02	-0.620	5.06	-1.017
0.025	-	-	6.01	-0.262	5.05	-0.841	5.12	-1.249
0.01	-	-	-	-	5.07	-1.099	5.17	-1.396
0	6.01	-0.147	6.02	-0.313	5.10	-1.341	5.49	-1.682

TABLE A.12b

MAXIMUM DEFORMATIONS OF SINGLE-DEGREE-OF-FREEDOM BILINEAR SYSTEMS

2 Percent Critical Damping; El Centro Earthquake

k_2/k_1	$u_y/u_o = 0.75$		$u_y/u_o = 0.50$		$u_y/u_o = 0.25$		$u_y/u_o = 0.10$	
	t_{max}	u_{max}	t_{max}	u_{max}	t_{max}	u_{max}	t_{max}	u_{max}
				f = 0.10 cps				
1.0	28.23	17.09	28.23	17.09	28.23	17.09	28.23	17.09
0.9	28.23	16.45	13.78	-15.40	13.78	-14.79	24.80	-15.33
0.8	28.23	16.03	13.78	-15.25	14.26	-14.06	24.93	-14.59
0.7	28.23	15.78	13.78	-15.06	14.26	-13.49	14.26	-13.51
0.6	13.78	-15.69	13.78	-14.84	14.26	-12.84	14.26	-12.04
0.5	13.78	-15.74	14.26	-14.79	14.26	-12.45	10.56	10.19
0.4	13.78	-15.78	14.26	-14.55	14.26	-11.29	10.80	10.32
0.3	13.78	-15.83	14.26	-14.35	24.93	-11.06	10.80	10.27
0.2	13.78	-15.87	24.80	-15.00	24.89	-12.10	10.80	9.82
0.1	13.78	-15.92	24.85	-14.90	24.93	-11.94	10.86	8.98
0	13.78	-15.96	24.85	-14.73	24.93	-12.01	10.86	7.69
				f = 1/3 cps				
1.0	13.62	18.04	13.62	18.04	13.62	18.04	13.62	18.04
0.9	9.30	-16.69	5.04	15.58	5.04	15.65	5.06	16.05
0.8	6.72	-15.93	5.04	15.50	5.10	15.45	5.15	16.00
0.7	4.98	15.77	5.06	15.35	5.15	15.05	5.22	15.54
0.6	4.98	15.80	5.06	15.16	5.16	14.47	5.28	14.43
0.5	5.04	15.85	5.06	14.91	5.22	13.73	5.34	12.55
0.4	5.04	15.90	5.10	14.61	5.22	12.70	3.96	-11.12
0.3	5.04	15.96	5.10	14.22	5.22	11.42	4.01	-11.11
0.2	5.04	16.01	5.10	13.75	3.78	-10.66	4.02	-10.16
0.1	5.04	16.06	5.10	13.22	3.84	-10.74	4.08	- 8.29
0	5.04	16.12	5.10	12.62	3.90	-10.77	2.55	8.22
				f = 1 cps				
1.0	4.95	-5.97	4.95	-5.97	4.95	-5.97	4.95	-5.97
0.9	4.50	5.95	4.52	5.59	4.53	5.30	4.56	5.32
0.8	4.50	5.94	4.53	5.22	4.57	4.57	4.59	4.37
0.7	4.52	5.92	4.53	4.92	4.59	3.92	4.62	3.45
0.6	4.52	5.92	4.53	4.69	4.59	3.50	6.11	3.07
0.5	4.52	5.91	4.53	4.59	4.59	3.30	6.18	3.09
0.4	4.52	5.91	4.53	4.60	3.06	-3.39	6.24	2.99
0.3	4.52	5.90	4.53	4.66	3.06	-3.70	3.08	-3.15
0.2	4.52	5.90	4.53	4.72	3.06	-4.17	3.09	-3.77
0.1	4.52	5.89	4.53	4.73	3.08	-4.30	3.15	-4.10
0	4.52	5.89	4.56	4.64	3.09	-4.45	5.79	-5.09
				f = 5 cps				
1.0	2.78	-0.423	2.78	-0.423	2.78	-0.423	2.78	-0.423
0.9	2.78	-0.423	2.79	-0.399	2.79	-0.395	2.80	-0.415
0.8	2.78	-0.425	2.79	-0.378	2.80	-0.362	2.81	-0.376
0.7	2.78	-0.427	2.79	-0.360	2.60	-0.355	2.60	-0.393
0.6	2.78	-0.428	2.59	-0.357	2.61	-0.359	2.61	-0.512
0.5	2.78	-0.429	2.59	-0.358	2.61	-0.368	2.63	-0.648
0.4	2.78	-0.430	2.79	-0.363	2.62	-0.373	2.66	-0.673
0.3	2.79	-0.433	2.79	-0.394	2.63	-0.362	2.54	0.641
0.2	2.79	-0.435	2.80	-0.440	2.82	-0.328	2.42	-0.867
0.1	2.79	-0.438	2.80	-0.455	2.50	0.354	2.26	1.089
0	2.79	-0.441	3.01	-0.479	2.51	0.476	2.42	-1.490

APPENDIX B

EXPRESSIONS FOR RESPONSE OF A SINGLE-DEGREE-OF-FREEDOM SYSTEM

The solution of Eq. 2.4 presented in the body of this report may be expressed by means of Duhamel's integral as

$$u(t) = e^{-\beta pt}\left[u(0)\cos p_d t + \frac{1}{\sqrt{1-\beta^2}}\left(\frac{\dot u(0)}{p} + \beta\, u(0)\right)\sin p_d t\right]$$
$$- \frac{1}{p\sqrt{1-\beta^2}}\int_0^t \ddot y(\tau)\, e^{-\beta p(t-\tau)} \sin\left[p_d(t-\tau)\right] d\tau \qquad (B.1)$$

where t is time, τ is a variable of integration, and p_d, the circular natural frequency of the damped system, is related to the corresponding undamped frequency by the equation

$$p_d = p\sqrt{1-\beta^2} \qquad (B.2)$$

The acceleration function $\ddot y(\tau)$ is considered to be sectionally continuous in the interval between 0 and t.

The expression for the relative velocity, $\dot u$, is obtained by differentiating Eq. B.1 with respect to time

$$\dot u(t) = e^{-\beta pt}\left[\dot u(0)\cos p_d t - \frac{1}{\sqrt{1-\beta^2}}\left(pu(0) + \beta\, \dot u(0)\right)\sin p_d t\right]$$
$$- \int_0^t \ddot y(\tau)\, e^{-\beta p(t-\tau)}\left[\cos[p_d(t-\tau)] - \frac{\beta}{\sqrt{1-\beta^2}}\sin[p_d(t-\tau)]\right] d\tau \qquad (B.3)$$

The absolute acceleration of the mass, $\ddot x$, is obtained by a further differentiation or, more conveniently, directly from Eq. (2.2) by substituting for u and $\dot u$ the expressions given by Eqs. B.1 and B.3. The resulting expression is

$$\ddot{x}(t) = e^{-\beta pt}\left[\ddot{x}(0)\cos p_d t - \frac{1}{\sqrt{1-\beta^2}}\left((1-2\beta^2)\, p\dot{u}(0) - \beta\, p^2 u(0)\right)\sin p_d t\right]$$

$$+ p\int_0^t \ddot{y}(\tau)\, e^{-\beta p(t-\tau)}\left[\frac{1-2\beta^2}{\sqrt{1-\beta^2}}\sin[p_d(t-\tau)] + 2\beta\cos[p_d(t-\tau)]\right]d\tau \quad (B.4)$$

The remaining response quantities, $\ddot{u}$, $\dot{x}$ and x, can now be determined from the equations

$$\ddot{u} = \ddot{x} - \ddot{y},\ \dot{u} = \dot{x} - \dot{y},\ \text{and } u = x - y,$$

respectively. Note that before $\dot{x}$ and x can be evaluated, the quantities $\dot{y}$ and y must be determined by integrating the prescribed acceleration function.

If the ground motion is specified as a velocity-time function, the relative displacement, u, the absolute and relative velocities, $\dot{x}$ and $\dot{u}$, and the absolute acceleration, $\ddot{x}$, can be expressed directly in terms of the prescribed function as follows. The expression for u is obtained by integrating the last term in Eq. B.1 by parts and combining terms,

$$u(t) = e^{-\beta pt}\left[u(0)\cos p_d t + \frac{1}{\sqrt{1-\beta^2}}\left(\frac{\dot{x}(0)}{p} + \beta\, u(0)\right)\sin p_d t\right]$$

$$- \int_0^t \dot{y}(\tau)\, e^{-\beta p(t-\tau)}\left[\cos[p_d(t-\tau)] - \frac{\beta}{\sqrt{1-\beta^2}}\sin[p_d(t-\tau)]\right]d\tau \quad (B.5)$$

Differentiation of this equation with respect to t gives,

$$\dot{x}(t) = e^{-\beta pt}\left[\dot{x}(0)\cos p_d t - \frac{1}{\sqrt{1-\beta^2}}\left(pu(0) + \beta\,\dot{x}(0)\right)\sin p_d t\right]$$

$$+ p\int_0^t \dot{y}(\tau)\, e^{-\beta p(t-\tau)}\left[\frac{1-2\beta^2}{\sqrt{1-\beta^2}}\sin[p_d(t-\tau)] + 2\beta\cos[p_d(t-\tau)]\right]d\tau \quad (B.6)$$

With $\dot{x}$ known, the relative velocity can be determined from $\dot{x} = \dot{u} + \dot{y}$. Finally, $\ddot{x}$ can be determined from Eq. 2.2.

If the ground motion is specified as a displacement function, the absolute displacement of the mass can be expressed in terms of the prescribed function by integrating Eq. B.5 by parts. The result is

$$x(t) = e^{-\beta pt}\left[x(0)\cos p_d t + \frac{1}{\sqrt{1-\beta^2}}\left(\frac{\dot{x}(0)}{p} + \beta[x(0) - 2y(0)]\right)\sin p_d t\right]$$

$$+ p\int_0^t y(\tau)\, e^{-\beta p(t-\tau)}\left[\frac{1-2\beta^2}{\sqrt{1-\beta^2}}\sin[p_d(t-\tau)] + 2\beta\cos[p_d(t-\tau)]\right]d\tau \qquad \text{(B.7)}$$

Differentiation of this equation gives the absolute velocity,

$$\dot{x}(t) = 2\beta p\, y(t)$$

$$+ e^{-\beta pt}\left[[\dot{x}(0) - 2\beta p y(0)]\cos p_d t - \frac{1}{\sqrt{1-\beta^2}}\left(p[x(0) - 2\beta^2 y(0)] + \beta\,\dot{x}(0)\right)\sin p_d t\right]$$

$$+ p^2\int_0^t y(\tau)\, e^{-\beta p(t-\tau)}\left[(1-4\beta^2)\cos[p_d(t-\tau)] - \frac{\beta(3-4\beta^2)}{\sqrt{1-\beta^2}}\sin[p_d(t-\tau)]\right]d\tau \qquad \text{(B.8)}$$

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